Learning About Voter Rationality*

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Abstract

We model the accountability relationship between voters and politicians to clarify what can and can’t be learned about voter rationality from existing evidence from the behavior literature. We make two key points. First, we show that evidence on the electoral consequences of natural disasters and economic shocks—typically interpreted as evidence for voter irrationality—is consistent with a canonical model with rational voters. Second, we show that the evidence on the electoral consequences of disaster response—typically interpreted as evidence for voter rationality—is consistent with the same model with irrational voters. Hence, neither body of evidence can adjudicate between rational and irrational voting behavior. We also derive new hypotheses that can better guide future empirical work attempting to assess voter rationality.

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The literature on voter behavior has long been interested in evaluating voters’ competence to fulfill their electoral function. The early literature on voter competence was concerned with whether or not voters were sufficiently informed to make good decisions (Campbell et al., 1960; Fair, 1978; Kinder and Sears, 1985; Popkin, 1991; Sniderman, Brody and Tetlock, 1993; Lupia, 1994; Delli Carpini and Keeter, 1996). A more recent literature is concerned with another aspect of the competence of voter decision making—specifically, voter rationality.

To examine whether voters behave rationally, that literature has turned its attention to how election outcomes change in response to various shocks to voter welfare. One strand examines the electoral consequences of natural disasters and government responses to them (Abney and Hill, 1966; Achen and Bartels, 2004; Healy and Malhotra, 2010; Healy, Malhotra and Mo, 2010; Bechtel and Hainmueller, 2011; Gasper and Reeves, 2011; Cole, Healy and Werker, 2012; Huber, Hill and Lenz, 2012; Chen, 2013). Another strand examines the electoral consequences of economic shocks (Ebeid and Rodden, 2006; Wolfers, 2009; Leigh, 2009; Kayser and Peress, 2012).

The literature finds heterogeneous effects and offers conflicting interpretations. Some studies find that incumbent electoral fortunes suffer following negative events outside the control of policy-makers (Achen and Bartels, 2004; Wolfers, 2009; Leigh, 2009; Healy, Malhotra and Mo, 2010). These results are typically interpreted as evidence of voter irrationality. The basic argument is that, if voters are rational, then incumbents’ electoral fortunes should be unaffected by shocks to outcomes outside of the incumbents’ control. Others studies find various results interpreted as evidence of voter rationality. One set of studies finds that voters do not seem to respond to irrelevant shocks (Abney and Hill, 1966; Ebeid and Rodden, 2006; Kayser and Peress, 2012). Other studies find that the negative electoral consequences of shocks to voter welfare are mitigated once they control for the quality of the government’s response to the crisis (Healy and Malhotra, 2010; Bechtel and Hainmueller, 2011; Gasper and Reeves, 2011; Cole, Healy and Werker, 2012).

The purpose of this paper is to use a formal model of the relationship between voters and politicians to clarify what we can and can’t learn about voter rationality from these various types of evidence. To this end, we make two key points. First, we show that the putative evidence for voter irrationality is consistent with a canonical model with rational voters. Second, we show that the putative evidence for voter rationality is consistent with the same model with irrational voters. Hence, neither body of evidence can adjudicate between rational and irrational voting behavior. We also offer new theoretical hypotheses that can better guide future empirical work attempting to assess voter rationality.
1 Our Argument

It is quite straightforward to see why the evidence on the positive electoral returns to successful disaster response—typically interpreted as evidence of voter rationality—is also consistent with irrational voting behavior. So let’s start with that point. The folk notion of rational voting seems to simply be that voters should be positively responsive to good governance outcomes. But, as Fearon (1999) pointed out, rational voters actually behave in a way that is quite specific—they make optimal, forward looking decisions given the available information. Most positively responsive voting behavior is not optimal in this way. Hence, it is straightforward to write down a model with highly irrational voters (e.g., voters who reward football victories), but who also respond positively to effective disaster response. Indeed, a voter who simply votes for the incumbent whenever life seems to be going well, for whatever reason, is consistent with the empirical evidence.

More subtle is our claim that evidence that incumbent electoral fortunes suffer following a natural disaster or economic downturn—typically interpreted as evidence of voter irrationality—is actually consistent with a model with rational voters. Clearly, rational voters do not punish politicians for events outside their control. So how can incumbent electoral fortunes systematically suffer following natural disasters or economics shocks if voters are rational? To show how this works, we start by examining two types of shocks to voter welfare.

The first type of shock is what we refer to as a non-interactive shock. A non-interactive shock is an event totally outside the control of the incumbent that affects voter welfare, but does not interact in any way with the quality of governance. An example might be a football loss by the home team (Healy, Malhotra and Mo, 2010).

The second type of shock is what we refer to as an interactive shock. An interactive shock is also an event totally outside the control of the incumbent. However, it interacts with the quality of governance to affect voter welfare. An example might be a hurricane—the event is an act of god, but the damage depends on the quality of infrastructure investment, emergency preparedness, and so on.

The standard intuition holds that, if voters are rational, then incumbent electoral fortunes should not respond to exogenous, observable shocks. We show that this intuition is correct only for the case of non-interactive shocks.\(^1\) Indeed, the model positively predicts

\(^1\)Cole, Healy and Werker (2012) have already pointed out that, with rational voters, incumbent electoral fortunes should respond to shocks if those shocks can’t be observed by voters, since voters don’t know that the shock to their welfare wasn’t due to the quality of governance. We agree, but believe that observability is the more natural assumption, especially in the case of natural disasters.
that, on average, incumbent electoral fortunes will suffer following large interactive shocks. Hence, a negative empirical relationship between natural disasters (or economic shocks) and incumbent electoral fortunes does not entail the conclusion that voters are irrational.

Importantly, the decline in expected incumbent electoral fortunes following interactive shocks does not derive from voters irrationally punishing politicians for events outside of their control. Rather, the mechanism is informational. Interactive shocks make voter welfare more informative about the quality of incumbent politicians. For instance, suppose incumbents are responsible for infrastructure. During normal times there may be little information about how good a job the incumbent did. But during a hurricane or tornado, the voters learn a lot about whether the incumbent did a good job overseeing infrastructure maintenance. Similarly, economic shocks may reveal extra information about the quality of regulatory infrastructure, fiscal management, and so on.

Why does an increase in unbiased information hurt expected incumbent electoral fortunes? On average, in our model, incumbents are advantaged in elections. This advantage arises endogenously through electoral selection (Samuelson, 1984; Zaller, 1998; Ashworth and Bueno de Mesquita, 2008). As such, the less new information the voters get, the less likely they are to learn enough to make them want to replace their incumbents. When there is lots of information available (e.g., following a disaster), it becomes more possible for a piece of information to be sufficiently informative to overcome the voters’ initial (rational) tendency to support the incumbent. Thus, even information that is on average accurate is bad for incumbents. (While the intuitions we provide here are about probability of winning, we show that all our results also hold when we use expected vote share as the measure of incumbent electoral fortunes.)

The mechanism underlying our argument is substantively motivated and is consistent with empirical scholarship showing that voters are more responsive to outcomes in high information environments than in low information environments (Berry and Howell, 2007; Ferraz and Finan, 2008; Snyder and Strömberg, 2010). The model also generates new empirical predictions which could be tested against data. First, the model predicts that disasters (or other interactive shocks) increase the expected electoral fortunes of incumbents whom, ex ante, the voters believe are low quality and decrease the expected electoral fortunes of incumbents whom, ex ante, the voters believe are high quality. Second, the model predicts that public opinion about incumbent competence should be more variable following disasters (or other interactive shocks)—though we also show that moving from this result on public opinion to a result on the variability of election outcomes is not straightforward. These predictions suggest new ways in which an empirical researcher could adjudicate between
the irrational voter hypothesis and our rational, informational account.

Before turning to the analysis, it is important to note that we are not arguing for or
against voter rationality. Our purpose is, rather, to use canonical models to show that the
existing empirical literature has been over-interpreted on both sides—it simply does not
answer the question one way or the other.

2 The Model

We study a canonical, career concerns model of electoral accountability (Lohmann, 1998;
Persson and Tabellini, 2000; Ashworth, 2005; Ashworth and Bueno de Mesquita, 2006;

An electoral district is made up of a continuum of voters and four politicians: \(d_1, d_2,\)
\(r_1,\) and \(r_2.\) The basic timeline of the game is as follows.

1. There is an initial election between \(d_1\) and \(r_1.\)
2. Nature determines a shock to the state of the world. Then the winner governs in the
   first governance period, choosing effort \(a_1 \in \mathbb{R}^+.\)
3. There is a second election between the winner of the initial election and the challenger
   of the other “party” (i.e., \(d_1\) against \(r_2\) or \(r_1\) against \(d_2\)—sadly, there is never an \(r_2d_2\)
election).
4. Nature determines a shock to the state of the world. Then the winner governs in the
   second governance period, choosing effort \(a_2 \in \mathbb{R}^+.\)
5. The game ends.

Each politician, \(j,\) has a quality \(\theta_j\) that is drawn from a normal distribution with mean
zero and variance \(\sigma^2_{\theta}\). Throughout we will use the following notation:

\[m_{j,h} = \mathbb{E}[\theta_j|h],\]

where \(h\) refers to an information set of the voters. In particular, we write \(h = 1\) for the
information set immediately prior to the first election and \(h = 2\) for the information set
immediately prior to the second election.

In the initial election, voters receive a public signal about candidate \(j:\)

\[s_{j,1} = \theta_j + \eta_{j,1},\]
where the $\eta$'s are independently drawn from a normal distribution with mean zero and variance $\sigma^2_\eta$. (To reduce notational clutter, we do not model a similar informative signal at the second election. No results would be changed by doing so.)

In each governance period, the governance outcome is determined by four factors: the quality of the incumbent, the incumbent’s effort, the shock to the state of the world, and random luck. The voter observes the shock to the state of the world, but not the random luck or the incumbent’s quality or effort.

The random luck is given by $\epsilon_t$, which is distributed normally with mean zero and variance $\sigma^2_\epsilon$. The state of the world is $\omega_t$, which is a random variable drawn from the set $\Omega$ according to a measure $\mu$.

Let $\theta_{I,t}$ be the competence of the incumbent in office in governance period $t$. Then, the governance outcome is:

$$\pi_t \equiv \alpha(\omega_t) + \beta(\omega_t)(\theta_{I,t} + a_t) + \epsilon_t.$$  

The functions $\alpha : \Omega \to \mathbb{R}$ and $\beta : \Omega \to [\beta_1, \beta_2] \subset \mathbb{R}_+$ allow us to use the state, $\omega$, to capture various types of shocks. We refer to a change to $\omega$ that changes the value of the function $\alpha$, but not the value of the function $\beta$, as a pure non-interactive shock. Similarly, a change to $\omega$ that changes the value of the function $\beta$, but not the value of the function $\alpha$, constitutes a pure interactive shock. Of course, a change to $\omega$ could also change both of these values.

Each voter is indexed by $b$, a bias in favor of party $d$. These biases are distributed normally with mean zero and variance 1. If the politician in office following election $t$ is from party $d$, then voter $b$’s payoff in that period is:

$$\pi_t + b.$$  

If the politician in office following election $t$ is from party $r$, then voter $b$’s payoff in that period is:

$$\pi_t.$$  

A voter’s total payoff is simply the sum of his per-period payoffs.

A politician receives a benefit $B > 0$ for each period she holds office. In addition, a politician in office in period $t$ who chooses effort $a_t$ bears an (additively separable) cost $c(a_t) = \frac{1}{2}a_t^2$. Politicians not in office in a period get payoff 0 for that period. A politician’s total payoff is simply the sum of her per-period payoffs.

So that we can prove results about a cross-section of districts, we assume there are a
continuum of districts independently playing the game described above. Since behavior in all districts is the same, we will focus on a particular district for the analysis and return to the continuum interpretation in the discussion of empirical results.

2.1 Equilibrium

The intuition for equilibrium play is as follows.

Since there is a continuum of voters, no voter is ever pivotal. As is standard in models of elections with two candidates, we assume voters vote sincerely. The winner of each election will be whichever candidate gets the support of the median voter—i.e., the voter with bias $b = 0$.

At the first election, the median voter elects whichever candidate generates a better signal, since this candidate provides better expected payoffs in the first policy-making stage and increases the expected quality of the politician in office in the second policy-making stage.

At the first governance period, the incumbent chooses an action to balance the benefits associated with an increased probability of reelection and the costs of effort. How much increased effort increases the probability of reelection depends on both the state ($\omega$) and what the voter learned about the incumbent in the first election. Hence, effort varies across incumbents (with different first-election signals) and across states.

In the second election, the median voter reelects the incumbent if and only if he expects the incumbent to be of higher ability than the challenger. Since the voters’ beliefs about the ability of the incumbent are increasing in the incumbent’s performance in the first governance period, this implies that the incumbent is reelected if and only if performance in the first governance period is good enough. Exactly how good performance has to be depends on the first election signal (since this also affects the voters’ beliefs about the incumbent’s quality), the state of the world (since this affects how informative performance is about ability), and the voters’ conjecture about how hard the incumbent worked (since this affects the extent to which voters credit good performance to ability vs. effort).

Finally, in the second governance period, there are no electoral incentives, so the politician in office exerts no effort. As such, the outcome is determined by the state, the shock, and the ability of the politician in office.

The following result summarizes some critical features of the equilibrium. A complete equilibrium characterization (including a proof of the proposition) is in Appendix A.2.

**Proposition 1** The following facts hold in equilibrium:
1. Candidate $i$ defeats candidate $j$ in the first election if $m_{i,1} > m_{j,1}$.

2. Let $m_{I,1}$ be the expected ability of the winner of the first election (the incumbent), conditional on the first election signal. For any state $\omega$, the effort chosen in the first governance period, $a^*_1$, is single-peaked in $m_{I,1}$ and maximized at $m_{I,1} = 0$.

3. Let $m_{I,2}$ be the expected ability of the winner of the first election, conditional on the first election signal and the outcome of the first governance period. The incumbent is reelected if and only if $m_{I,2} \geq 0$.

4. In the second governance period, the politician in office chooses effort 0.

The fact that $a^*_1$ is single-peaked in $m_{I,1}$ (point 2 of Proposition 1) will be particularly important in the sequel. So let’s see why it is true.

The incumbent is reelected if her expected ability is sufficiently high (i.e., if $m_{I,2} \geq 0$). The better an incumbent’s performance in the first governance period, the higher quality voters believe she is. Thus, the incumbent exerts effort in order to improve performance, which she hopes will convince voters she is of high enough ability to merit reelection.

Given this, what kinds of incumbents have strong incentives to exert effort? Only those who believe that they are close to the cutoff between reelection and replacement. An incumbent who enters the first governance period with very high expected ability (high $m_{I,1}$) exerts little effort because she is virtually certain to gain reelection anyway. And an incumbent who enters the first governance period with very low expected ability (low $m_{I,1}$) exerts little effort because she is virtually certain to lose anyway. Incumbents who enter the first governance period with expected ability close to the reelection threshold (close to zero) have the strongest incentives to exert effort. As such, effort is single-peaked and maximized when the incumbent thinks she is most likely to face a tight reelection contest ($m_{I,1} = 0$).

### 2.2 Incumbent Electoral Fortunes

There are two ways to think about the idea of electoral fortunes: expected vote share in the second election and expected likelihood of winning reelection. We will show that interactive shocks are bad for incumbent electoral fortunes in both senses. To get started, we need to simply calculate these two quantities for a given incumbent.

At the beginning of the first governance period the voters’ beliefs are that the incumbent has quality that is normally distributed with mean $m_{I,1}$ and variance $\sigma^2_1$. (See Appendix
A.1 for a calculation of these quantities.) Suppose the state of the world is $\omega$ in the first governance period. Then the voters know that performance in the first governance period is

$$\pi_1 = \alpha(\omega) + \beta(\omega)(\theta_{I,1} + a^*_1) + \epsilon_1.$$

In equilibrium the voters correctly conjecture that effort is $a^*_1$. Hence, the voters can filter out the state and their conjecture about the incumbent’s effort to extract an unbiased signal of the incumbent’s ability from performance in the first governance period. To see this, notice that

$$\frac{\pi_1 - \alpha(\omega) - \beta(\omega)a^*_1}{\beta(\omega)} = \theta_{I,1} + \frac{\epsilon_1}{\beta(\omega)}$$

is a normally distributed random variable with mean $\theta_{I,1}$ and variance $\sigma^2_1 + \frac{\sigma^2}{\beta(\omega)}$. As such, standard results on Bayesian updating imply that the voters’ posterior beliefs about the incumbent’s quality are normally distributed with a mean given by a weighted average of the (adjusted) signal and the voters’ prior (from after the first election):

$$m_{I,2} = \lambda(\omega) \left( \frac{\pi_1 - \alpha(\omega) - \beta(\omega)a^*_1}{\beta(\omega)} \right) + (1 - \lambda(\omega))m_{I,1},$$

(1)

where

$$\lambda(\omega) = \frac{\sigma^2_1}{\sigma^2_1 + \frac{\sigma^2}{\beta(\omega)}}.$$

Since $\pi_1$ is a normally distributed random variable, the affine updating rule described in Equation 1 makes $m_{I,2}$ itself a normally distributed random variable. The law of iterated expectations says that its mean is just $m_{I,1}$—regardless of the kind of shock, the expected value of voters’ posterior expectation of the incumbent’s ability equals their prior expectation of the incumbent’s ability. The variance of $m_{I,2}$, denoted $v(\omega)$, follows from the usual formula:

$$v(\omega) = \lambda(\omega)^2 \cdot \text{var} \left( \frac{\pi_1 - \alpha(\omega) - \beta(\omega)a^*_1}{\beta(\omega)} \right) = \frac{\sigma^4_1}{\sigma^2_1 + \frac{\sigma^2}{\beta(\omega)^2}}.$$

(2)

There are three important facts to notice in this updating.

First, before using the first period outcome $\pi_1$ to update their beliefs about the incumbent’s ability, the voters subtract $\alpha(\omega)$ from it. This is because $\alpha(\omega)$ represents the direct welfare consequences of state $\omega$ that are entirely unrelated to the incumbent’s quality. Thus, the voters, here, do not irrationally blame the incumbent for bad outcomes for which the incumbent is not responsible. Indeed, the voters fully, rationally ignore those negative
consequences.

Second, and in much the same way, the voters subtract $\beta(\omega)a^*_1$ from $\pi_1$ before updating their beliefs. That is, rational voters force themselves to ignore how hard the incumbent worked in response to the shock when making a vote choice. They do this not because the incumbent is not responsible for her own effort choices, but because that effort choice is not directly informative about future performance. And future performance is what a rational voter cares about. As Ashworth and Bueno de Mesquita (forthcoming) point out, the voters might actually be better off if they could commit to irrationally punishing incumbents for bad behavior. But, as Fearon (1999) shows, they can’t.

Third, the variance of $m_{I,2}$ differs across shocks. These differences reflect differences in information across states. In particular, when $\beta(\omega)$ is larger, $v(\omega)$ is larger. This is because large interactive shocks make performance more responsive to, and thus more informative about, incumbent ability. And that means voters’ beliefs are likely to move further from their priors.

What we actually want to know is how the probability of the incumbent winning reelection differs across states of the world. We calculate this based on the distribution of the random variable $m_{I,2}$ conditional on the state $\omega$ and the belief at the beginning of the first governance period, $m_{I,1}$.

As stated in point 3 of Proposition 1, the median voter (who is unbiased) will support the incumbent if $m_{I,2} \geq 0$. Given this, the probability the incumbent is reelected, given $m_{I,1}$, is just the probability that the random variable $m_{I,2}$ ends up with a realization greater than zero. Figure 1 shows this probability for two different values of $m_{I,1}$, one greater than zero (so that the incumbent is ahead going into the first governance period) and the other less than zero (so that the incumbent is behind going into the first governance period). Since the conditional distribution of $m_{I,2}$ is normal with mean $m_{I,1}$ and variance $v(\omega)$, the reelection probability is:

$$\text{Pr}(m_{I,2} \geq 0|m_{I,1},\omega) = 1 - \Phi \left( \frac{-m_{I,1}}{\sqrt{v(\omega)}} \right) = \Phi \left( \frac{m_{I,1}}{\sqrt{v(\omega)}} \right).$$

The intuition for how information affects this probability of victory is easy to see in an extreme case. Suppose the first governance period revealed no information. Then, in every district where the incumbent was ahead, she would win reelection and in every district where the incumbent was behind she would lose reelection. Now suppose the first governance period becomes informative. In some districts where the incumbent was ahead, the new information would harm her and she would lose the election. And in some districts,
where the incumbent was behind, the new information would help her and she would win
the election. One can see the point more generally by noting that the final term in Equation
3 is increasing in $v(\omega)$ if $m_{I,1} < 0$ and decreasing in $v(\omega)$ if $m_{I,1} > 0$.

A similar logic holds for vote share. To see how to derive an incumbent’s expected vote
share, first consider the case when $d_1$ is the incumbent. Voter $b$ votes for the incumbent if

$$m_{d,2} + b \geq 0.$$  

Since $b$ is distributed normally with mean 0 and variance 1, the share of voters who vote
for the incumbent $d_1$ is:

$$1 - \Phi(-m_{d,2}).$$

If $r_1$ is the incumbent, a voter $b$ votes for her if:

$$m_{r,2} \geq b.$$  

Given this, the share of voters who vote for the incumbent $r_1$ is:

$$\Phi(m_{r,2}) = 1 - \Phi(-m_{r,2}).$$

Hence, in either case, the vote share of the incumbent, given the voters’ posterior belief
about the incumbent $m_{I,2}$, is

$$1 - \Phi(-m_{I,2}).$$

We can find the expected vote share of the incumbent by integrating the vote share given
a posterior expected ability \(m_{I,2}\) against the prior distribution of these beliefs. Recall, in
a state \(\omega\), the prior distribution of voter posterior mean beliefs has mean \(m_{I,1}\) and variance
\(v(\omega)\). Hence, the expected vote share of an incumbent who entered the first governance
period with expected ability \(m_{I,1}\), given a state \(\omega\), is:

\[
E[\text{Incumbent Vote Share}| m_{I,1}, \omega] = \int [1 - \Phi(-m_{I,2})] \frac{1}{\sqrt{v(\omega)}} \phi \left( \frac{m_{I,2} - m_{I,1}}{\sqrt{v(\omega)}} \right) dm_{I,2}. \tag{4}
\]

As with the probability of reelection, this expected vote share is decreasing in \(v(\omega)\) (so that
responsiveness is bad for expected vote share) when incumbents are ahead (i.e., \(m_{I,1} > 0\))
and is increasing in \(v(\omega)\) when incumbents are behind (i.e., \(m_{I,1} < 0\)).

2.3 Cross-Sectional Distribution of Incumbent Electoral Fortunes

We calculated the probability of reelection and the expected vote share, given a state \(\omega\), of
incumbents who enter the first governance stage with expected ability \(m_{I,1}\). This allowed
us to see how interactive shocks affect the electoral fortunes of a particular incumbent.
Whether such shocks are good or bad for an incumbent depends on whether she enters the
first governance period ahead (\(m_{I,1} > 0\)) or behind (\(m_{I,1} < 0\)). But empirical results are
about the central tendency across all incumbents. Thus, to relate our theoretical findings
to empirical scholarship, we want to think about what happens to the average incumbent,
integrating across the cross-section of districts. To do so, we need to think about the first
election, which determines the cross-sectional distribution of \(m_{I,1}\).

In the first election, the median voter will support whichever candidate generates the
better campaign signal. This means that the expected quality of the winning candidate in
district \(d\) is the maximum of two normally distributed random variables. As such, its
distribution is the distribution of the first order statistic of two normally distributed random
variables. This distribution is obviously “better” than the prior distribution of these random
variables.

Given this, think about the cross-sectional distribution (across districts) of voter mean
beliefs about candidate quality prior to the first governance period. In all districts, voters
believe that the future challenger has expected quality zero. In each district, the voters’
beliefs about the incumbent’s expected quality (\(m_{I,1}\)) depend on the first electoral signals.
In most districts, the voters will think that the incumbent is of higher expected quality
than the challenger. This is because, in most districts, the incumbent will have had a first
period electoral signal greater than zero (since it is the better of two mean-zero draws).
Figure 2: The cross-sectional distribution of expected ability of the winners of the first election is the distribution of the maximum of two draws from the prior distribution of candidate quality.

Appealing to the law of large numbers, Figure 2 shows the cross-sectional distributions of expected incumbent abilities. Incumbents with expected quality greater than zero are “ahead” of future challengers and incumbents with expected quality less than zero are “behind”. The key is that, in most districts, the incumbent is ahead.

We formalize this intuition with Lemma 2 in appendix A.3.

Why is this important? As we’ve seen, interactive shocks are bad for the electoral fortunes of incumbents who enter the first governance period ahead and are good for the electoral fortunes of incumbents who enter the first governance period behind. Since most incumbents go into the first governance period ahead of the future challenger, anything that reveals extra information about incumbents is bad for the average electoral fortunes of incumbents, integrating across the cross-section of districts.

This intuition drives several key results below.

3 Shocks and Incumbent Electoral Fortunes

As we indicated at the outset, we will consider two special cases of shocks to highlight the intuition of what we can learn about voter rationality from different kinds of empirically relevant shocks to voter welfare. In the first instance, we consider non-interactive shocks—i.e. when the value of $\alpha$ changes with $\omega$, but the value of $\beta$ does not. In the second instance, we consider interactive shocks—i.e., when the value of $\beta$ changes with $\omega$, whether or not the value of $\alpha$ does. After doing so we will apply these results to the literature on natural
disasters, incumbent electoral fortunes, and voter rationality.

3.1 Non-interactive Shocks

The case of non-interactive shocks is particularly simple. Since $\beta(\omega)$ is constant in $\omega$, the right-hand sides of Equations 3 and 4 are constant. This immediately implies:

**Proposition 2** Suppose shocks to $\omega$ affect the value of $\alpha$ but not the value of $\beta$. Then both the probability an incumbent is reelected and expected incumbent vote share are independent of the realization of $\omega$. This holds both unconditionally and conditional on beliefs about the incumbent’s ability prior to the first governance period ($m_{1,1}$).

**Proof.** Follows immediately from Equations 3 and 4. ■

The intuition is straightforward. The voter can observe the state, and knows that the mean of the distribution of outcomes is shifted by $\alpha(\omega)$. It is thus a simple matter to adjust her observation of performance to get an unbiased estimate of the incumbent’s true ability $\theta$. (Formally, this adjustment involves subtracting $\alpha(\omega)$ from $\pi$.) Moreover, the precision of the estimate is unaffected by the shock. Thus there is nothing left to affect the updating.

It this were all there was to the story of shocks to voter welfare, the inference of voter irrationality drawn from the empirical literature would be clearly correct. Rational voters do not respond to observable, non-interactive shocks. However, the assumption that welfare shocks do not interact with the quality of governance is strong. And, as we will see, this assumption is critical for the validity of the inference.

3.2 Interactive Shocks

The case of interactive shocks is more subtle. Consider two states, $\omega'$ and $\omega''$ such that $\beta(\omega') > \beta(\omega'')$. This says that voter welfare is more responsive to politician quality in state $\omega'$ than in state $\omega''$. We refer to $\omega'$ as the “more responsive” state.

Recall from Equation 3, the probability that the incumbent is reelected in state $\omega$ is:

$$1 - \Phi \left( \frac{-m_{1,1}}{\sqrt{v(\omega)}} \right),$$

where $v(\omega)$ reflects the variance of the prior distribution of the voter’s beliefs about the incumbent’s ability after the first governance period.

It is straightforward from Equation 2 that $\beta(\omega') > \beta(\omega'')$ implies that $v(\omega') > v(\omega'')$. As we’ve discussed, this reflects the fact that there is more information available about
Figure 3: The left-hand panel shows that more information hurts incumbents who are ahead. The right-hand panel shows that more information helps incumbents who are behind.

incumbent quality when voter welfare is more responsive to that quality.

So is reelection more likely under the more or the less responsive state? The answer depends on the voters’ belief entering the first governance stage, \( m_{I,1} \). Consider Figure 3. In the left-hand panel, the blue (smaller variance) distribution represents the distribution of posterior beliefs if voter welfare is less responsive to incumbent quality. The red (higher variance) distribution represents the distribution of posterior beliefs if voter welfare is more responsive to incumbent quality. In this panel, the incumbent is ahead prior to the first governance period (i.e., \( m_{I,1} > 0 \)). In the less responsive state, the probability of incumbent reelection is the sum of the areas of regions 1 and 2. In the more responsive state, the probability of incumbent reelection is the sum of the areas of regions 2 and 3. The fact that region 1 has larger area than region 3 shows that, when the incumbent is ahead going into the first governance period, the probability of reelection is higher when the voter’s welfare is less responsive to incumbent quality.

Now consider the right-hand panel of Figure 3. Again the blue (lower variance) distribution represents the less responsive state and the red (higher variance) distribution represents the more responsive state. The only difference is that now the incumbent is behind prior to the first governance period (i.e., \( m_{I,1} < 0 \)). In the less responsive state, the probability of incumbent reelection is the sum of the areas of regions \( i \) and \( ii \). In the more responsive state, the probability of incumbent reelection is the sum of the areas of regions \( ii \) and \( iii \). The fact that region \( i \) has smaller area than region \( iii \) shows that, when the incumbent is behind going into the first governance period, the probability of reelection is lower in the less responsive state than in the more responsive state.
We’ve seen that some incumbents’ expected electoral fortunes are better in the more responsive state and others’ expected electoral fortunes are better in the less responsive state. But we want to know, on average, how responsiveness affects incumbent electoral fortunes.

To do so, think about the cross-section of districts, each of which behaves as described above. As we show in Lemma 2 in the appendix, the cross-sectional distribution of $m_{I,1}$ is better than (in the sense of the monotone likelihood ratio order) the prior distribution of candidate quality. One implication of this is that, on average, incumbents are ahead going into the first governance period. The reason, of course, is electoral selection. Incumbents have already won an election. Thus, the majority of incumbents have shown themselves to be higher expected quality than the average candidate.

So we’ve seen two facts. First, the more responsive state helps the expected electoral fortunes of incumbents who are behind and hurts the expected electoral fortunes of incumbents who are ahead. Second, most incumbents are ahead. As formalized in the following result, combining these two facts leads to the conclusion that on average, the expected electoral fortunes of incumbents suffer the more responsive voter welfare is to incumbent quality.

**Proposition 3** Suppose $\omega', \omega'' \in \Omega$ are states such that $\beta(\omega') > \beta(\omega'')$. Regardless of the values of $\alpha(\omega')$ and $\alpha(\omega'')$, the cross-sectional average probability of incumbent reelection is greater in state $\omega''$ than in state $\omega'$.

Proposition 3 shows our key result for one interpretation of incumbent electoral fortunes, namely the likelihood of winning reelection. For the same reasons, the result holds for the other interpretation, expected vote share. Expected vote share is decreasing in $v(\omega)$ (so that responsiveness is bad for expected vote share) when incumbents are ahead and decreasing in $v(\omega)$ when incumbents are behind. Since most incumbents are ahead, on average, responsiveness reduces expected incumbent vote share, as formalized in the next result.

**Proposition 4** Suppose $\omega', \omega'' \in \Omega$ are states such that $\beta(\omega') > \beta(\omega'')$. Regardless of the values of $\alpha(\omega')$ and $\alpha(\omega'')$, the cross-sectional average incumbent vote share is greater in state $\omega''$ than in state $\omega'$.  

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4 Reinterpreting the Evidence

In this section we use the model to assess the extent to which existing evidence on voter rationality or irrationality entails the conclusions drawn by the behavioral literature. We start by considering the literature that purports to show evidence for voter irrationality by linking natural disasters to incumbent electoral fortunes. We show that our model, with fully rational voters, is consistent with the findings in that literature. Hence, we claim, this literature’s findings do not entail its conclusions.

We then consider the follow-up literature that purports to show evidence for voter rationality by linking incumbent responses to natural disasters to electoral fortunes. We first show that the findings in this literature actually appear inconsistent with our model. Moreover, we argue, they are consistent with a version of our model in which voters behave irrationally. Hence, those findings do not constitute prima facie evidence for voter rationality. That said, we don’t think these findings should be viewed as compelling evidence one way or the other because, as Cole, Healy and Werker (2012) point out, regressions exploiting variation in the quality of incumbent responses to disaster are not well identified.

4.1 Natural Disasters and Incumbent Electoral Fortunes

Of course, there is a long tradition of attempts to assess voter rationality in the political behavior literature. Much of that literature simply regresses incumbent electoral fortunes on various measures of voter welfare (e.g., economic variables as in Kramer, 1971; Fair, 1978; Lewis-Beck, 1990; Erikson, 1989, 1990; Ebeid and Rodden, 2006; Duch and Stevenson, 2008). The idea is that, if voters are rational, incumbent electoral fortunes should respond positively to voter welfare insofar as voter welfare is affected by the incumbent.

The natural disasters literature is an important advance over this existing literature. The performance of government vis-a-vis voter welfare is endogenous to incumbents’ expectations of electoral outcomes. For instance, an incumbent expecting a particularly tight reelection contest might have particularly strong incentives to work on behalf of voters. (Indeed, this is precisely why $a_1^*$ is maximized at $m_{I,1} = 0$ in our model.) Hence, from the perspective of making causal claims about the relationship between performance and electoral outcomes, regressions of the sort described above are confounded. Natural disasters, on the other hand, are exogenous shocks. It is worth noting (and will be important later) that, while natural disasters themselves are exogenous, their consequences may not be, to the extent that those consequences derive from an interaction with prior government decisions—e.g., how well infrastructure is constructed and maintained. (Similar findings
exist for exogenous economic shocks, for instance Wolfers (2009). We believe our argument similarly applies to those findings.

A key strand of this literature argues that, because natural disasters are outside the control of incumbents, if incumbent electoral fortunes systematically suffer following natural disasters, this is evidence of voter irrationality. Achen and Bartels (2004, pp.7–8) write, in their seminal paper, “To the extent that voters engage in sophisticated attributions of responsibility they should be entirely unresponsive to natural disasters, at least on average; to the extent that they engage in blind retrospection, they should exhibit ‘systematic attribution errors’.”

Our model suggests this interpretation is not warranted. In our view, many natural disasters are best interpreted as shocks with both a non-interactive and an interactive component. To be sure, a tornado or drought has a direct negative effect on voter welfare. And as Proposition 2 shows, this effect ought to have no impact on incumbent electoral fortunes if voters are rational. But the impact of tornados and droughts on voter welfare also interacts with the quality of governance. Voters will suffer less following such a disaster if infrastructure is well maintained, relief plans are in place, and citizens were competently educated by public agencies. Hence, during times of disaster, voters learn more about the quality of their government than they otherwise would. This means voters will be more responsive to government performance—good or bad—during times of disaster. In those places where the government performs well (relative to expectations), this increased information will particularly help the incumbent. In places where the government performs poorly (relative to expectations), this increased information will particularly harm the incumbent. But, as Propositions 3 and 4 show, on average incumbent electoral fortunes will be harmed, not because incumbents perform poorly on average, but because increased information is simply bad on average for incumbents who start ahead of their challengers.

It is worth pausing to comment on another important paper in this literature. Bechtel and Hainmueller (2011) use a difference-in-differences design to study the effect of flooding on incumbent vote share in a German election.2 For our purposes, two key facts come out of their study. First, the incumbent party started the election behind the challengers in the polls. Second, the incumbent party performed better electorally, on average, in districts that experienced flooding than in districts that did not. This finding seems in contradiction

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2One potential issue, which we will abstract away from here, is the following. Bechtel and Hainmueller (2011) measure whether a district experienced a flood using a variety of measures. Some of these seem to capture the idea of a true exogenous shock. But others, like whether the levies were breached, are not only measures of whether an event that interacts with government quality occurred, but a measure of government quality itself, which is endogenous.
with other empirical results that suggest that natural disasters harm incumbent electoral fortunes. Bechtel and Hainmueller (2011) attempt to resolve this tension by arguing that the boost to incumbent electoral fortunes in flooded districts must represent voter gratitude for an effective disaster response.

This could certainly be the case. But our model offers two arguments for why the evidence does not entail this specific interpretation. First, if the disaster response was indeed surprisingly effective, our model would also predict improved incumbent electoral fortunes. (This is just the statement that the probability of reelection is increasing in the outcome.) But the reason is very different than in Bechtel and Hainmueller’s (2011) gratitude interpretation. In our model, voters do not vote to reward or punish bad past behavior. They vote in a forward looking way, based on expectations of future performance. (Indeed, Fearon (1999) shows, quite generally, this is what rational voters must do.) Thus, the electoral returns to an unusually effective response need not be due to gratitude or a logic of reward and punishment. They could result from good governance outcomes providing information about the quality of the incumbent and, thus, the expected quality of future governance.

Second, and more importantly, our model predicts positive electoral returns to flooding on average for an incumbent who is behind. As Bechtel and Hainmueller (2011) document, the floods in Germany were an interactive shock—their effect on voter welfare depended critically on the quality of government preparedness. Hence, districts that experienced floods got more information about the incumbent than districts that did not. To see that this increased information is good for the expected electoral fortunes of an incumbent who enters the first governance period behind, inspect the probability of reelection in Equation 3. An incumbent is behind in our model if $m_{I,1}$ is less than zero (which is the prior belief about the challenger’s quality). The probability of reelection for such an incumbent is increasing in $v$, which, recall, is a measure of the amount of information the voter will have after observing governance outcomes. That is, the more information the voters get about an incumbent who is behind, the better her electoral fortunes, even though on average she performs as expected. Information is good for incumbents who are behind because they need to find a way to convince voters to change their views significantly. Hence, even absent exceptional performance, our model predicts better electoral outcomes for incumbents in districts that experienced flooding than in districts that did not.

4.2 Incumbent Responses to Natural Disasters

Bechtel and Hainmueller (2011) offer the quality of government response to a natural dis-
aster as an interpretation for their findings about the relationship between the occurrence of a natural disaster and incumbent electoral fortunes. Others, like Healy and Malhotra (2009, 2010) have gone one step further, including measures of government response in regressions relating incumbent electoral fortunes to natural disasters. They find a positive relationship. Healy and Malhotra (2010, 2013) interpret this type of finding as evidence of voter rationality.

Before turning to the question of whether this type of evidence supports the conclusion of voter rationality, we pause to consider an interpretive issue in relating our model to the evidence. It is not clear whether we should interpret observed relief following a natural disaster as effort \(a\) or the overall quality of governance \(\pi\). On the one hand, the empirical literature interprets voter responses to relief as rewards for effort, suggesting the former interpretation. On the other hand, clearly disaster relief is a function of the effort, ability, and luck of the incumbent, suggesting the latter interpretation. As we will see, on either interpretation, our model suggests that these types of empirical findings do not entail the conclusion of voter rationality. We take the two interpretations in turn.

**Relief as Effort** Suppose we adopt the interpretation that effort is the correct analogue in our model to empirically observed disaster relief. Then, for the empirical findings to support the conclusion that voters are rational, we would want two things to be true. First, we would want our model, with rational voters, to predict the observed relationship—increased effort predicts increased probability of reelection. Second, we would want plausible models with irrational voters to predict a different relationship. As we show below, such is not the case. In particular, our model with rational voters predicts a negative relationship between incumbent effort and expected incumbent electoral fortunes.

Why is this the case? Voters are trying to select good types (reelect if and only if \(m_{I,1} \geq 0\)). Since, prior to the first governance period, \(m_{I,2}\) is a random variable with mean \(m_{I,1}\), the incumbent’s reelection probability is strictly increasing in \(m_{I,1}\). That is, incumbents who enter the first governance period with higher expected ability are more likely to gain reelection. But, as shown in Proposition 1, effort is single-peaked in the incumbent’s expected ability and maximized at \(m_{I,1} = 0\).

These two facts suggest that there might be no relationship between effort and electoral fortunes. The negative relationship comes from the fact that most incumbents are better than average. Hence, for most incumbents who don’t exert high effort because they don’t face serious electoral threat, the reason is that they are way ahead, not way behind. This means that incumbents who work hard expect close elections, while most incumbents who
don’t work hard expect safe elections. Thus, a negative relationship emerges between effort and electoral fortunes on average, as formalized in the next result.

**Proposition 5** For any $\omega \in \Omega$, in equilibrium, the expected probability of reelection action is decreasing in the action. That is, let $I$ be an indicator function that takes the value 1 if the incumbent is reelected and the value 0 if the incumbent is not reelected. Then, the function $E[I|\omega, a_1^\ast(\omega, m_{I,1})]$ is decreasing in $a_1^\ast(\omega, m_{I,1})$.

This result establishes that evidence showing a positive relationship between effort and reelection probabilities is not evidence of voter rationality, since at least one canonical model with rational voters predicts a negative relationship. There is a longstanding intuition that increased effort by incumbents should lead to increased probability of reelection (Key, 1966; Fiorina, 1981). We are suggesting it is wrong when voters are rational. Why?

The intuition comes from thinking about voters rewarding politicians for working hard on their behalf. But, as Fearon (1999) points out, this sort of logic is actually inconsistent with voter rationality, which requires that voters vote based on expectations of future performance. Given this, we should not expect a simple reward-punishment logic to govern reelection and effort. Rather, we must think carefully about what the source of variation is that drives higher observed effort. Here we are suggesting that one important source of that variation is the incumbent’s own expectations about her reelection prospects. Incumbents work hard when they feel under threat (though not so much threat that they give up). Thus, from the perspective of the analyst, there is information in the observation that an incumbent works hard—we should expect that she faces a close election.

**Relief as Overall Performance** Suppose we instead adopt that interpretation that disaster relief reflects overall performance ($\pi$), not just effort. Again, for a positive relationship between performance and electoral fortunes to constitute evidence of voter rationality, we want two things to be true. First, our model with rational voters should predict this relationship. Second, plausible models with irrational voters should predict a different relationship.

Unlike with the effort interpretation, under the overall performance interpretation, the first condition is clearly true in our model. In particular, the incumbent in our model is reelected if and only if $m_{I,2}$ is greater than zero. It is clear from Equation 5 that $m_{I,2}$ is strictly increasing in $\pi_1$. Hence, better performance leads to better electoral fortunes.

The issue here is the second condition. A positive relationship between overall performance and ability also emerges from many models with irrational voters. Indeed, there is a sense in which it is hard to think of a plausible model of voters in which this is not the
case. Any model in which the behavior of voters is weakly monotone in their welfare—i.e.,
where they are more likely to vote for the incumbent the better off they feel—will pro-
duce a positive relationship between government performance and electoral outcomes. Such
weak monotonicity follows from models with simple behavioral voters who set an arbitrary
pain-pleasure threshold (Achen and Bartels, 2004), voters who fail to filter out confounding
information (Ashworth and Bueno de Mesquita, forthcoming), or a variety of models with
aspiration-based adaptive voters (Bendor et al., 2011; Bendor, Kumar and Siegel, 2010;
Andonie and Diermeier, 2012; Diermeier and Li, 2013). Since almost any plausible model
of voters—whether rational or irrational—predicts a positive relationship between performance and reelection, evidence of such a relationship does not constitute evidence of voter rationality.

5 New Hypotheses

Our model shows that the existing empirical evidence does not entail the conclusions about
voter rationality drawn by the literature. Importantly, the model also offers some additional
empirical implications which might suggest more fruitful paths forward for empiricists in-
terested in this question. Here we discuss two such implications.

The first implication has to do with the effect of interactive shocks on public opinion. Imagine a survey instrument that allows the researcher to measure the public’s assessment of an incumbent’s competence, purged of ideological (and other) factors. The model yields two hypotheses about the relationship between interactive shocks and such a measure of public opinion. First, interactive shocks have no systematic effect on the mean public assessment of incumbent competence. This is because rational voters always extract an unbiased signal from the available information and use the signal to update their beliefs properly. Second, interactive shocks increase the variance of the mean district-level public assessment of incumbent competence. This is because in districts with larger interactive shocks voters have more information. These intuitions are formalized in the next result.

Proposition 6

1. \( E[m_{I,2} - m_{I,1} | \omega] = 0 \) for all \( \omega \in \Omega \).

2. For any \( \omega', \omega'' \in \Omega \),

\[
\text{var}[m_{I,2} - m_{I,1} | \omega'] > \text{var}[m_{I,2} - m_{I,1} | \omega'']
\]
if and only if $\beta(\omega') > \beta(\omega'')$.

Let’s think about how one might operationalize Proposition 6 empirically. Imagine panel data on the survey instrument suggested above. For each district, the researcher observes two years of public opinion. Further, the researcher can divide districts into two bins: those that experienced large interactive shocks and those that didn’t. Point 1 of the proposition suggests that, averaging across districts, there should be no systematic change in public opinion in either bin. Point 2 of the proposition suggests that the variance of the changes in public opinion will be larger in the first bin (districts that experience large interactive shocks) than in the second bin (districts that did not experience large interactive shocks). Of course, with a more continuous measure of the size of shocks, one could implement similar ideas with a more flexible interaction.

One might think that the results in Proposition 6 translate straightforwardly into analogous results on electoral fortunes measured as vote share. But that can’t be right. Indeed, Proposition 4 shows directly that point 1 of Proposition 6 does not hold for vote share—interactive shocks lead to a systematic decrease in expected vote share for incumbents. For similar reasons, having to do with the non-linear relationship between public opinion and vote share, there is also no straightforward relationship between the variance of public opinion and the variance of vote share.

While point 1 of Proposition 6 does not extend to incumbent electoral fortunes, the model does in fact offer a more subtle hypothesis about the relationship between interactive shocks and electoral fortunes. In particular, as highlighted in the discussion surrounding Figure 3, we predict heterogeneous effects depending on whether the incumbent is ahead or behind. Shocks that increase the responsiveness of voter welfare to incumbent quality are expected, on average, to hurt incumbents who are ahead and to help incumbents who are behind. We formalize this fact in the next result.

**Proposition 7** Suppose $\omega', \omega'' \in \Omega$ are states such that $\beta(\omega') > \beta(\omega'')$. Regardless of the values of $\alpha(\omega')$ and $\alpha(\omega'')$, both of the following are true:

1. An incumbent who enters the governance period ahead $(m_{I,1} > 0)$ has a greater probability of reelection and a greater expected vote share in state $\omega''$ than in state $\omega'$.

2. An incumbent who enters the governance period behind $(m_{I,1} < 0)$ has a greater probability of reelection and a greater expected vote share in state $\omega'$ than in state $\omega''$.

**Proof.** The result is immediate from Equations 2 and 3.
6 Conclusion

A longstanding literature in political behavior seeks to evaluate voter competence. Concern with important matters of identification has led recent strands of the literature to turn its attention to natural disasters and economics shocks as sources of plausibly exogenous variation in voter welfare. However, the literature reaches contradictory conclusions.

We argue that neither of the key empirical findings in the existing behavioral literature entails the conclusions drawn by that literature. One set of studies finds that incumbent electoral fortunes seem to systematically suffer following disasters. These findings have typically been interpreted as evidence of voter irrationality. However, we show that they are entirely consistent with a canonical model of electoral agency with rational voters. Disasters increase voter information about incumbents. And such information, even if unbiased, is bad for incumbents, who are typically ahead of challengers because of prior electoral selection.

Another set of studies finds that incumbent electoral fortunes are helped by effective disaster response. These findings, while less well identified (since disaster response may be endogenous to anticipated electoral competitiveness) have typically been interpreted as evidence of voter rationality. However, we show that they are entirely consistent with a canonical model of electoral agency with irrational (or rational) voters. This is because voter rationality pins down voter behavior more strongly than is typically supposed. Rational voters aren’t just positively responsive to government performance. They use information optimally to maximize future payoffs. Hence, many forms of positive responsiveness are in fact irrational behavior.

Our argument, thus, suggests that the empirical literature is significantly less informative about voter rationality than has been supposed. Further, our results show that learning about voter rationality is more difficult than has been previously appreciated, both because the way electoral fortunes respond to exogenous shocks under rational voting is in fact subtle and because certain kinds of irrational behavior are consistent with folk notions of rationality.

Nonetheless, we are able to suggest several hypotheses that differentiate responses to shocks that are consistent with voter rationality from responses that are inconsistent with voter rationality. First, our model of rational voters predicts that public opinion about incumbent competence should become more variable following disasters. Second, our model of rational voters predicts heterogeneous effects on electoral fortunes conditional on the ex ante electoral prospects of particular incumbents.
All that said, it is perhaps also worth pausing to reflect on whether the assessment of voter rationality deserves the central place it occupies in the literature. Traditionally, the behavioral literature has been interested in voter rationality because it was thought that only rational voters were competent to fulfill their democratic function. But a growing literature points out ways in which certain types of voter irrationality at least sometimes actually can improve democratic performance (Ashworth and Bueno de Mesquita, forthcoming; Diermeier and Li, 2013; Levy and Razin, 2013). Hence, in addition to remaining unsettled for reasons pointed out in this paper, it also seems that the question of whether or not voters are rational is less important for key normative debates than the amount of attention paid to it might suggest.
A Proofs

A.1 Belief calculations

We use the standard appeal to the law of large numbers for a continuum of iid random variables to equate the cross-sectional distribution with the prior distribution of a single instance.

In any given district, a candidate, \( j \), generated a campaign signal, \( s_{j,1} = \theta_j + \eta_{j,1} \). This signal is normally distributed with mean \( \theta_j \). Since the voters’ prior about \( j \)’s quality was normal with mean zero, standard results say that the voters’ posterior beliefs are normal with mean

\[ m_{j,1} = \lambda_1 s_{j,1} \]

and variance

\[ \sigma_1^2 = \lambda_1 \sigma_\eta^2, \]

with \( \lambda_1 = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\theta^2} \). Moreover, standard results on Bayesian updating of normal random variables imply that the prior distribution of \( m_{j,1} \) is normal with mean 0 and variance

\[ \sigma_1^2 = \frac{(\sigma_\theta^2)^2}{\sigma_\theta^2 + \sigma_\eta^2}. \]

A.2 Equilibrium Characterization

In this subsection, we characterize a pure strategy perfect Bayesian equilibrium. In the course of doing so, we will also provide a proof of Proposition 1.

At the final governance period, whichever politician is in office will choose effort \( a_2 = 0 \), since at this point, payoffs are strictly decreasing in effort. (This is point 4 of Proposition 1.)

At the final election, standard results imply that the voters will have a posterior distribution over the quality of the incumbent that is normal; its mean is \( m_{I,2} \) and its variance is \( \sigma_2^2 \). Let \( \bar{\alpha} \) and \( \bar{\beta} \) be the expected values of \( \alpha(\omega) \) and \( \beta(\omega) \), respectively, under \( \mu \). The median voter’s payoff at date 2 has expected value \( \bar{\alpha} + \bar{\beta} m_{I,2} \) if the incumbent is reelected and \( \bar{\alpha} \) if the challenger is elected. Thus the voter will reelect the incumbent if and only if \( m_{I,2} \geq 0 \). (This is point 3 of Proposition 1.)

As shown in Section A.1, at the beginning of the first governance period the voters’ beliefs are that the incumbent has quality that is normally distributed with mean \( m_{I,1} \) and variance \( \sigma_1^2 \). Suppose the state of the world is \( \omega \) in the first governance period. Further,
suppose the voters conjecture that the incumbent takes action $a_1^*$. Then the voters believe that the first period outcome, $\pi_1$, is $\alpha(\omega) + \beta(\omega)(\theta_{I,1} + a_1^*) + \epsilon_1$. Thus, from the voters’ perspective

$$\frac{\pi_1 - \alpha(\omega) - \beta(\omega)a_1^*}{\beta(\omega)}$$

is a normally distributed random variable with mean $\theta_{I,1}$. As such, standard results on Bayesian updating imply that the voters’ posterior beliefs about the incumbent’s quality are normally distributed with mean

$$m_{I,2} = \lambda(\omega) \left( \frac{\pi_1 - \alpha(\omega) - \beta(\omega)a_1^*}{\beta(\omega)} \right) + (1 - \lambda(\omega))m_{I,1}, \quad (5)$$

where

$$\lambda(\omega) = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_{\epsilon}^2}. \beta(\omega)^2$$

If the incumbent actually chooses effort $a_1$, then, conditional on an $\omega$, $\pi_1 = \alpha(\omega) + \beta(\omega)(\theta_{I,1} + a_1) + \epsilon_1$. Substitute this into Equation 5, and recall that the median voter reelects if and only if $m_{I,2} \geq 0$, to see that, conditional on $\omega_1$, if the incumbent chooses effort $a_1$, she is reelected if:

$$a_1 - a_1^* + \frac{1 - \lambda(\omega)}{\lambda(\omega)}m_{I,1} + \theta_{I,1} + \frac{\epsilon_1}{\beta(\omega)} \geq 0.$$  

From the incumbents’ perspective $\theta_{I,1}$ is itself a normally distributed random variable with mean $m_{I,1}$. This implies that, from the incumbents’ perspective, the left-hand side is distributed normally with mean $\frac{m_{I,1}}{\lambda(\omega)} + a_1 - a_1^*$ and variance

$$\sigma^2_\pi(\omega) = \sigma_1^2 + \frac{\sigma_{\epsilon}^2}{\beta(\omega)^2}. \beta(\omega)^2.$$  

As such, the incumbent believes that if she chooses $a_1$, she is reelected with probability:

$$1 - \Phi \left( \frac{-\frac{m_{I,1}}{\lambda(\omega)} - (a_1 - a_1^*)}{\sigma_\pi(\omega)} \right).$$

Hence, she chooses effort to solve:

$$\max_{a_1} \left[ 1 - \Phi \left( \frac{-\frac{m_{I,1}}{\lambda(\omega)} - (a_1 - a_1^*)}{\sigma_\pi(\omega)} \right) \right] B - \frac{a_1^2}{2}. \quad (6)$$
The first-order condition is:
\[ \phi \left( \frac{m_{I,1}}{\lambda(\omega)} - (a_1 - a_1^*) \right) \frac{B}{\sigma_\pi(\omega)} - a_1 = 0. \]

Part 3 of Lemma 1 (below) implies that this first-order condition in fact characterizes the optimal choice.

In equilibrium, the voters’ conjecture and the incumbent’s effort must be the same. Imposing this rational expectations requirement, we have:
\[ a_1^*(m_{I,1}, \omega) = \phi \left( \frac{m_{I,1}}{\lambda(\omega)} \right) \frac{B}{\sigma_\pi(\omega)}. \]  
(7)

(This establishes point 2 of Proposition 1.)

Now consider the first election. The voter can choose between two candidates. He has a posterior on the quality of each candidate based on the campaign signal. These posteriors are normal, with means \( m_{d,1} \) and \( m_{r,1} \). The expected payoff to electing a candidate with expected ability \( m_{j,1} \) is the sum of two factors:

1. Her expected performance in the first governance period.
2. The expected performance of the winner of the next election in the second governance period, given that she is one of the candidates.

The first factor is
\[ \mathbb{E}_\mu[\alpha(\omega) + \beta(\omega)(m_{j,1} + a_1^*(m_{j,1}, \omega))]. \]  
(8)

To see that this expectation is increasing in \( m_{j,1} \), it suffices to see that function inside the expectation is increasing for any \( \omega \). This follows directly from part 4 of Lemma 1 (below).

To calculate the second term, note that our analysis above indicates that candidate \( j \) will win the second election if and only if the voter’s posterior beliefs about her ability following the first governance period \( (m_{j,2}) \), are greater than zero. Hence, the expected second-governance period payoff of electing a candidate of expected ability \( m_{j,1} \) in the first election is
\[ \overline{\alpha} + \overline{\beta}\mathbb{E}[\max\{m_{j,2}, 0\}|m_{j,1}]. \]

This term is clearly increasing in \( m_{j,1} \).

The median voter’s expected payoff from election candidate \( j \) is the sum of two components, each of which is increasing in \( m_{j,1} \). Thus, in the first election, the median voter
will support whichever candidate generated the better campaign signal. (This is point 1 of Proposition 1.)

Finally, we state and prove the Lemma that has been referred to twice above. At several points, its proof uses the fact that $\phi'(x) = -x\phi(x)$ REF.

**Lemma 1** Suppose

$$B \leq \sqrt{2\pi e} \left( \frac{\sigma_\theta^2 \sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2} \right),$$

and define the function $h$ by

$$h(x) = \phi \left( \frac{-x}{\sigma_\pi(\omega)} \right) \frac{B}{\sigma_\pi(\omega)}.$$

Then:

1. $|\phi(x)x| \leq \frac{1}{\sqrt{2\pi e}}$.

2. $|h'(x)| \leq \frac{\sigma_1^2}{\sigma_\pi(\omega)^2} < 1$.

3. The first-period incumbent’s objective function in 6 is concave in $a_1$.

4. The expected first-period public good provision in 8 is increasing in $m_{j,1}$.

**Proof.**

1. The function $x \mapsto x\phi(x)$ is zero at $x = 0$, and it approaches 0 as $x$ tends to either $\infty$ or $-\infty$, since the Gauss kernel tends to zero faster than any polynomial. The derivative is $\phi(x) - x^2\phi(x)$, so the critical points are 1 and $-1$. At each of these points, the absolute value of the function is

$$\frac{1}{\sqrt{2\pi e}} e^{-(1/2)}.$$

2. Observe first that

$$|h'(x)| = \left| -\phi' \left( \frac{-x}{\sigma_\pi(\omega)} \right) \frac{B}{\sigma_\pi(\omega)^2} \right| \leq \left| \phi \left( \frac{-x}{\sigma_\pi(\omega)^2} \right) \left( \frac{-x}{\sigma_\pi(\omega)^2} \right) \right| \cdot \left| \frac{B}{\sigma_\pi(\omega)^2} \right|.$$

From the bound in part 1 and the hypothesized bound on $B$, we have

$$\left| \phi \left( \frac{-x}{\sigma_\pi(\omega)^2} \right) \left( \frac{-x}{\sigma_\pi(\omega)^2} \right) \right| \cdot \left| \frac{B}{\sigma_\pi(\omega)^2} \right| \leq \frac{1}{\sqrt{2\pi e}} \cdot \frac{\sqrt{2\pi e} \left( \frac{\sigma_\theta^2 \sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2} \right)}{\sigma_\pi(\omega)^2} \cdot \frac{\sigma_1^2}{\sigma_\pi(\omega)^2}.$$
Since $\sigma_\pi(\omega)^2 = \sigma_1^2 + \frac{\sigma_2^2}{\sigma(\omega)^2}$, we have $|h'(x)| \leq \frac{\sigma^2}{\sigma_\pi(\omega)^2} < 1$, as required.

3. The derivative of the objective function in 6 is

$$
\phi \left( \frac{-m_{l,1}}{\lambda(\omega)} - (a_1 - a_1^*) \right) \frac{B}{\sigma_\pi(\omega)} - a_1 = h \left( \frac{m_{l,1}}{\lambda(\omega)} + (a_1 - a_1^*) \right) - a_1.
$$

Thus part 2 implies the second derivative is non-positive.

4. For each $\omega$, first-period public good provision is a positive affine transformation of

$$
m_{j,1} + a_1^*(m_{j,1}, \omega) = m_{j,1} + h(m_{j,1}/\lambda(\omega)).
$$

Differentiate with respect to $m_{j,1}$ to get

$$
1 + h'(m_{j,1}/\lambda(\omega)) \frac{1}{\lambda(\omega)} \geq 1 - \frac{\sigma_1^2}{\sigma_\pi(\omega)^2} \frac{1}{\lambda(\omega)} = 0.
$$

A.3 Proofs of Numbered Results

The proofs of Propositions 3 and 4 share a common structure, based on the following three Lemmas.

Lemma 2 Let $f$ denote the cross-sectional density of expected qualities of the winners of the first election. Then the likelihood ratio

$$
\ell(m) \equiv \frac{f(m)}{f(-m)}
$$

is strictly increasing in $m$ and satisfies $\ell(m) \geq 1$ for any $m \geq 0$.

Proof. Since the median voter has bias $b = 0$, he elects the candidate in the first election with greater expected quality. This implies that $m_{l,1} = \max\{m_{l,1}, m_{r,1}\}$. Let $f$ denote the prior density of $m_{l,1}$. Since it is the density of the first-order statistic of two normally distributed random variables, it is given by:

$$
f(m) = 2\phi \left( \frac{m}{\sigma_1} \right) \Phi \left( \frac{m}{\sigma_1} \right).
$$
Since \( \phi \) is symmetric and \( \Phi(-x) = 1 - \Phi(x) \), we have:

\[
\ell(m) = \phi \left( \frac{m}{\sigma_1} \right) \Phi \left( \frac{m}{\sigma_1} \right) \phi \left( -\frac{m}{\sigma_1} \right) \Phi \left( -\frac{m}{\sigma_1} \right) \frac{\Phi \left( \frac{m}{\sigma_1} \right)}{1 - \Phi \left( \frac{m}{\sigma_1} \right)}.
\]

This expression is increasing in \( m \), and is equal to 1 at \( m = 0 \).

**Lemma 3** Suppose \( h \) is a function with that satisfies \( h(-x) = (-1)^k h(x) \) for some positive integer \( k \). Then, for any function \( g \),

\[
\int_{-\infty}^{\infty} g(x) h(x) \, dx = \int_{0}^{\infty} \left[ g(x) + (-1)^k g(-x) \right] h(x) \, dx.
\]

**Proof.** Split the integral into two parts:

\[
\int_{-\infty}^{\infty} g(x) h(x) \, dx = \int_{-\infty}^{0} g(x) h(x) \, dx + \int_{0}^{\infty} g(x) h(x) \, dx.
\]

In the first integral, make the change of variables \( x \mapsto -x \) to get:

\[
\int_{-\infty}^{0} g(x) h(x) \, dx = \int_{0}^{\infty} g(-x) h(-x) \, dx.
\]

Since \( h(-x) = (-1)^k h(x) \), we have

\[
\int_{-\infty}^{\infty} g(x) h(x) \, dx = \int_{0}^{\infty} (-1)^k g(-x) h(x) \, dx + \int_{0}^{\infty} g(x) h(x) \, dx.
\]

**Lemma 4** Suppose \( h \) is a function satisfying the following properties:

1. \( h(x) = -h(-x) \)
2. \( h(x) < 0 \) for \( x > 0 \).

If \( f(x) > f(-x) \) for all \( x > 0 \), then

\[
\int_{-\infty}^{\infty} h(x) f(x) \, dx < 0.
\]
Proof. From Lemma 3 with \( k = 1 \), we have

\[
\int_{-\infty}^{\infty} h(x) f(x) \, dx = \int_{0}^{\infty} h(x) \left[ f(x) - f(-x) \right] \, dx,
\]

which is negative because, for \( x > 0 \), we have \( h(x) \) negative and \( f(x) > f(-x) \). ■

Proof of Proposition 3. The cross-sectional average reelection probability in state \( \omega \) is

\[
\int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{-m_{I,1}}{\sqrt{v(\omega)}} \right) \right] f(m_{I,1}) \, dm_{I,1}.
\]

We need to show that this integral is decreasing in \( \sqrt{v(\omega)} \).

Differentiate the integral with respect to \( \sqrt{v(\omega)} \) to get

\[
\int_{-\infty}^{\infty} -\phi \left( \frac{-m_{I,1}}{\sqrt{v(\omega)}} \right) \left( \frac{m_{I,1}}{\sqrt{v(\omega)}} \right) f(m_{I,1}) \, dm_{I,1}.
\]

Now note that the function \( h(m_{I,1}) = -\phi \left( \frac{-m_{I,1}}{\sqrt{v(\omega)}} \right) \left( \frac{m_{I,1}}{\sqrt{v(\omega)}} \right) \) has the two properties from Lemma 4 and that \( f(m_{I,1}) > f(-m_{I,1}) \) by Lemma 2. Hence Lemma 4 implies the integral is negative, as required. ■

To prove proposition 4, we need the following definitions. Let

\[
V(v, m_{I,1}) = \int_{-\infty}^{\infty} \left[ 1 - \Phi(-m_{I,2}) \right] \frac{1}{\sqrt{v}} \phi \left( \frac{m_{I,2} - m_{I,1}}{\sqrt{v}} \right) \, dm_{I,2}.
\]

Conditional on an \( m_{I,1} \) and state \( \omega \), the expected incumbent vote share is \( V(v(\omega), m_{I,1}) \).

Further define \( \Delta(m_{I,1}) \) as the difference in expected vote share the two states:

\[
\Delta(m_{I,1}) \equiv V(v(\omega''), m^I) - V(v(\omega'), m^I).
\]

Lemma 5

1. If \( m_{I,1} > 0 \), then \( \Delta(m_{I,1}) < 0 \).

2. \( \Delta(m_{I,1}) = -\Delta(-m_{I,1}) \).

Proof. It will be useful to make the change of variables \( y \mapsto \frac{m^I - m^L}{\sigma} \). This implies that

\[
m^I \mapsto y\sigma + m^L \quad \text{and that} \quad dm^I \mapsto \sigma dy.
\]

Hence, we can rewrite the integral above as:

\[
V(v, m_{I,1}) = \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( y + \sqrt{v} m_{I,1} \right) \right] \phi(y) \, dy.
\]

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Differentiate to get:

\[
\frac{\partial V}{\partial v}(v, m_{I,1}) = \frac{m_{I,1}}{2\sqrt{v}} \int_{-\infty}^{\infty} -\phi(y + \sqrt{v} m_{I,1}) \phi(y) \, dy.
\] (9)

Since \( \int_{-\infty}^{\infty} -\phi(y + \sqrt{v} m_{I,1}) \phi(y) \, dy > 0 \), we have

\[
\text{sgn} \left( \frac{\partial V}{\partial v}(v, m_{I,1}) \right) = -\text{sgn}(m_{I,1}).
\]

Using the fundamental theorem of calculus, we can write:

\[
\Delta(m_{I,1}) = \int_{v(w')}^{v(\omega'')} \frac{\partial V}{\partial v}(v, m_{I,1}) \, dv.
\]

From Equation 9, this is:

\[
\Delta(m_{I,1}) = -\frac{m_{I,1}}{2\sqrt{v}} \int_{v(w')}^{v(\omega'')} \int_{-\infty}^{\infty} \phi(y + \sqrt{v} m_{I,1}) \phi(y) \, dy \, dv.
\]

Using Lemma 3 in the first and third lines and symmetry of the normal density in the second, we have:

\[
\int_{-\infty}^{\infty} \phi(y + \sqrt{v} m_{I,1}) \phi(y) \, dy = \int_{-\infty}^{\infty} \phi(y - \sqrt{v} m_{I,1}) \phi(y) \, dy.
\]

**Proof of Proposition 4.** The cross-sectional average of the difference in vote share in the good and bad states is:

\[
\int_{-\infty}^{\infty} \Delta(m') f(m') \, dm'.
\]

Lemma 5 implies that \( \Delta \) has both properties from Lemma 4. Hence, the result follows from Lemmas 2, 4, and 5.
Proof of Proposition 5. We are interested in the following conditional expectation:

\[ E[\text{Probability Reelection}|a_1, \omega] = \mathbb{E} \left[ \Phi \left( \frac{m_{I,1}}{\sqrt{v(\omega)}} \right) \left| a_1 = a_1^*(m_{I,1}, \omega), \omega \right. \right] . \]

Let \( z(a, \omega) \) be the \( m_{I,1} > 0 \) that yields \( a \) as the equilibrium effort given the state \( \omega \). That is, \( z(a, \omega) \) is the unique positive solution to \( a_1^*(z, \omega) = a \).

Now, suppose that, in state \( \omega \), the incumbent takes action \( a \). Along the equilibrium path, the incumbent is either of expected ability \( z(a, \omega) \) or \( -z(a, \omega) \). From the perspective of the analyst, the conditional probability that the incumbent is of type \( z(a, \omega) \) is \( f(z(a, \omega)) \) and the conditional probability that the incumbent is of type \( -z(a, \omega) \) is the complement.

Given this we can write:

\[ E[\text{Probability Reelection}|a_1, \omega] = \Phi \left( -z(a, \omega) \right) f(-z(a, \omega)) + \Phi \left( z(a, \omega) \right) f(z(a, \omega)) \]

Recall that \( \ell(x) = \frac{f(x)}{f(-x)} \) to rewrite the conditional expectation as:

\[ E[\text{Probability Reelection}|a_1, \omega] = \frac{1}{\ell(z(a, \omega)) + 1} + \Phi \left( \frac{z(a, \omega)}{\sqrt{v(\omega)}} \right) \frac{\ell(z(a, \omega)) - 1}{\ell(z(a, \omega)) + 1} \]

Differentiate with respect to \( a \) to get:

\[
\frac{2\Phi \left( \frac{z(a, \omega)}{\sqrt{v(\omega)}} \right) - 1}{(\ell(z(a, \omega)) + 1)^2} \ell'(z(a, \omega)) + \frac{1}{\sqrt{v(\omega)}} \phi \left( \frac{z(a, \omega)}{\sqrt{v(\omega)}} \right) \frac{\ell(z(a, \omega)) - 1}{\ell(z(a, \omega)) + 1} \frac{\partial z(a, \omega)}{\partial a}.
\]

Each term in the brackets is positive. To see this, first note that \( z(a, \omega) > 0 \) implies that \( 2\Phi(z(a, \omega)) - 1 > 0 \). Second, Lemma 2 implies that \( \ell' > 0 \), and, since \( z(a, \omega) > 0 \), that \( \ell(z(a, \omega)) > 1 \).

Equation 7 implies that \( \frac{\partial z}{\partial a} < 0 \). Thus, the conditional expectation is decreasing in \( a \).

\[ \blacksquare \]

Proof of Proposition 6.

1. Follows from the law of iterated expectations.
2. For a fixed \( m_{I,1} \), the difference, \( m_{I,2} - m_{I,1} \), is normal with mean zero and variance \( v(\omega) \). Because this variance is the same for all \( m_{I,1} \), the cross-sectional average of the variance is \( v(\omega) \). Now the result follows from Equation 2.
References


