Force, Wealth, and Elections*

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Abstract

Some countries, "failed states," are unable to establish any kind of peaceful order. A few maintain order without holding elections. Many celebrate peaceful elections in which opposition is either not allowed at all or not given a chance to win. Finally, in some countries elections are competitive and peaceful. During most of modern history, civil peace was maintained when one political force consolidated its military power and potential challengers acquiesced to elections in which they had no chance to win: there is nothing new about "electoral authoritarianism." The shadow of violence fades only when people are wealthy enough that they do not want to bear the costs of fighting to increase their incomes. The paper analyzes why civil peace is frequently difficult to establish, why most often it emerges under the dominance of one party, and why some rulers allow competitive elections and leave office when they lose.

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"The 'normal' exercise of hegemony on the now classical terrain of parliamentary regime is characterized by a combination of force and consent, which balance each other reciprocally, without force predominating excessively over consent. ... Between consent and force stands corruption/fraud (which is characteristic of certain situations when it is hard to exercise the hegemonic function, and when the use of force is too risky)." (Antonio Gramsci, Prison Notebooks)

1 Introduction

Some countries, "failed states," are unable to establish any kind of peaceful order. A few maintain order without holding elections. Many celebrate peaceful elections in which opposition is either not allowed at all or not given a chance to win. Finally, in some countries, those that we identify as democracies, elections are competitive and peaceful. Consolidation of state power is often a protracted process and even when the state becomes sufficiently powerful to ward off potential challengers, elections take place under the shadow of violence. During most of modern history, civil peace was maintained when some political forces consolidated their military power to the point that potential challengers were sufficiently intimidated to acquiesce to elections in which they had little of a chance to win: there is nothing new about "electoral authoritarianism." The shadow of violence fades only when people are wealthy enough not to care much about whatever they can gain by fighting.

The purpose of this analysis is to understand why civil peace is frequently difficult to establish, why most often it emerges under the dominance of a single political force, and finally why some rulers allow competitive elections and leave office when they lose.

Here is the core of the argument. Parties participate in elections to determine some policy over which they have conflicting interests. The probability that the current incumbent would win is $p$. Having observed the result of an election, parties decide whether to respect the outcome or to try to impose their will by force. The probability that the incumbent would prevail in a violent conflict is $q$. Hence, political actors face two lotteries, with different stakes and different probabilities.

A general conclusion follows already from this generic formulation. A party that loses an election may be prone to revert to force if it expects the defeat to be permanent or just indefinite, yet it may respect the result if it believes that at some future time it will have a chance to win. Hence, the mechanism by which elections induce peace is that they enable inter-temporal horizons (Przeworski 1991, 2005). A peaceful
outcome, however, requires at least one of two conditions. One is that outcomes of elections are obeyed only if electoral chances reflect relative military prowess, which means that if one party dominates militarily it must also dominate electorally. Already Herodotus (quoted in Bryce 1921: 25-26) thought that in a democracy "physical force of the citizens coincides (broadly speaking) with their voting power," while Condorcet (1986: 11) observed that in the ancient, brutal times, "for the good of peace and general utility, it was necessary to place authority where the force was." Yet the relation between military force and electoral chances becomes less important when people value less whatever they can acquire by fighting. Hence, if conflicts concern income, peace is easier to maintain in wealthier societies.

Note that in this barebone sketch the military strength of the ruler is the only exogenous feature of the environment in which incumbents make decisions whether to hold an election or rule without them, whether or not to allow opposition, whether or not to give the opposition a chance to win, and whether or not to yield power if they happen to lose: exactly the features by which we distinguish political regimes. I argue below that this environment is richer, namely, that we should distinguish the coercive power of the state from the coercive power of the particular incumbents. But this rudimentary formulation is sufficient to see that regimes are structured by the relations of physical force and the kind of elections they hold, if any, is endogenous. Regimes are autocratic when rulers can govern by force alone; they are authoritarian when rulers can muster enough force to hold elections which they are certain to win; they are democratic when rulers are compelled to tolerate a chance that they may lose and are forced to leave when they do.

The paper is structured as follows. The section that follows summarizes some historical patterns and offers some intuitions. The central intuitions are further developed in a simple framework and the model is placed in the context of the relevant literature. The subsequent section is an analysis of the conditions under which elections are peaceful when everyone shares the belief about the military prowess of the incumbent. This model is then extended to situations in which these beliefs diverge, so that some political actors may miscalculate. A brief summary and some caveats close the paper.

2 Some history and intuitions

One striking pattern visible in examining political history of several countries from 1788 until recently is that many experienced long periods during which successive governments were elected and completed their
constitutionally specified terms, interrupted by periods during which governments, elected or not, were repeatedly overthrown by force.\(^1\) To get a flavor of such histories, here are some examples (Continuous lines with a value of 1 indicate years that belong to periods of completed terms and with a value of 0 otherwise; successful coups are indicated by isolated points with value of 1 or 2):

\[\text{Completed terms and coups in Ecuador}\]

\[\text{Completed terms and coups in Thailand}\]

Here is an intuition of how such patterns may emerge. Several political actors contest political power. At some time one contender, thought

\(^1\) All the data used in this section are from Przeworski et al. (2007). Please note that the data are still being cleaned.
by everyone to have military power \( q(0) \), becomes the ruler. If the potential challengers believe that the military power of the incumbent is low, one of them revolts, seeking to overthrow the incumbent by force. What it often meant in 19th century Latin America was that a local caudillo, who called himself "general," recruited a bunch of his peones ("soldiers") and invaded the capital or claimed independence for his region. If the incumbent is defeated, the challenger becomes the incumbent, with \( q = q(0) \). But if the incumbent defends himself successfully, everyone updates upward their beliefs about his military prowess. When these beliefs pass some critical value, that is, when the potential opponents have learned that an incumbent is proficient in defending himself, they are willing to participate in elections in which they do not have much of a chance to win. A period of peace thus ensues. But this is peace supported by the threat of violence.

Hence, we should expect elections to be often non-competitive and the opposition to be sufficiently intimidated to obey their results. While some voices claim that we are currently witnessing an emergence of a qualitatively new phenomenon, "electoral authoritarianism," such regimes were the prevalent form of political organization throughout history. The idea of an official government list submitted to voters for a plebiscitary approval was present already in France under the Directorate (Crook 1996), used under Restoration, and perfected under Napoleon III (Zeldin 1958). The Spanish monarchy gained in this way such a complete control over voters that between 1876 and 1917 it was able to orchestrate a system in which governments alternated in every elections according to a pre-arranged agreement between parties. The same was true in Portugal between 1851 and 1869. Promoting government candidates was not a transgression but a duty of public officials: the French Prime Minister, de Vilèlle, issued in 1822 a circular instructing "All those who are members of my ministry must, to keep their jobs, contribute within the limits of their right to the election of M.P.s sincerely attached to the government" (quoted in Zeldin 1958: 79). Partisan use of public administration was ubiquitous in Latin America as well as in Europe. Following Chile after 1831 (about which see Valenzuela 1995), several Latin American countries established stable systems of succession in which incumbent presidents completed their terms, faithfully obeying term limits, chose their successors and used governmental power to assure their victory at the polls. The stability of such systems of oligarchical pluralism – Chile between 1831 and 1891 and again until 1924, Nicaragua between 1856 and 1890, Brazil between 1894 and 1930, Argentina between 1897 and 1916, Uruguay between 1898 and 1932, Mexico between 1934 and 2000 – was remarkable. Indeed, in the entire history of Latin America only
three incumbent presidents who presented themselves for reelection ever lost. As Halperin-Donghi (1973: 116) observed, “Among the many ways of overthrowing the government practiced in postrevolutionary Spanish America, defeat at the polls was conspicuously absent.” Nothing is new about Putinism.\(^2\)

This history poses an intriguing question about civil wars in Africa, namely, whether they are exceptionally frequent or just reflect the length of period since independence. After all, civil wars were frequent in 19th century Latin America, and many among them were related to elections (Posada-Carbó 1994, Alonso 2000, Malamud 2000, Sabato and Littieri 2003, Sabato 2008).\(^3\) Here is a comparison of average proportion of countries with completed electoral terms on the two continents, by years since independence.

<table>
<thead>
<tr>
<th>Years since independence</th>
<th>Latin America</th>
<th>Africa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td>0.65</td>
</tr>
<tr>
<td>20</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>

If we treat completed terms as an indication of civil peace, Africa appears not to have been less peaceful than Latin America during the first fifty years of independence. True, many African countries maintained peace under one-party rule, while Latin American elections tended to be contested. But these patterns suggest that consolidation of any kind of stable political regime may just take time and there may be nothing

\(^2\)Here is how the governor of Murmansk, Jurij Jewdokimow, described the practices of his own party, Only Russia, in the municipal election of March 15, 2009: "Employees of municipal enterprises are being forces to staff mailboxes with fliers calling for voting for the 'proper' candidate. Pre-school teachers have to distribute to parents agitational materials...." (Gazeta Wyborcza, Warsaw, Poland, March 17, 2009).

\(^3\)Malamud (2000: 9) observes that "... an important part of the revolutions, civil wars, and other types of coups were often related to electoral processes, in order to attempt to change the rules of the game before voting took place or to modify the popular verdict."
specific about Africa. Moreover, the role of ethnicity in sparking conflicts in Africa becomes doubtful. Perhaps the mechanism is the same as it was in Latin America – local "caudillos" invade the capital or proclaim independence of their region – only that in Africa local happens to coincide with "ethnic."

Why, however, would periods of lasting peace be shattered by violence, which happened in several Latin American countries after 1925 and in Africa in the late 1980s? The most frequent reason, I suspect, is that when entrenched incumbents lose elections they do not accept the defeat. Another possibility is that having won several consecutive elections incumbents become overconfident about their power. Finally, it may well be that the partisan postures of the military change exogenously.

The outcomes of elections are eloquent in showing that electoral defeats of incumbents have been historically rare and peaceful alternations in office even more so. Table 1 summarizes outcomes of elections in which the office of the chief executive was at stake and the subsequent events. The "incumbent" is not necessarily the same person: he or she may be a member of the same party or an otherwise designated successor. "Winner," as well, may be a person or a party. Note that if the incumbent won, he is the winner; if the incumbent lost, the winner is someone else. "Assumed indirectly" stands for sequences of events in which the winner assumed office but only after someone else – the loser or a third party – held it unconstitutionally in the immediate aftermath of an election. "Assumed," whether directly or not, indicates that the winner held office for at least one year, but not necessarily that he completed the constitutionally specified term.

<table>
<thead>
<tr>
<th>Incumbent</th>
<th>Winner</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>assumed</td>
<td>did not</td>
</tr>
<tr>
<td></td>
<td>directly</td>
<td>indirectly</td>
</tr>
<tr>
<td>won</td>
<td>1999</td>
<td>9</td>
</tr>
<tr>
<td>lost</td>
<td>473</td>
<td>19</td>
</tr>
<tr>
<td>total ran</td>
<td>2472</td>
<td>28</td>
</tr>
<tr>
<td>did not run</td>
<td>84</td>
<td>6</td>
</tr>
<tr>
<td>unclear</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>2571</td>
<td>37</td>
</tr>
</tbody>
</table>

Most of the time, in $2472/2648 = 0.93$ cases, results of elections were obeyed: someone won, someone lost, and the winner assumed office.
Yet most striking is the frequency with which incumbents won elections: 2,103 out of 2,648 cases in which they ran, which gives $p = 0.79$, and 4:1 odds of winning. In the remaining 176 cases, 0.07 of elections, the winner either never made it into office or had to overcome some usurper first.

These data suggest that parties are willing to obey outcomes of elections even when current incumbents enjoy an overwhelming advantage. Given the estimate of $p$, the expected tenure in office is five electoral terms, implying that partisan alternations have been infrequent. Note that partisan alternations are peaceful only if the incumbent loses and the winner assumes office, which occurred in $473/2648 = 0.18$ cases, one in 5.6 elections. Moreover, even though the first partisan alternation in history occurred in the United States in 1801, peaceful alternations have been rare until the last quarter of the past century.

There is also strong evidence that the frequency of peaceful alternations rises steeply in per capita income. The intuitive explanation is that when incomes are higher people care less about increasing them through violence, and if the cost of violence is constant, above some income level they obey even if they lose (Benhabib and Przeworski 2006, Przeworski 2005).
3 Generic model and related literature

To develop the central intuitions, consider the relation between electoral chances and the relations of military force in a highly simplified context.

Suppose two parties are in conflict over some policy. The policy can be determined by an election, in which one party has a probability $p$ of winning, or by fighting, in which this party has a probability $q$ of prevailing. If the policy is determined by the election, this party gets $W$ when it wins and $L$ when it loses. In an armed conflict, it gets $W^*$ when it wins and $L^*$ when it loses. Because in elections both parties must move toward the center, it must be true that $W^* > W > L > L^*$. Hence, this party faces two lotteries, with expected values given by

$$E_p U(Election) = pW + (1-p)L$$

and

$$E_q U(Fight) = qW^* + (1-q)L^*.$$  

Some conclusions are already obvious. If the party expects to gain more from fighting than from electoral victory, $E_q U(Fight) > W$, it reverts to force even if it would win the election, so it may not even bother to hold the election as the incumbent. Simple algebra shows that this will be true if $q > (W - L^*)/(W^* - L^*) \equiv \overline{q}$. In turn, the party rebels against the outcome of elections when it loses if $q > (L - L^*)/(W^* - L^*) \equiv \underline{q}$. Hence if $q > \overline{q}$, this party always fights, if $q < q < \overline{q}$, it fights when it loses but not when it wins the election, and if $q < \underline{q}$, it obeys the result of the election even when it loses.

Comparing the two lotteries ex ante shows that this party prefers the electoral lottery if

$$p > \frac{L^* - L}{W^* - L^*} + q \frac{W^* - L^*}{W - L}.$$
Because the expression multiplying $q$ is positive, this party prefers elections only if its chance of winning increases with its prowess to prevail in a violent conflict. And because the same is true for the other party, elections are preferred by both parties only if electoral chances remain within some interval that moves with the relative military force of the contenders. Such an interval and the associated outcomes are illustrated in Figure 1. Whether this interval exists and, if it does exist, its locus and its shape depend on the ideal points of the particular pair of contenders and on the electoral platforms they would offer.

![Figure 1: The relation between $q$, $p$, and the outcomes.](image)

To understand the role of intertemporal considerations, suppose now that the party which lost the current election knows that it may win the next one with probability $p$. If this party obeys the result of the current election, it gets $L$ during the current period and can expect to get the value of the electoral lottery, discounted at $0 < \rho < 1$, during the next period. Hence, the value of obeying in case of electoral defeat is

$$V_p(Obey) = L + \rho[pW + (1 - p)L].$$

The alternative is to rebel and face the conflict lottery, the results of which last during the two periods, so that

$$V_q(Fight) = (1 + \rho)[qW^* + (1 - q)L^*].$$

The condition for the electoral loser to obey is now
Hence, a party that would not tolerate an electoral loss if the world ended at the current period, may accept the defeat and take its chances in the next election if it has some positive chance to win. And because the same is true again of the other party, a peaceful equilibrium may ensue when parties think in intertemporal terms.

Now, in the model that follows the instrument under the control of each incumbent is the probability with which it wins the next election, while the electoral platforms and thus the resulting policies are taken as fixed. This approach differs from several models of dynamic bargaining, in which $p$ is taken as fixed and the winner of the current election either unilaterally chooses the electoral platform (as in Alessina 1988 as well as Dixit, Grossman, and Gal 2000, where there is no rebellion alternative, or as in Fearon 2006, where there is) or makes a policy offer which the opposition can accept or reject (as in Powell 1999, Londregan and Vindigni 2006, or Little 2009, where rejecting is equivalent to rebelling). Because parties care only about the expected values, it should not make no logical difference whether one treats the probability that the incumbent wins as given and the policy as an instrument or the policy platform as fixed and the probability of reelection as an instrument. But my focus on $p$ is premeditated.

The focus on policy choice entails an image of elections in which parties propose policies, voters choose parties, and parties assume or leave office according to the verdict of the polls. Incumbent advantage, for example, is explained in some such models (Banks and Sundaram. 1993) by the risk aversion of voters, who prefer the same platform offered by already known candidates. But the exclusive focus on the voters’ choice between policy platforms is blinding. There is much more to elections than voter’s choice: incumbents intimidate, manipulate the rules, and quite often revert to fraud in order to secure their victory. Unfortunately, the relative importance of platforms, of the threat of force, rules, and fraud in shaping the outcomes of elections cannot be in general distinguished empirically. Yet even if parties choose platforms and voters choose among parties, these choices are constrained and, I argue, the most binding constraint are the relations of physical force. That force is not manifest does not mean that it is absent: as someone (Sartre?) observed, "that streets are peaceful does not mean there is no violence." Thus, the intent of the paper is programmatic: to insert the study of elections in a context in which political forces have more instruments than just the choice of policies.
Finally, one may also consider the possibility that the dictator who comes into power by force does not implement his ideal policy but makes some concessions, as in Grossman and Noh (1990), McGuire and Olson (1996), Ginkel and Smith (1999), Bertocchi and Spagat (2001), or Gandhi and Przeworski (2006). But as long as the dictator gets more than he would have received as an electoral winner – and why else would anyone want to be a dictator? – this modification would not change much.

4 When can elections be peaceful?

4.1 Assumptions

There is a set of potential contenders for power, \( j \in \{A, B, \ldots\} \). Contenders have preferences over policy \( x \in R \). Each contender is characterized by an ideal point, \( x^*_j \), drawn from some distribution \( f(x^*_j) \) with the support \( x \leq 0 \leq \pi \) and the mean \( \mu \). Utilities depend on the distance between the ideal and the implemented policy, specifically, \( U_j(x^*_j, x) = -d(x^*_j, x) = -|x^*_j - x| \). If a violent conflict occurs, the loser also bears the cost of being dominated by force, which is \( c > 0 \). The process occurs over discrete time, \( t = 0, 1, \ldots \).

At some initial time, \( t = 0 \), nature draws a pair of contenders and an election occurs, won by party \( I \) and lost by party \( O \). In this election, as in all subsequent ones, parties offer platforms \( x_j \). Regardless of the view of electoral competition one takes, it must be true that the electoral platforms are located somewhere between their ideal points. For

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4 Clearly, a historically fascinating question is what political forces fight about. My impression is they have typically fought over wealth, religion, and pure power, and that in many concrete situations these motives are hard to distinguish. Unfortunately, this topic is too complex to be discussed here.

5 Several results reported below depend on the distance between the ideal points of particular pairs of contenders. These results hold for a class of concave utility functions but risk aversion would make the conditions for peaceful elections less stringent.

6 The limitation of the model to two contenders at a time has costs: the model cannot handle situations in which two parties compete in elections while some third political force wages armed conflict.

7 However one thinks of electoral competition – whether parties care only about winning or also about the welfare of their constituents, whether they know everything or only something, whether there are two or more of them, whether they compete in one dimension or in several – they can win only if they propose platforms somewhere in the political center. See Downs (1957) for victory-seeking parties in one dimension, Lindbeck and Weibull (1987) for a model with uncertainty, Wittman (****) for policy-oriented parties, Austen-Smith (2000) for a model with multiple parties, Roemer (2001) as well as Osborne and Slivinsky (1997) for competition in several dimensions.
simplicity, I assume that parties propose platforms\(^8\)

\[
x_j = \frac{x_j^*}{\overline{x} - \underline{x}}
\]

In each election some rules determine who is the winner, given the outcome of voting.

Once the outcome of the election is revealed, each contender decides whether to obey its result or to use force ("revolt" or "rebel" or simply "fight."). If the election is not followed by a revolt, the winning platform is realized and a new election is held.\(^9\) If a violent conflict occurs, the winner implements his ideal policy and may or may not hold another election. Specifically, if the current incumbent would rebel regardless of the outcome of an election, no elections are held. If no election takes place, revolts occur with the periodicity of elections. The loser of the conflict returns to the pool of contenders (or disappears: it makes no difference as long as the number of contenders is sufficiently large) and a new contender takes his place.

If an election is to be held, the incumbent chooses the probability with which he will win this election, \(p_I\). This probability depends on the platforms \(\{x_I, x_O\}\) but also on a set of instruments, \(e_I\), discussed below, that are controlled by the incumbent. Hence, \(p_I = p(x_I, x_O; e_I)\). The technology \(e\) is such that the incumbent can generate any \(p_I \in [0, 1]\).

Each incumbent is characterized by the military capacity \(q_I\), which is probability that the incumbent defeats any challenger in the eventuality of a revolt. This capacity is not observed but everyone, including the incumbent, believes that it is \(q\), which need not equal \(q_I\). Each new incumbent who enters power by force is believed to have \(q = q(0)\), not necessarily the same for different entrants. Each time the current incumbent defeats a new challenger, all contenders, including the incumbent, update their beliefs according to

\[
q(n + 1) \geq q(n),
\]

\(^8\)I also assume that \(x_I \neq x_O\): platforms do not converge and elections make a difference. If they do converge, \(p\) plays no role in decisions whether to obey or revolt. Hence, this model is more general than those of Przeworski (2005) and Benhabib and Przeworski (2006). In turn, it is shown below that as long as the platforms bear some monotonic relation to the ideal points, the specifics of the electoral equilibrium do not matter for the qualitative conclusions.

\(^9\)As will become apparent below, platforms are credible because probabilities of winning are chosen optimally by incumbents given the platforms of both parties and parties care only about expected values, which depend on these probabilities and the platforms.
where \( n \) is the number of challengers the incumbent already defeated. The specific updating rule does not matter at this point.\(^{10}\)

Everyone is forward-looking but myopic, in a specific sense that actors do not anticipate that their beliefs may change. As a consequence, when calculating the expected value of revolt, the contenders do not consider the possibility that if the current incumbent defeats successive revolts, elections may occur without revolts.

The first strategic problem of the incumbent is thus to decide with what probability of winning he wants to run the election, given that the opposition may rebel if this probability is excessively high. But unless \( p = 1 \), the opposition may win and become the new incumbent. Hence, while choosing his own probability of winning, the current incumbent must anticipate the probability that the opposition would choose as the incumbent. Because the same is true for the current opposition, both the current incumbent and the current opposition can determine the values they can expect to receive by obeying. Once these values are determined, the incumbent and the opposition decide whether to obey or to rebel. The timing diagram is attached at the end.

Let \( j \in \{I, O\} \) now indicate the current and \( k \in \{I, O\} \) the hypothetical incumbency status. The strategy of each contender is an action \( a_j^k \in \{\text{Obey}, \text{Rebel}\} \), \( \{o, r\} \) for short, and the probability with which they win as incumbents, \( p_j^I \).

### 4.2 Value functions

To ease the language, it is useful to distinguish the contenders independently from their incumbency status. Let the current pair of contenders be \( j \in \{L, R\} \) and their incumbency status \( k \in \{I, O\} \). In principle, values must be defined for each party in each incumbency status given its own actions and the actions of the other party in each incumbency status. These values are thus \( V_j^k(a_L^I, a_O^O; a_R^O) \). For example, one such value is \( V_L^I(o_L^I, r_O^O; r_R^O, o_R^O) \): the value of \( L \) as the incumbent if he obeys as an incumbent and rebels in opposition given that \( R \) rebels as an incumbent and obeys in opposition. There are \( 2 \times 2^4 = 32 \) such values for each party. I first focus on situations in which the actions of both parties in both incumbency statuses are the same and denote the values in such situations by \( V_j(a), a \in \{o, r\} \).

Letting \( p_j \) be the probability party \( j \) wins the next election, the values for party \( L \) when both parties obey in both incumbency statuses are then given by

\[^{10}\text{For example, let } q \sim \text{Beta}(a, b; n) \text{ and let the updating process be Bayesian, with the expected value } E(q|n) = (a + n)/(a + b + n).\]
\[ V^I_L(o) = U_L(x_L) + \rho p_L V^I_L(o) + \rho(1 - p_L)V^O_L(o) \]  

\[ V^O_L(o) = U_L(x_R) + \rho(1 - p_R)V^I_L(o) + \rho p_R V^O_L(o) \]  

If \( L \) is the incumbent and both parties obey, \( L \) gets \( U_L(x_L) \) during the current period and then faces an electoral lottery in which it remains the incumbent with the probability \( p_L \) and becomes the opposition with the probability \( (1 - p_L) \). In turn, if \( L \) is currently in opposition, it gets \( U_L(x_R) \) during the current period and faces a lottery in which it may become the incumbent with the probability \( 1 - p_R \), chosen by party \( R \).

Solving for the present values yields

\[ V^I_L(o) = \frac{U_L(x_L) + \rho(1 - p_L)V^O_L(o)}{1 - \rho p_L} \]  

\[ V^O_L(o) = \frac{U_L(x_R) + \rho(1 - p_R)V^I_L(o)}{1 - \rho p_R} \]  

and making the substitutions

\[ V^I_L(o) = \frac{(1 - \rho p_R)U_L(x_L) + \rho(1 - p_L)U_L(x_R)}{(1 - \rho) \left[(1 - \rho p_R) + \rho(1 - p_L)\right]} \]  

\[ V^O_L(o) = \frac{\rho(1 - p_R)U_L(x_L) + (1 - \rho p_L)U_L(x_R)}{(1 - \rho) \left[(1 - \rho p_R) + \rho(1 - p_L)\right]} \]

Note that if \( x_L = x_R = x \), that is, parties adopt the same platform, the probabilities of winning elections do not matter and \( V^I_L(o) = V^O_L(o) = U_L(x)/(1 - \rho) \).

To derive the value of rebelling, think as follows. If \( L \) wins the current rebellion, it gets \( U_L(x^*_L) \) during the current period and a discounted continuation value. If it loses, it is out of power and \( R \) is in, so that during this period \( L \) gets \( U_L(x^*_R) \). But \( R \) can be deposed and replaced by someone else. And because the someone else can be deposed as well, \( L \) can expect that over the long run it will be governed by a contender with the expected ideal policy \( x^*_j = \mu \). Because losers in a rebellion also suffer a cost \( c \) in each period after they had lost, \( L \) can thus expect to get approximately \( U(\mu - c) \) if he rebels and is defeated.\(^{11}\) Thus,

\[ V_L(r) = \frac{q U_L(x^*_L) + (1 - q) U_L(\mu - c)/(1 - \rho)}{1 - \rho q} \]

\(^{11}\)One can either think that a defeat in a rebellion costs the loser some instantaneous quantity \( C = \sum \rho^t c \), so that \( c = (1 - \rho)C \) or that being ruled by force costs \( c \) per period.
But \( U_L(x_L^*) = 0 \). Hence,

\[
V_L(r) = \frac{1 - q}{1 - \rho} \frac{U_L(\mu - c)}{1 - \rho}
\]  

(7)

Analogously for \( R \), skipping some steps,

\[
V'_R(o) = \frac{U_R(x_R) + \rho(1 - p_R)V'_R(o)}{1 - \rho p_R}
\]  

(8)

\[
V'_R(o) = \frac{U_R(x_L) + \rho(1 - p_L)V'_R(o)}{1 - \rho p_L}
\]  

(9)

\[
V_R(r) = \frac{q}{1 - \rho(1 - q)} \frac{U_R(\mu - c)}{1 - \rho}
\]  

(10)

## 4.3 Equilibrium

Each contender must make three decisions, namely, to choose \( p \) and actions as the incumbent and as the opposition. Formally, the problem of the current incumbent is

\[
\max_{p_I, a_I^I, a_I^O} V_I(p_I, a_I^I, a_I^O; a_O^I, a_O^O, p_O)
\]

s.t. \( V_O = \max_{p_O, a_O^I, a_O^O} V_O(p_O, a_O^I, a_O^O; a_I^I, a_I^O, p_I) \)  

(11)

and the problem of the current opposition is symmetrical.

Each current incumbent solves this problem anticipating the reaction of the current opposition to each of its actions. Hence, each party is a Stackelberg leader as the incumbent. The solutions do not depend on past history, because only the current value of \( q \) enters in determining these values. Hence, the solutions to this dynamic game are pairs of subgame-perfect asymmetric Nash-Markov equilibria.

Characterizing these equilibria requires several steps.

(1) The current incumbent \( I \) knows the value of \( p_I \) for which the current opposition \( O \) is indifferent between obeying and rebelling. Let this value be \( \max p_I \). This value depends, in turn, on the value of \( p \) that the current opposition chooses as the incumbent, \( p_O \). Because \( \max p_I \) is the best reply to any \( p_O \), it must be true that \( p_I = \max p_I \). And the current opposition cannot commit itself not to use \( \max p_O \), which is the best reply to any \( p_I \). Hence, this part of the equilibrium is \{\( \max p_I, \max p_O \}\}. Because the functions \( \max p_I(p_O) \) and \( \max p_O(p_I) \) are linear, this equilibrium is unique, although not necessarily bounded by \([0, 1]\). If either
max \( p \geq 1, p = 1 \), and the incumbent establishes a one-party system. If either \( max \ p < 0, p = 0 \), and the opposition rebels.

(2) While by construction the current opposition is indifferent between obeying and rebelling when \( p_I = \max p_I \), these values are not sufficient to guarantee that the incumbent obeys or that the opposition would obey as the incumbent. To decide whether to obey or rebel the incumbent must consider what action the opposition would take as the incumbent as well as its own action in response to each action of the opposition as the incumbent. Potentially, there are nine values to compare, but the current incumbent knows the best reply of the opposition to its actions as the incumbent, \( a^O_O(a^I_I) \), the action of the opposition as the incumbent given the action of the current incumbent, \( a^I_O(a^I_I) \), as well as his own reaction in opposition in response to the action of the current opposition as the incumbent, \( a^O_I(a^O_O) \).

Because all these best replies are known, the incumbent must compare only two values: \( V_I(o^I_I, p_I; o^O_O(a^I_I), a^I_O(a^I_I)) \) and \( V_I(r^I_I; a^O_O(r^I_I), a^I_O(r^I_I), a^O_I(a^I_I)) \). Hence, each incumbent compares the values of obeying and rebelling given the anticipated actions of the current opposition as the opposition and as the eventual incumbent as well as his own actions when in opposition. Because the same reasoning applies to the current opposition as the incumbent, these comparisons determine the actions \( a^I_I \) and because all other actions are direct or indirect functions of \( a^I_I \), they are all determined. It may seem farfetched that anyone would engage in such calculations but this is precisely what the Algerian government must have done when it annulled the second round of the 1992 election which the Islamic party, FIS, appeared to be winning.

**Definition 1** The equilibrium consists of two pairs of actions \( \{a^I_I, a^O_O\} \) that are best replies to each other and, if elections are held, of probabilities with which incumbents win elections \( \{\max p_I, \max p_O\} \). Hence, the equilibrium is characterized by \( \{(a^I_I, a^O_O), (a^O_O, a^I_I); \max p_I, \max p_O\} \).

### 4.4 Conditions for an electoral equilibrium

We can now characterize different equilibria that are of substantive interest and identify the conditions under which they occur. First the definitions:

\[V_I(o^I_I, o^O_O; a^I_I, a^O_O), V_I(o^I_I, o^O_O; r^I_I, o^O_O), V_I(o^I_I, o^O_O; r^I_I, r^O_O), V_I(r^I_I, o^O_O; a^I_I, a^O_O), V_I(r^I_I, o^O_O; r^I_I, a^O_O), V_I(r^I_I, o^O_O; r^I_I, r^O_O), V_I(r^I_I, r^O_O; o^I_I, o^O_O), V_I(r^I_I, r^O_O; o^I_I, r^O_O), V_I(r^I_I, r^O_O; r^I_I, o^O_O), V_I(r^I_I, r^O_O; r^I_I, r^O_O)\]
Definition 2 An equilibrium is peaceful if at least one contender obeys in both incumbency statuses. Otherwise it is violent.

Note that an equilibrium is peaceful either if both parties obey in both incumbency statuses or if one party imposes itself by force and the other party accepts it without fighting.

Definition 3 An peaceful equilibrium is electoral if both parties obey results of elections in both incumbency statuses.

Hence peaceful electoral equilibria are a subset of peaceful equilibria in which elections are held. Yet events called "elections" can be held even when the opposition has no chance to win.

Definition 4 An electoral equilibrium is competitive if $\max p_j < 1$ for both parties.

Each of these equilibria occurs within some range of the relative military strength of the incumbent, $q$. To identify these ranges, we need to determine first the thresholds that define them and then to examine under what conditions these ranges are non-empty. In the rest of this section, it is assumed that $q$ characterizes the current incumbent whether he wins or loses an election, that is, as the incumbent and in opposition. Also, just as a convention, let $\overline{x} \leq x_I^* < \mu < x_O^* \leq \overline{x}$, where $\mu$ is the mean of the distribution of $x_j^*$.

Lemma 1 (1) If $q'' < q < q^*$, the current incumbent holds an election and the current opposition obeys. (2) If $q^{**} < q < q'$, the current opposition would hold an election as the incumbent and the current incumbent would obey.

Proof. All longer proofs are in the Appendix. $\blacksquare$

Here is the intuition. The current incumbent, which is party $I$, holds an election implementing $\max p_I$, unless it can safely impose itself by force, which is true when $q > q^*$. The opposition, party $O$, may not obey the result even when $I$ is ready to abdicate after one term, that is, when $\max p_I = 0$, if $I$ is militarily weak, namely, if $q < q''$. The reason is that when $I$ is so weak, $O$ does not want to suffer from policy $x_I$ even during one period. The same is true when $O$ is the incumbent. $O$ would hold an election if $I$ is sufficiently strong, $q > q^{**}$ (remember that $q$ characterizes the original incumbent in and out of office). In turn, $I$ would rebel even if $\max p_O = 0$ when it is sufficiently strong, namely, if $q > q'$.\(^{13}\)

\(^{13}\)One may think that this result is due to the fact that $p \in [0, 1]$ while $x_I$ is fixed and that a peaceful equilibrium could be always reached if the incumbent were allowed to make policy compromises. There are, however, conditions on $q$ when this is not true: given $q$, the incumbent would rather rebel than make such compromises.
Lemma 2 A sufficiently extreme current incumbent, \( x_I < -c \), establishes a one-party system, setting \( \max p_I = 1 \), at some \( q^- \leq q^* \). The corresponding condition for the current opposition is that \( \max p_O = 1 \) at some \( q^- > q^{**} \) if \( x_O > c \).

To remind, a non-competitive electoral equilibrium is not the same as a peaceful equilibrium in which one party imposes itself by force and the other obeys. Even in one-party systems, the incumbent who conducts elections gets only the value of his electoral platform, while if the incumbent rules by force it gets its ideal policy. Hence, the incumbent may rebel even if it is sufficiently strong to maintain a one-party rule. Note that, following the pioneering study by Gandhi (2009), there is rapidly accumulating evidence that dictatorships that hold elections are less exploitative than those that do not.

Proposition 1 just puts these partial results together.

Proposition 1 If \( q \) does not depend on outcomes of elections and if \( q^{**} < q < q^* \), incumbents hold elections, implementing \( \{\max p_I, \max p_O\} \). Election results are obeyed by both parties in opposition if \( q^- < q < q^* \). The electoral equilibrium is competitive if \( q^- < q < q^- \).

Suppose that \( 0 < q^{**} < q'' < q^- < q^- < q' < q^* < 1 \). Both incumbents hold elections, if \( q^{**} < q < q^* \). Both parties obey the outcome of the election, that is, the electoral equilibrium is peaceful if \( q^{**} < q'' < q < q' < q^* \). A peaceful electoral equilibrium is competitive if \( q^{**} < q'' < q^- < q < q^- < q' < q^* \).

Which equilibria exist depends on the relative ordering of these threshold values. Consider first the relation between \( q^* \) and \( q^{**} \).

Lemma 3 A necessary condition for \( q^{**} < q^* \) is that the range of ideal points from which the contenders are drawn, \( D(x_j^*) = \bar{x} - \underline{x} \), is not too large. In particular, if a defeat in a revolt is costless, \( c = 0 \), then \( q^{**} < q^* \) only if \( D(x_j^*) < \frac{2q^*}{1-q^*} \). If \( c > 0 \), this range is broader and it also depends on the policy distance between the particular contenders.

Hence, an electoral equilibrium does not exist if the polity from which the contenders are drawn is highly polarized and the costs of fighting are low. The reason the range of support of ideal points, \( D(x_j^*) \), matters is that defeated contenders must expect that the current victor may be in turn deposed by someone with more extreme preferences. In turn, \( q^* \to 1 \) as \( x_I^* \to \mu \) and \( q^{**} \to 0 \) as \( x_O^* \to \mu \), which implies that if both the incumbent and the opposition are sufficiently moderate (their ideal points are close to the mean ideal point), they never rebel as incumbents.
Now, it is obvious that $q' < q^*$ and $q^{**} < q''$: because each party is worse off in opposition than as the incumbent, it is willing to take the gamble of violent conflict with a lower chance of prevailing. But it may or may not be true that $q' > q''$. Suppose that $q^{**} < q' < q'' < q^*$. If $q'' < q < q^*$, the current incumbent holds an election and the current opposition obeys, but the current incumbent does not obey when in opposition. Hence, elections are peaceful only as long as the current incumbent continues to win. In turn, if $q' < q < q''$, both parties hold elections as incumbents but neither obeys as opposition.

Whether peaceful electoral equilibria exist depends on the ideal points of the particular pairs of contenders. Specifically, it can be shown that $q' \to 1$ and $q'' \to 0$ as the two parties become less extreme. Hence,

**Lemma 4** $q' > q''$ if at least one contender is moderate.

The two lemmas imply that a polity can hold peaceful elections if the range of policy preferences is not too large and if one of the particular contenders is not too extreme in relation to the other. Moreover, we know that elections are competitive if $q^{**} < q < q'$. Conversely, the same values of $q$ can lead to different – peaceful or violent – equilibria, depending on the degree of political polarization and the postures of the current contenders.

**Proposition 2** A peaceful electoral equilibrium exists if the polity is not too polarized and if at least one party is moderate.

Note that as long as the electoral platforms are some monotonic function of the ideal points all the above results hold for any electoral equilibrium in which platforms do not converge.

To get a sense of the dynamic process, assume that the initial state is $q(0) < q'' < q^*$, in which the current incumbent holds an election but the opposition rebels. If the incumbent defeats some number of consecutive revolts, the challengers learn that it is stronger than they had thought and the process moves to a state $q'' < q < q^*$, in which election results are obeyed. Here is a numerical example.

**Example 1** Suppose $q(0) = 0.4$, $q'' = 0.54$, $q^* = 0.75$, the incumbent is $A$, the opposition $B$. $A$ holds an election and the opposition rebels. Suppose that $A$ defeats $B$, $q$ increases to $q(1) = 0.5$,\footnote{This will be true if $q \sim \text{Beta}(2, 3; n)$, generating posteriors $E(q|n) = (2+n)/(2+3+n)$.} and a new contender, $C$, emerges to take $B$’s place. $A$ holds another election and a revolt occurs...
again. Now C defeats A, becomes the new incumbent, with beliefs about his power reverting to $q(0)$, and D is the new opposition. This process continues until the incumbent $L$ defeats two successive contenders and when the current challenger is $R$, $q$ rises to $q(2) = 0.55 > q^*$. Given $q(2)$, an election occurs with $\max p_L = 0.8, \max p_R = 0.76$ and everyone obeys. The incumbent $L$ wins $1/(1 - \max p_L) = 5$ successive elections.

### 4.5 Other equilibria

Suppose that $q > q^*$ or $q < q^{**}$, so that either the current incumbent rebels or the current opposition would rebel as the incumbent. If the opposition obeys when the current incumbent rebels, the opposition gets $U_O(x^*_I)/(1 - \rho)$; if the current incumbent obeys in opposition when the current opposition rebels as the incumbent, the current incumbent would get $U_I(x^*_O)/(1 - \rho)$. If they rebel, they get their expected values of revolt.

**Lemma 5** If costs of defeat are relatively high and if the rebelling incumbent is relatively moderate, the opposition accepts the rule of the incumbent without elections.

The intuition is that if either contender rebels in opposition and is defeated, someone else may revolt and replace the current ruler. If the current ruler is moderate, it may be better for the opposition to tolerate this ruler than to risk being defeated at the cost $c$ and face the prospect of a more extreme ruler.

Now, suppose that the current incumbent knows that it will obey if the current opposition rebels as the incumbent. Will the current incumbent obey as the incumbent?

**Lemma 6** If the costs of defeat are relatively high and if the current opposition is relatively moderate, the current incumbent obeys even if the current opposition would rebel as the incumbent and the current incumbent would obey in opposition. If costs are lower or the opposition is more extreme, the current incumbent rebels as the incumbent.

We can now establish the conditions when equilibria are peaceful and violent.

**Proposition 3** If the power of the incumbent is very high or very low, $q > q^*$ or $q < q^{**}$, and (1) if the party that rebels is moderate, equilibrium is peaceful, (2) if the party that rebels is more extreme, the equilibrium is violent.
This proposition says that if one of the parties is militarily strong and moderate in terms of its policy preferences, it imposes its rule by the mere threat of force without having to apply it, because the opposition does not want to engage in a potentially costly fight that may result in a more extreme ruler taking power. If the current incumbent is more extreme, however, then the opposition takes the risk of rebelling if the incumbent rebels.

4.6 Partisanship of the military

The analysis above was based on the assumption that the military power of the incumbent is not affected by outcomes of elections. But one can easily imagine that an electoral defeat changes the actual relations of force and the beliefs about them.

Let the first time when an election does not generate a revolt be \( t = T \), such that everyone believes that \( q^{**} < q'' < q' < q = q_T < q^* \). Under these conditions the current incumbent holds an election \( (q_T < q^* ) \) and the opposition obeys \( (q_T > q'' ) \) but if the opposition were to win it would hold an election \( (q_T > q^{**} ) \) and the former incumbent would rebel \( (q_T > q' ) \). As long as the incumbent wins, there is no reason for beliefs to change, so that the incumbent could repeatedly hold elections which the opposition would obey. But suppose the incumbent loses. What are the contenders to believe about their respective military strength? This question matters because there may be situations in which elections are peaceful only if the same party always wins, so that partisan alternation in office cannot be peaceful.

Consider two extreme possibilities. In one, beliefs do not change: in all subsequent elections not followed by a revolt everyone continues to believe that the original incumbent’s strength is \( q_T \). This is what we assumed thus far. At the other extreme, all winners of elections are believed to have the same coercive power. Hence, if the original incumbent entered with \( q_T \) and is defeated, everyone believes that the power of the new government is \( q_T \).

Let \( s \) be the, common knowledge, loss of coercive power that an incumbent suffers from a defeat in an election, so that when it loses everyone believes that its strength is \( q_T - s \). If an electoral defeat does not affect relations of military power, \( s = 0 \). In turn, if all winners of elections have the same coercive power, the defeated incumbent knows that the power of the new government must be \( q_T \) and, because it must be true that \( 1 - (q_T - s) = q_T, s = 2q_T - 1 \). Hence, \( 0 \leq s \leq 2q_T - 1 \) characterizes the loss of coercive power experienced by a government defeated in an election.
An electoral defeat may reduce the coercive power of the incumbent because it enables citizens to coordinate their resistance. In Fearon’s (2006) view, an electoral defeat reveals to isolated individuals that some large numbers of others are dissatisfied with the incumbent and thus provides a signal that a coordinated revolt would be effective when the incumbent seeks to remain in power. Tucker (2009) applied this logic to explain "color revolutions." An alternative mechanism is that electoral defeats induce splits within the ruling block (Magaloni 2007). When a ruler holds an election and is unable to secure a victory, members of the coercive apparatus must envisage the possibility that the ruler may fall and they may be held individually responsible for repression. Hence, they become hesitant to take this risk (Przeworski 1988): after General Pinochet lost the 1989 plebiscite to extend his term, the other members of the Chilean junta refused to override the result by force (Barros 2002).

While either or both mechanisms may be at work, I interpret the loss of power in terms of the postures of the repressive apparatus. Thus, incumbents lose no power when the repressive apparatus is completely partisan, standing behind the same party in or out of office. In contrast, all elected incumbents have the same power when the coercive forces are perfectly neutral. Under this interpretation, \( s \) measures the partisan neutrality of the coercive forces (or, if they are divided along partisan lines, the balance of force between them).

Suppose the military are unconditionally partisan, \( s = 0 \). The incumbent loses an election and is now in opposition, so that under the conditions stipulated above it rebels. In turn, suppose that the military are perfectly constitutionalist, \( s = 2q_T - 1 \). Now it may be true that even though \( q^{**} < q'' < q' < q_T < q^* \), when the military shift their support the condition becomes \( q^{**} < q'' < q_T - s < q' < q^* \) and the defeated incumbent obeys.

There is thus a range of \( s \) such that results of all elections are obeyed, so that partisan alternations are peaceful.

**Proposition 4** (1) If the posture of the military is such that \( q_T - q' \leq s \leq 2q_T - 1 \), the incumbent which enters with \( q_T \) obeys the result of an election in which it is defeated. (2) The party that entered as opposition accepts its electoral defeat as the incumbent if \( s \leq q_T - q'' \). (3) Hence, if \( q_T - q' \leq s \leq q_T - q'' \), all subsequent alternations are peaceful. (4) This state is absorbing.

**Proof.** Obvious. Note only that this state is absorbing because once elections are obeyed, the contenders do not change, so that although threshold values depend on the ideal points of particular contenders, they also remain the same.
Example 2 Suppose the incumbent $L$ loses an election when $q_T = 0.55$ and $q' = 0.5$. Then if the military continue to support the defeated incumbent, $s < 0.05$, the incumbent rebels. If, however, the military are sufficiently neutral, $0.05 < s < 0.10$, the incumbent accepts the defeat. In turn, if the former opposition loses a subsequent election, it accepts the defeat as long as $q'' \leq 0.55 - s$.

Partisanship of the coercive apparatus thus places a wedge between the coercive power of the state and the repressive capacity of the particular incumbents. States may be highly effective in organizing and monopolizing the coercive force, yet those who wield this force may be non-partisan, constitutionalist, so that the control over the repressive apparatus rests in the hands of elected civilian governments: such states are characterized here by a high $q$ and a high $s$. For example, General Fidel Ramos in the Philippines supported Corazon Aquino after President Ferdinand Marcos was caught at having committed fraud in the election of 1985 and peaceful competitive elections ensued. In turn, other states may be able to utilize powerful repressive forces for partisan purposes: they have a high $q$ and a low $s$. An example is the Russian army executing the order of President Boris Yeltsin to shell the parliament. But some states are just coercively weak. "Failed states" are precisely those whose military power can be easily contested: they have a low $q$ and thus necessarily a low $s$. In such states the electoral incumbents cannot prevent armed challenges even if the military forces are partisan, simply because the state as an organization cannot muster enough physical force.

Facing "unreliable" coercive apparatus, however, the incumbent is less prone to hold competitive elections.

Proposition 5 When an electoral defeat of the incumbent causes his military power to fall, the incumbent does not hold elections at $q^*(s) < q < q^*$.

Here is the intuition. The incumbent knows that if he loses an election, in opposition he will obtain the equivalent of the value of rebellion. But this value will reflect the loss of his coercive power. Hence, if the incumbent was exactly indifferent between holding an election and rebelling with his original power intact – the equality that defines $q^*$ – he will be indifferent at a lower value of $q$ when he faces the prospect of losing $s$. Moreover, the larger the $s$, the lower the $q^*(s)$. Hence, rulers in states that have a centralized, effective coercive apparatus but who fear that their control would be eroded if they lose an election are less inclined to hold elections at all.
4.7 Income dependence

Suppose that the utility functions $U_j(x^*_j, x; y, c)$ are of the form $-d_j(x^*_j, x)/y - \delta c$, where $\delta$ indicates whether a contender lost in a violent conflict. Intuitively, this assumption says that the distance to the ideal point matters less in countries with higher per capita income, $y$.

**Proposition 6** Given $U_j(x^*_j, x; y, c) = -d_j(x^*_j, x)/y - \delta c$, $q^* = q^*(y)$ and $q^{**} = q^{**}(y)$, where $\partial q^*/\partial y > 0$ and $\partial q^{**}/\partial y < 0$. Moreover, $\lim_{y \to \infty} q^* = 1$ and $\lim_{y \to \infty} q^{**} = 0$.

As income increases, the incumbent obeys when he has more military power and the opposition obeys when it believes that the incumbent has less military power. When income is sufficiently high, both obey regardless of their military power. Note that because both thresholds depend on $x^*_j$, some contenders may obey while others would rebel, at the same income level. But the shadow of violence fades in economically developed societies.

The intuition behind this result is that when the contenders are wealthier, they value less whatever they can gain by violence. Because violence is potentially costly, at some income level the cost of fighting becomes larger than its marginal benefit (Przeworski 2006).

Here is a story. There was an election in Costa Rica in 1948, when that country had per capita income of about $1,500 (1985 PPP, from PWT5.6). The election was technically tied: the two candidates received almost the same number of votes and there were widespread allegations of fraud, so that it was impossible to determine who in fact did win. It was not clear who should decide, but the Congress took it upon itself to declare as the winner the candidate who officially received somewhat fewer votes. A civil was ensued, in which about 3000 people were killed. At another time, there was an election in another country. The election was technically tied: the two candidates received almost the same number of votes and there were widespread allegations of fraud, so that it was impossible to determine who in fact did win. It was not clear who should decide, but the Supreme Court, appointed in part by one of the candidates father, took it upon itself to declare as the winner the candidate who officially received somewhat fewer votes. Then everyone drove home in their SUVs to cultivate their gardens. They had SUVs and gardens because this country has per capita income of about $20,000.
5 Why elections may be violent even if they need not be?

5.1 A general framework

As in many models of inefficient conflict, elections may not be peaceful when beliefs differ. While Londregan and Vindigni (2006) argue that votes can be used to estimate the military strength, this can be true only if the vote count is perceived as accurate. And there are reasons to believe that often it is not. While the logical implications of differing beliefs may be easy to intuit, the mechanisms that generate violence merit scrutiny.

Elections must inextricably follow some rules that regulate who can vote, whether voting is direct or indirect, secret or public, compulsory or voluntary, how votes are aggregated, and so on. And rules affect outcomes. Even minute details, such as the color of ballots, location of the polling places, or the day of the week when voting takes place can affect the result. Hence, elections are inextricably manipulated. Manipulation, however, can be more or less blatant. Somehow we feel that carving electoral districts in the form of a salamander is excessive, while making districts nicely square does not raise anyone’s eyebrows. Manipulation is a matter of degree, which is represented here by $m$. Think of $p_I(v, m)$ as the probability that an incumbent who enjoys support of $v$ percent of the citizens given $\{x_I, x_O\}$, wins the election having manipulated the rules to the extent $m$.

Manipulation consists of establishing rules under which elections are conducted. Hence, it is visible. But its consequences are not always easy to determine. Even political scientists need to revert to simulations in order to identify the effects of electoral systems. Consequences of rules are a matter of judgement. In Nicaragua the opposition decided at the last moment to boycott the election of 1984, believing that the Sandinista government had manipulated the rules to the point that the opposition had no chance. Yet in 1990 it won under the same rules.

Manipulation is not the same as fraud. Fraud entails violating rules, however biased they may be. Breaking into the office of the opposition

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15 Note that even “the opposition” can be a product of manipulation. For example, the incumbent may allow some parties to participate and ban others (Lust-Okar 2005: 27-28).

16 A question arises as to why an incumbent who can manipulate the rules and commit fraud would still want to move the electoral platform from his ideal point toward the center. The reason is that manipulation works only up to a point while fraud may backfire (see below).

17 On the difficulties of defining fraud, see Annino (1995: 15-18).
party to steal its secrets is fraud because it violates a general prohibition against burglary. Buying votes constitutes fraud when it is prohibited by specific rules. So is casting votes of people whose spirits have passed to a better world. And setting rules and breaking rules are subject to different reactions. The same physical act – a campaign contribution – has different political consequences when it is permitted by law and when it is illegal: “institutional facts have some autonomy with regard to brute facts” (Sánchez-Cuenca 2003: 81-82). The technology of fraud is highly varied (Simpser 2006, Lehoucq 2003), but in almost all of its forms fraudulent activity is clandestine.

Manipulation and fraud are substitutes: the same probability may be generated by a lot of manipulation with little fraud or by little manipulation with extensive fraud. But to some extent they are also complements, because rules can be manipulated to facilitate fraud. Let \( f \) stand for the degree of fraud. The probability that the incumbent wins is then \( p_I = p(v; m, f) \), increasing both in \( m \) and in \( f \).\(^{18}\) The difference is that, even if their consequences may be difficult to assess, rules are explicit and public, so that they are known before the election occurs, while fraud can be detected before the election (rigging voter rolls), during (preventing some people from voting), and after (while counting). Hence, one can expect that pre-election violence occurs when the opposition believes that the incumbent gives it too little of a chance to win (For a model in which smaller parties are more inclined to use violence, see Chaturvedi 2005). In turn, while post-election violence may be a belated reaction to the defeat caused by manipulation, it is more likely to result from discovery of fraud. Note that in Magaloni’s (2007) model, if fraud is unobservable and there is more than one opposition party, one of them may allege fraud even if there was none.

The fact is that, as Malamud (2000: 9) observes with regard to nineteenth-century Latin America, "... an important part of the revolutions, civil wars, and other types of coups were often related to electoral processes, in order to attempt to change the rules of the game before voting took place or to modify the popular verdict." Note, however, that even if elections concentrate violence, the inference that they cause violence is perilous. Schorr Newman (2010) shows that incidence of politically motivated violence increases in the proximity of elections:

Yet other modes of succession may be even more violent. As Rapoport and Weinberg (2000) observe, political succession may be just a violent

\(^{18}\)Gandhi and Przeworski (2009) use the function \( p(m, f) = m(1 + f^\alpha), \alpha \in (0, 1) \), with \( \partial p/\partial f = amf^{\alpha-1} > 0 \).
process whatever its modalities are. After all, Pinochet’s Chile or contemporary Saudi Arabia experienced a high level of political violence without holding elections. Hence, to identify the causal effect of elections on political violence, one has to ask about counterfactuals, and the answer is not obvious.

5.2 Analysis

Assume that the stage game has the following structure: (1) The incumbent decides whether to hold an election. Because we want to analyze the situation in which elections are be peaceful, assume that \( q < q^* \), that is, that he does hold them. (2) The incumbent chooses the extent of manipulation, \( m \). He can choose \( m \) such that \( p_I = p(v, m) \leq \max p_I \) or such that \( p_I = p(v, m) > \max p_I \). (2.1) If the opposition concludes that \( p_I = p(v, m) \leq \max p_I \), it participates in the election and the incumbent chooses the degree of fraud, \( f \), which again can be such that \( p_I = p(v; m, f) \leq \max p_I \) or such that \( p_I = p(v; m, f) > \max p_I \). (2.1.1) If the opposition believes that \( p_I = p(v; m, f) \leq \max p_I \), it obeys the outcome. (2.1.2) If the opposition believes that \( p_I = p(v; m, f) > \max p_I \), it rises in protest. (2.2) If the opposition concludes that \( p_I = p(v, m) > \max p_I \), it threatens to rebel. The incumbent then has two choices: (2.2.1) Reduce \( m \) and bring \( p_I = \max p_I \) down to \( \max p_I \), in which case the game enters into stage (2.1), or (2.2.2) maintain \( p_I > \max p_I \), and face a rebellion. Hence, electoral violence can occur either if the incumbent manipulates excessively or if he is caught having committed excessive fraud.

Suppose that beliefs about the military power of the incumbent differ: specifically, the incumbent believes that his strength is \( q^I > q_T \), where \( q_T \) is what the opposition believes. Consider first the manipulation stage. The incumbent chooses \( p_I = \max p_I(q_I) \), which is the value of \( p_I \) that would make the opposition indifferent had it believed that \( q = q_I \). But if the opposition believes that \( q = q_T < q_I \), the opposition rebels given \( p_I = \max p_I(q_I) \). Say the opposition threatens to boycott the election. There are two questions to investigate: (1) How much is the incumbent willing to reduce \( m \) and thus \( p_I \)? (2) What are the conditions for elections to be peaceful given divergent beliefs about \( q \)?

Proposition 7 When beliefs about \( q \) differ, \( q_I > q_T \), the incumbent is willing to reduce his probability of winning to \( \max p_I(q_T) < \max p_I(q_I) \) and the opposition obeys if (1) \( q_I \) is not too high and (2) the incumbent is not too extreme. The divergence of beliefs, \( q_I - q_T \), has only a small effect.

Corollary 1 Given that \( p_I = \max p_I(q_T) \) and that \( dp_I/dm > 0 \), there
is a maximum degree of manipulation, \( m^* \), under which the opposition participates in elections.

Given this corollary, to study fraud we can now use the model of Gandhi and Przeworski (2009), in which the degree of manipulation is constrained exogenously but the incumbent optimally chooses the extent of fraud, \( f^* \), thought of as the proportion of votes the incumbent falsifies.

Now, even if particular acts of fraud can be detected, the extent of fraud is not observable. Suppose that the opposition believes that, after all the manipulation, some proportion \( v_I(m^*) \) is certain to vote for the incumbent and some proportion \( v_O(m^*) \) is certain to vote against him. If the probability that the voters whose intentions are not known vote one way or another is uniformly distributed, the probability that the incumbent wins is \( p_I = \frac{0.5 - F(v_O)}{1 - F(v_O) - F(v_I)} \). Suppose that \( v_I = 0.4 \) and \( v_O = 0.3 \), so that \( p_I = 2/3 \). The results of the election are announced and the incumbent is declared to have won 60 percent of the vote. The opposition cannot tell if this result is accurate or fabricated. It may actually calculate that the expected vote share of the incumbent was 60 percent \((Ev_I = v_I + p_I \ast (1 - v_I - v_O))\) but it does not know what proportion of the 0.2 share the incumbent claims to have won among the voters whose intentions were unknown were cast as such. Even with the best monitoring technology, this is simply unknowable: think of 2000!

This is not to say that monitoring technology does not matter. Let \( r(f) \) be the probability that fraud of magnitude \( f \) is detected, with \( r'(f) > 0, r''(f) > 0 \). Given that the opposition is indifferent between obeying and rebelling when the extent of manipulation is \( m^* \), if the opposition detects fraud it rebels. Hence, \( r(f) \) is the probability that the opposition rebels if the incumbent declares himself to be the winner.

The results of Gandhi and Przeworski (2009) now hold in the present framework:

**Proposition 8** The extent of fraud increases in the degree of manipulation until the probability of detection becomes dissuasive. Formally, \( \frac{\partial f^*}{\partial m} \geq 0 \) if \( r(f^*) \leq r^*(q, s, \rho) \), where \( r^* \) is some critical value.

**Proposition 9** Incumbents with higher military power commit more fraud, \( \frac{\partial f^*}{\partial q} > 0 \).

**Proposition 10** The optimal degree of fraud is lower when the coercive forces are less partisan, \( \frac{\partial f^*}{\partial s} < 0 \).

**Proof.** See Gandhi and Przeworski (2009) \( \blacksquare \)
Note that if they can get away with it, that is, if they do not fear that fraud would be detected, incumbents commit some fraud even if they have a good chance to win without it. Secondly, contrary to Schedler (2002: 46), the capacity to repress is what enables fraud: an incumbent who is less afraid to be overthrown if fraud would be detected, engages in more of it. Finally, if the military are less partisan, that is, if they turn against an incumbent caught at fraud, the incumbent uses less of it.

6 Elections, violence, and competition

Why are conflicts processed through elections at all? Fearon (2006) is correct to point out that the desire for peace is not sufficient to justify elections. If everyone knows everything, then they also know the expected value of the policy outcome $x$ associated with $q$. Why then hold elections, rather than simply agree to $x = E_q(x_t)$, $\forall t$? Moreover, if utility functions are concave in $x$, agreeing to a fixed $x$ is superior in welfare terms to policies chosen by alternating parties (Alesina 1988). Hence, additional reasons must be adduced to understand why to be peaceful conflicts must be processed by elections. Przeworski (2005) argues that $x$ could not be completely specified and rulers would use their residual power to deviate from any agreement. Fearon (2006) sees elections as a device to coordinate revolt in case the incumbent abuses his power, that is, exceeds $x = E(x_t)$. Londregan and Vindigni (2006) think that elections are a cheap way to learn the value of $q$.\footnote{This was already the view of Simmel (1950: 241-2): "because the voting individuals are considered to be equals, the majority has the physical power to coerce the minority.... The voting serves the purpose of avoiding the immediate contest of forces and finding out its potential result by counting votes, so that the minority can convince itself that its actual resistance would be of no avail."} But, in the end, all these are arguments to the effect that elections are a peaceful way of processing conflicts that otherwise may have or would have been violent. As Bobbio (1984: 156) put it, "What is democracy other than a set of rules ... for the solution of conflicts without bloodshed."

Yet one must be careful not to pose this question in functionalist terms. The answer cannot be that we have elections because they are in some way good, because they generate peace or some other efficient outcomes. Elections occur either because incumbents want to hold them or because incumbents who do not hold them are often deposed. Take the extreme and the most puzzling situation: elections without opposition, elections in which the incumbents are certain to win, so that no one is selected as their result. We have seen that within some range of repressive power incumbents hold such elections because the opposition
does not revolt when they hold them but would revolt if they did not hold them. The reason is that incumbents who hold non-competitive elections still behave as electoral winners, that is, they set policies less favorable to them than incumbents who rule by sheer force. Hence, the only possible answer to this question is that we have elections because most incumbents want to hold elections.

Whether the results of elections regulate government change depends on the balance of forces between the contesting sides. Countries suffer from civil strife when the state does not have the capacity to deter armed challenges and political forces are polarized in terms of their policy preferences. When the rulers are militarily strong but moderate, they can sustain order without holding elections. Elections occur when the polity is not highly polarized and neither the current incumbent nor the potential challengers have an overwhelming military advantage. Their results are obeyed if the military are non-partisan or if both parties are moderate or if the country has a high income. When beliefs about the power of the incumbent diverge, incumbents may attempt to engage in manipulation which the opposition judges to be excessive or may commit fraud, provoking a rebellion.

We should not be surprised if incumbents repeatedly win. Peaceful elections are thus not necessarily, indeed they are rarely, competitive. Most peaceful elections are neither "fair," nor "genuine," nor "democratic," to use the language of election monitoring agencies. Militarily strong but moderate political parties can maintain themselves in office holding "elections" without any opposition or maintaining an overwhelming chance to win, with a passive consent of the potential challengers. Only when the military power is balanced or the military are constitutionalist or when incomes are high are elections competitive and peaceful.

Thus elections occur in the shadow of violence. The power to use violence is a political power. But the relations of physical force do not uniquely determine the outcomes. What one does with power depends on what one wants to achieve and what one fears from one’s opponents. When political forces are moderate about what they want, they expose themselves to peaceful competition even if one of them enjoys military dominance.

The glaring weakness of this model, and of the more general approach

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20 European Union observers want elections to be "open and fair," OSCE wants them to be "genuine," while the Declaration of Principles for International Elections Observation of the Carter Center and NDI wants them to be "genuine and democratic." Everyone also wants elections to be non-violent. See respectively European Commission (2008), OSCE (2007), Carter Center (2005).
it implements, is that the repressive agents are not treated as a full-fledged strategic actor. Obviously, the military intervene in politics not only on behalf or behest of civilians but at times for their own reasons. In a way, the central implication of this analysis is that to understand political regimes, one should look not at elections but elsewhere, namely, at relations of military force. While this is obviously not the place to do it, I am not persuaded by analyses which treat civilian-military relations as a principal-agent problem, in which civilians pay the military to engage in repression and the military revolt when the civilians do not satisfy the participation constraint (Acemoglu, Ticchi, and Vindigni 2008). The coercive apparatus that does day-to-day repressing are not "the military" but various kinds of police forces. Yet, paradoxically, almost all coups are made by the military, not the police. Moreover, about one half of the coups after 1950 were initiated from lower ranks of the military hierarchy (Singh 2009), indicating that relations within the military hierarchy, perhaps simple impatience to reach higher ranks, may motivate coups independently of civil-military relations. All I can say is that partisan postures of different repressive apparatuses and the relations among them are still largely unexplored (but see Davis and Pereira 2003).

Finally, while the violence analyzed here was treated as strategic, some of it must be spontaneous, unpremeditated, resulting simply from flaring tempers.

7 Appendix: Proofs

7.1 Proposition 1

7.1.1 Assumptions

The following assumptions simplify the algebra and reduce the proliferation of cases:

Assumptions: (i) Without a loss of generality, assume that the distribution \( f(x^*_j) \) is symmetric, so that \( \mu = 0 \), and with some loss of generality that (ii) \( D = \pi - x > 1 \) and that (iii) all \( L \)'s are drawn from \([x,0)\) and all \( R \)'s from \((0,\pi]\).

7.1.2 Lemma 1

Consider the first part, in which \( L \) is the incumbent. If \( L \) were to lose an election, with probability \( 1 - p_L \), it would be pushed in opposition to indifference between obeying and rebelling. Hence, \( L \) obeys as the incumbent if
\[ V_L^I(o) = \frac{U_L(x_L) + \rho(1 - p_L)V_L(r)}{1 - \rho p_L} > V_L(r), \]

or if

\[ U_L(x_L) > \frac{1 - q}{1 - \rho q} U_L(\mu - c), \]

of if

\[ q < \frac{U_L(\mu - c) - U_L(x_L)}{U_L(\mu - c) - \rho U_L(x_L)} \equiv q^*. \] (12)

Now, \( R \) rebels in opposition if \( V_R^O(o) < V_R(r) \) for all \( max p_L \geq 0 \). Substituting \( p_L = 0 \) in \( V_R^O(o) \) shows that \( R \) obeys if

\[ q > \frac{(1 - \rho)[(1 - \rho p_R)U_R(x_L) + \rho U_R(x_R)]}{(1 + \rho(1 - p_R))U_R(\mu - c) - \rho[(1 - \rho p_R)U_R(x_L) + \rho U_R(x_R)]} \equiv q''. \] (13)

Note that \( p_R = \text{max} p_R \), so that \( q'' \) is unique for a particular pair of contenders given \( \rho \) and \( c \).

The analysis is analogous for the second part. Substituting \( V_R(r) \) into \( V_R^I(o) \) shows that \( R \) obeys as an incumbent only if

\[ V_R^I(o) = \frac{U_R(x_R) + \rho(1 - p_R)V_R(r)}{1 - \rho p_R} > V_R(r) \]

or if

\[ U_R(x_R) > \frac{q}{1 - \rho(1 - q)} U_R(\mu - c). \]

or if

\[ q > \frac{(1 - \rho)U_R(x_R)}{U_R(\mu - c) - \rho U_R(x_R)} \equiv q**. \] (14)

In turn, \( L \) obeys in opposition when \( \text{max} p_R \geq 0 \) if

\[ q < \frac{(1 + \rho(1 - p_L))U_L(\mu - c) - [(1 - \rho p_L)U_L(x_R) + \rho U_L(x_L)]}{(1 + \rho(1 - p_L))U_L(\mu - c) - \rho[(1 - \rho p_L)U_L(x_R) + \rho U_L(x_L)]} \equiv q'. \] (15)
7.1.3 Lemma 2

The \( p_L \) that satisfies \( V_R^O(o) \geq V_R(r) \) is

\[
p_L \leq 1 + \left( \frac{1}{\rho} - p_R \right) \frac{(1 - \rho(1 - q)) U_R(x_L) - q U_R(\mu - c)}{(1 - \rho(1 - q)) U_R(x_R) - q U_R(\mu - c)} \equiv \max p_L \tag{16}
\]

We already know that if \( q > q^\ast \), the denominator of the fraction is positive. Hence, \( \max p_L < 1 \) if the numerator is negative. Substituting the utilities shows that the value of the numerator is non-negative if

\[
q \geq \frac{U_R(x_L) - \rho U_R(x_L)}{U_R(\mu - c) - \rho U_R(x_L)} \equiv \hat{q} \tag{17}
\]

Note that if \( U_R(x_L) > U_R(\mu - c) \) then \( \hat{q} > 1 \), so that \( \max p_L < 1 \). But if \( U_R(x_L) < U_R(\mu - c) \) or if \( x_L < -c \), \( \hat{q} < 1 \) and the incumbent \( L \) institutes a one-party system at \( 1 > q \geq \hat{q} \). Hence, \( L \) chooses \( \max p_L = 1 \) when it is extreme in relation to cost of defeat in conflict.

In turn, the value \( p_R \) which satisfies \( V_L^O(o) \geq V_L(r) \) is

\[
p_R \leq 1 + \left( \frac{1}{\rho} - p_L \right) \frac{(1 - \rho q) U_L(x_R) - (1 - q) U_L(\mu - c)}{(1 - \rho q) U_L(x_L) - (1 - q) U_L(\mu - c)} \equiv \max p_R, \tag{18}
\]

and the analysis is analogous. \( R \) institutes a one-party system if

\[
q \leq \frac{U_L(\mu - c) - U_L(x_R)}{U_L(\mu - c) - \rho U_L(x_R)} \equiv q^{**}. \tag{19}
\]

Note that \( \hat{q}_R > 0 \) only if \( U_L(\mu - c) > U_L(x_R) \) or \( x_R > c \), meaning that \( R \) is extreme in the metric of the costs of defeat.

7.1.4 Proposition 1

Lemmas 1 and 2 imply Proposition 1.

7.2 Proposition 2

7.2.1 Lemma 3

Substituting the utilities into the respective expressions for \( q^\ast \) and \( q^{**} \), we learn that

**Condition 1** \( L \) obeys as the incumbent if \( q < \frac{1 - c/x_L}{1 + (D-1)(1-\rho) - c/x_L} \equiv q^\ast. \)

**Condition 2** \( R \) obeys as the incumbent if \( q > \frac{(D-1)(1-\rho)}{1 + (D-1)(1-\rho) + c/x_R} \equiv q^{**}. \)
Note that $\partial q^*/\partial x_L > 0$ and $q^* \to 1$ as $x_L \to 0$. Because $x_L = x_L^*/D$, an $L$ close to $\mu = 0$ never rebels as the incumbent (Remember that by assumption $x_L^* < \mu = 0$). The intuitive reason is that as $x_L^* \to \mu$, there is little to gain by rebelling. In turn, $\partial q^{**}/\partial x_R > 0$, so that $q^{**} \to 0$ as $x_R^* \to 0$, meaning again that if $R$ is very moderate he never rebels as the incumbent.

Comparing these conditions shows that $q^{**} < q^*$ if

$$(D - 1)^2(1 - \rho)^2 < 1 + cD\frac{x_L^* - x_R^* - cD}{x_L^*x_R^*}.$$  

Consider first the case when $c = 0$ and rewrite this condition as $D < 1 + \frac{1}{1-\rho}\sqrt{1},$ yielding $D(c = 0) < \frac{2}{1-\rho}.$ Now, let $c > 0$ and rewrite it as $D < 1 + \frac{1}{1-\rho}\sqrt{1 + g(D)}.$ Because $g(D) > 0, D(c > 0) > D(c = 0).$ Hence, if fighting is costly to the loser, the range of $q$ in which both parties can obey is larger.

While it was easier to keep track of who is who by labelling parties, the results for $L$ apply to any current incumbent and for $R$ to any current opposition. If $R$ is the incumbent, then these conditions apply by placing $-c/x_L$ for $c/x_R$ in Condition 1 and vice versa in Condition 2. Note only that if we took $R$ to be the incumbent, then $\partial q^{**}/\partial x_L < 0$ and $\partial q^*/\partial x_R^* < 0.$

7.2.2 Lemma 4

Examine first $q'$ in (15). As both $x_L^*$ and $x_R^*$ tend to $\mu$, $U_L(\mu - c) \to U_L(-c), U_L(x_R) \to 0, U_L(x_L) \to 0$, and $q' \to 1$. In turn, when both parties are equally moderate, $q'' \to 0$ in (13). Hence, when both parties are sufficiently moderate $q' > q''$. Now, when $L$’s ideal point remains fixed while $R$ becomes more extreme, $\partial q'/\partial x_R^* < 0$. In turn, when $R$’s ideal point is fixed while $L$ becomes more extreme, $\partial q''/\partial (-x_L^*) > 0$.

Finally, let $\alpha_j U_j(x_L) + \beta_j U_j(x_R) = X < 0$, where $j \in \{L, R\}$ and $\alpha, \beta$ are any non-negative weights. Lower values of $X$ are then associated with greater difference between $x_R^*$ and $x_L^*$. Taking the respective derivatives shows that $\partial q'/\partial (-X) < 0$ and $\partial q''/\partial (-X) > 0$, so that $q'$ declines while $q''$ increases as polarization increases.

7.2.3 Proposition 2

Lemmas 3 and 4 imply Proposition 2.

7.3 Proposition 3

7.3.1 Lemma 5

If $q < q^{**}$, $R$ rebels as the incumbent. If $L$ obeys in opposition in response to $R$ rebelling as the incumbent, $L$ can expect to get $U_L(x_R^*)$.
for ever. Hence, $L$ obeys if

$$U_L(x^*_R) > \frac{1 - q}{1 - \rho q} U_L(\mu - c). \quad (20)$$

Substituting utilities yields

$$x^*_L - x^*_R > \frac{1 - q}{1 - \rho q} (x^*_L - c).$$

This inequality defines $q^o < q^*$ such that if $q < q^o$, $L$ obeys in opposition when $R$ rebels as the incumbent.\footnote{We know that $q^o < q^*$ because the left-hand side of (17) is smaller than the left-hand side of (14) and the right-hand side declines in $q$.} Note that if $x^*_R > c$ this inequality is never satisfied.

If $q > q^*$, $L$ rebels as the incumbent. $R$ obeys if

$$x^*_L - x^*_R > \frac{q}{1 - \rho(1 - q)} (-x^*_R - c),$$

which defines $q^{oo} > q^{**}$ such that if $q > q^{oo}$, $R$ obeys in opposition when $L$ rebels as the incumbent. This inequality can be satisfied only if $-x^*_L < c$.

### 7.3.2 Lemma 6

Suppose now that $L$ is the incumbent and $L$ knows it would obey if $R$ was to rebel as the incumbent because $q < q^{**}$. The $L$ incumbent obeys if

$$\frac{U_L(x_L) + \rho(1 - p)L(x^*_R)/(1 - \rho)}{1 - \rho p_L} > V_L(r),$$

which can be rewritten as

$$[U_L(x_L) - \frac{1 - q}{1 - \rho q} U_L(\mu - c)] - \rho p_L [U_L(x^*_R) - \frac{1 - q}{1 - \rho q} U_L(\mu - c)] > \rho [U_L(x_L) - U_L(x^*_R)].$$

Now, the RHS of this inequality is positive and, given that $q < q^*$ so is the expression in the first bracket on the LHS. Let

$$U_L(x^*_R) - \frac{1 - q}{1 - \rho q} U_L(\mu - c) = A,$$

Substituting yields

$$(1 - \rho)[U_L(x_L) - \frac{1 - q}{1 - \rho q} U_L(\mu - c)] > -\rho(1 - p_L)A.$$
If $A > 0$, which is equivalent to $q < q'$, this is always true. If $A < 0$, this inequality is satisfied if

$$\frac{(1 - \rho)U_L(x_L) + \rho(1 - p_L)U_L(x^*_R)}{1 - \rho p_L} > \frac{1 - q}{1 - \rho q} U_L(\mu - c),$$

which, in turn, is true for some $q \in (q^o, q^*)$ but it can be true only if $x^*_R$ is not too high, that is, the current opposition that rebels as the incumbent is not too extreme.

Analogous reasoning leads to the conclusion that $R$ obeys as the incumbent knowing that it would obey in opposition when $L$ rebels as the incumbent if $q > q^{oo}$, but $R$ rebels as the incumbent at some $q \in (q^*, q^{oo})$ if $L$ is too extreme.

7.3.3 Proposition 3

Follows directly from the two Lemmas. There are several cases:

1. If $q < q^{**} < q^o < q^*$ or if $q^o < q < q^{**} < q^*$ and $R$ is moderate, the equilibrium is \{L obeys as the incumbent, R obeys in opposition; L obeys in opposition, R rebels as the incumbent\},

2. If $q^{**} < q^{oo} < q^* < q$ or if $q^{**} < q^* < q < q^{oo}$ and $L$ is moderate, \{L rebels as the incumbent, R obeys in opposition; L obeys in opposition, R obeys as the incumbent\}.

These two equilibria are peaceful but not electoral.

3. If $q^o < q < q^{**} < q^*$ and if $R$ is extreme, \{L obeys as the incumbent, R obeys in opposition; L rebels in opposition, R rebels as the incumbent\},

4. If $q^{**} < q^* < q < q^{oo}$ and if $L$ is extreme, \{L rebels as the incumbent, R rebels in opposition; L obeys in opposition, R obeys as the incumbent\}.

These equilibria are violent, respectively when $L$ or $R$ are the incumbents.

7.4 Proposition 5

Note that $q^*$ was defined implicitly by

$$\frac{U_L(x_L) + \rho(1 - p_L)V_L(r|q^*)}{1 - \rho p_L} = V_L(r|q^*).$$

But if the incumbent loses $s$ of his power upon electoral defeat,

$$\frac{U_L(x_L) + \rho(1 - p_L)V_L(r|q^* - s)}{1 - \rho p_L} < V_L(r|q^*).$$

Hence, he does not hold elections when $q^*(s) < q < q^*$. Moreover, because $\partial V_L(r|q^* - s)/\partial s < 0$, it must be true that $\partial q^*(s)/\partial s < 0$. 

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7.5 Proposition 6
If $U_j(x_j^*; x; y, c) = -d(x_j^*; x)/y - \delta c$, the incumbent $L$ obeys if

$$q < \frac{1 - cy/x_L}{1 + (D - 1)(1 - \rho) - cy/x_L} \equiv q^*(y),$$

and $\partial q^*(y)/\partial y > 0$, given that $x_L < 0, D > 1$.

In turn, the opposition $R$ would obey as an incumbent if

$$q > \frac{(D - 1)(1 - \rho)}{1 + (D - 1)(1 - \rho) + cy/x_R} \equiv q^{**}(y),$$

and $\partial q^{**}(y)/\partial y < 0$.

It is sufficient to inspect (13) and (15) to see that $q \to 0$ and $q^* \to 1$ as $y \to \infty$. Note that contenders with extreme ideal points may revolt given an income level under which moderate contenders would not.

It is easy to show that the same result hold with regard to $q'$ and $q''$.

7.6 Proposition 7
The incumbent $L$ believes that his strength is $q_I$ and he is told by the opposition that his strength is $q_T < q_I$ and that it will rebel if the incumbent runs the election with $\max p_L(q_I)$. Now, expecting that if the opposition wins it will operate elections with $\max p_R(q_T)$, $L$ asks himself what is the maximum $p_L$ that the opposition would accept. The answer is the $p_L$ that solves

$$V^O_R(o; \max p_R(q_T), p_L) = V_R(r; q_T)$$

which is given by (13) with $q = q_T$ and $p_R = \max p_R(q_T)$.

Now, given that the opposition rebels unless $p_L \leq \max p_L(q_T) < \max p_L(q_I)$, the incumbent asks himself if he is willing to accept $\max p_L(q_T)$. He is willing to do so if $p_L \geq \min p_L(q_I)$ given by $V^I_L(o; p_L, \max p_R(q_T)) \geq V_L(r; q_T)$. The solution is

$$p_L \geq 1 + \frac{1 - \max p_R(q_T)}{\rho} \left( \frac{(1 - \rho q)U_L(x_L) - (1 - q)U_L(\mu - c)}{(1 - \rho q)U_L(x_R) - (1 - q)U_L(\mu - c)} \right) \equiv \min p_L(q_I)$$

Hence, a compromise is possible if $\min p_L(q_I) \leq \max p_L(q_T)$. Tedious algebra shows that

Condition 3 $\min p_L(q_I) \leq \max p_L(q_T)$ if $\frac{1 - q_I}{1 - \rho q_I} - \frac{q_T}{(1 - q_T)} > \frac{1}{x_L^* - c(D - 1)(x_L^* + x_R^*)}$. 

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Now, $\frac{1-q_I - \frac{q_T}{1-\rho(1-q_T)}}{1-q_I} > 0$ if $q_I + q_T < 1$ or $q_T < 1 - q_T$. In turn, the right-hand side is positive if $x_R^* < -x_L^*$, meaning that the incumbent $L$ is more extreme (further from the mean) than the opposition $R$. Hence, we must consider the particular cases:

(1) If $x_R^* > -x_L^*$, meaning that $R$ is more extreme, and $q_I < 1 - q_T$, this condition is always satisfied. The incumbent does not feel strong, the opposition believes him to be weak and the opposition cares less about avoiding a fight.

(2) If $x_R^* > -x_L^*$, $R$ is more extreme, and $q_T > 1 - q_T$, this condition is satisfied only if $(1 - \rho) \frac{1-q_T - q_I}{(1-\rho(1-q_T))} > \frac{1}{x_L^*} \frac{(x_L^*-x_R^*)^2}{(D-1)(x_L^*+x_R^*)}$.

(3) If $x_R^* < -x_L^*$, $L$ is more extreme, and $q_I < 1 - q_T$, it is satisfied only if $(1 - \rho) \frac{1-q_T - q_I}{(1-\rho(1-q_T))} > \frac{1}{x_L^*} \frac{(x_L^*-x_R^*)^2}{(D-1)(x_L^*+x_R^*)}$.

(4) If $x_R^* < -x_L^*$, $L$ is more extreme, and $q_T > 1 - q_T$, the condition cannot be satisfied. In this case, the incumbent believes he is strong and he risks a lot by letting $R$ win elections.

Although the left-hand side of Condition (3), $LHS = \frac{(1-\rho)\frac{1-q_T - q_I}{(1-\rho(1-q_T))}}{(1-q_I)\frac{1-q_T - q_I}{(1-\rho(1-q_T))}}$, increases in the distance in the beliefs, let it be $k = q_I - q_T$, this distance matters little. Basically, a compromise is possible if the incumbent does not believe that he is very strong and if he is not too extreme in his preferences. The following figure illustrates the condition (3) for different values of $k$.

![Diagram](image-url)
8 References


Gandhi, Jennifer. 2009.


Tucker, Joshua. 2009.
