Super Tuesday: Campaign Finance and the Dynamics of Sequential Elections*

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Abstract:

I present a model of campaign finance in primary elections in which campaigns supply hard information about candidates' electability. Focusing on a class of equilibria in which informed voters vote according to the signal they observe, I show that bandwagons can arise in equilibrium when a third party is financing campaigns.

To address the controversy surrounding the timing of presidential primaries in the United States, I examine the welfare effects of making changes to the electoral calendar. For relatively low campaign costs, a calendar with a block of voters voting simultaneously early in the process, followed by the remaining voters voting consecutively, is optimal for voters and the party. This result provides a rationale for "Super Tuesdays" in U.S. presidential primaries. For higher campaign costs, a sequential calendar is optimal. Donors always prefer a sequential electoral calendar.

1. Introduction

"People don't lose campaigns. They run out of money and can't get their planes in the air. That's the reality."

-Robert Farmer, fundraiser for Michael Dukakis and Bill Clinton (quoted in Brown et al. '95[11])

Presidential nomination campaigns in the United States are lengthy processes. In 2008, the most recent, the first votes were cast in the Iowa caucus on January 3rd while the last votes were cast five full months later in Montana and South Dakota on June 3rd. During the course of the primary season candidates entered and left the race, and their popularity with voters and donors fluctuated as the process unfolded. Many of the phenomena of interest to both the popular media and academics are inherently dynamic in nature: candidates' momentum, the potential effects of changing the electoral calendar, what voters learn from early results and how information affects their voting decisions, etc.

Money plays an important role throughout the nomination process. Running a competitive campaign is very costly and candidates depend on donors to keep their bids alive. During the 2008 primaries, candidates for the Democratic nomination raised a staggering $787 million, while Republicans raised $477 million[1]. Donors learn about candidates as the primary season progresses and donations fluctuate through time as candidates' performance in early states informs future donation decisions. As the opening quote highlights, contenders typically know they have lost the election when they can no longer raise enough funds to continue campaigning competitively. Clearly, donors are major players in presidential primaries, and their behavior has a first-order impact on the dynamic phenomena mentioned above.

This paper presents a game-theoretic, microfounded model of primary elections which examines the role of campaign finance in determining the unfolding of presidential nomination campaigns. I take the view that campaigns are a means of providing information to the public (as in Coate '04[13] and Ashworth '06[4]), and that the election itself is an information aggregation mechanism through which information dispersed in the population is elicited in order to make the best possible choice of nominee (as in Feddersen and Pesendorfer '96[20]). Policy differences within a party are taken to be negligible and the information that is aggregated by the elections and revealed through campaigns is about the candidates' electability:

[1] Numbers from The Campaign Finance Institute. See http://www.cfinst.org/pr/prRelease.aspx?ReleaseID=205.

the qualities which determine how likely a candidate is to win the general election for the party.

Within this framework, donating to a political campaign is a way of increasing the amount of information available to voters. Thus, it is a way of helping the party select a better candidate and increase its chances of winning the general election. Given mixed evidence (Ansolabehere, de Figueiredo and Snyder '03[3]), I am agnostic as to the motivation of donors. In the main body of the paper I speak of a single special interest group (SIG) which sees donations as investments which will yield future benefits in the form of access, policy favors, agenda setting, or other services if the receiving candidate wins the general election. The SIG has an interest in helping the party select the most electable candidate because that is the group's only chance of benefiting from favorable policies. In Section 5, I present extensions of the model, with many special interest groups (Proposition 1) and with altruistic donors (Proposition 2), which lead to equivalent donor behavior. The model of altruistic donations is of special interest as it reconciles the small average size of individual donations (individuals are currently subject to a $2400 donation limit) with donor behavior which is responsive to circumstances in ways suggestive of the expectation that a particular donation will affect outcomes and/or elicit future favors. Thus, it contributes to the theoretical understanding of campaign finance.

The predictions of the model are in line with stylized facts observed in U.S. presidential primaries:

- Donors give gradually to candidates (McCarty and Rothemberg ’00[34]).
- Money follows electoral success (Aldrich ’80[1][2], Hinckley and Green ’96[27], Mayer ’96[31], Damore ’97[15]).
- Candidates drop out under financial duress (Mayer ’96 ch. 2[32], Norrander ’00[36], Haynes et al. ’04[26]).

Another point of interest is the effect of making changes in the electoral calendar. In the 2008 primaries, much controversy was sparked by Florida and Michigan’s decision to hold their primaries in January, ignoring the parties’ order that they be held no sooner than February 5th. Much speculation surrounded the largest Super Tuesday ever, held on February 5th, in which 22 states voted and over half of all delegates were pledged. Indeed, the trend toward frontloading, holding more events sooner in the primary season, has been a subject of debate since it began in 1988 (Busch and Mayer ’04[33]). Proposals for reform of the electoral calendar abound and include a national primary, voting in regional blocks, a scheduling lottery, and others (Smith and Springer ’09[39]).

Because the electoral calendar determines what information donors will have when deciding whether to fund a campaign, the model studied in this paper allows one to examine the welfare implications of adopting different electoral calendars. Donors prefer to have strictly sequential primaries so that the decision of whether to fund each campaign can be made individually, thus minimizing the expected cost of the process (Proposition 1). However, stakeholders who do not bear the cost of the campaign, such as voters and parties, prefer to have as many campaigns funded as possible. These stakeholders may be best served by electoral calendars which are 'lumpy' and force donors to choose whether to fund campaigns in groups. Under the right cost conditions, these electoral calendars will maximize the expected amount of donations made and, thus, the expected amount of information revealed before a nominee is selected (Theorem 5). These blocks of voters, voting simultaneously early in an election, are reminiscent of Super Tuesdays in U.S. presidential primaries. One of the main contributions of this paper is to provide a game theoretic rationale for the existence of Super Tuesdays. My conclusion is that a frontloaded or Super Tuesday calendar may be preferable to a sequential one if the cost of campaigning is low enough for competitive challengers to raise adequate funds for early primaries. Otherwise, a sequential election will be more effective at helping voters select the most competitive nominee.

1.1. Related Literature

Sequential elections were first studied in a game-theoretic setting by Dekel and Piccione '00[17]. Their main result is that equilibria of a simultaneous election game are also equilibria of all sequential versions of the game. Because voters condition their vote on being pivotal, it does not matter whether some information is revealed before a voter casts his ballot. This result left scholars to wonder under what conditions the dynamic phenomena mentioned in the introduction, especially momentum, might arise. Battaglini '05[8] shows that, if voting is costly, voters will abstain once a candidate takes a sufficiently large lead. Callander '07[12] shows that bandwagons can arise when voters prefer to vote for the eventual winner. Ali and Kartik '08[28] show that voting according to posterior beliefs is an equilibrium and can lead to herding. Gershkov and Szentes '09[21] present a model where voters must decide whether to acquire costly information prior to voting. They characterize voting mechanisms which maximize the quality of the decisions taken in equilibrium.

These papers have established a canonical model of sequential elections in which there are two candidates and two states of the world. Voters receive private signals about the true state of the world and their utility depends on whether the election selects the 'right' candidate. In this paper I adhere to this canonical framework as far as possible.

Two of the most influential works on the dynamics of primary elections in the political science literature are Bartels '88[7] and Aldrich '80[1]. Both present empirical and anecdotal evidence of momentum and other dynamic phenomena. Bartels emphasizes the role of the media in influencing voter preferences while mostly ignoring the role of money[2]. Aldrich focuses more on campaign finance and, in [2], he models momentum as explicitly arising from a feedback mechanism where electoral success increases donations which, in turn, make electoral success more likely. However, he stops short of explicitly modelling the decisions of voters and donors that are behind this feedback mechanism. Both Bartels' and Aldrich's work serve as a starting point for the modelling done in this paper, and one of my goals is to reconcile some of their arguments and evidence with the abstract literature on sequential voting.

While the effect of campaign spending on voting behavior (e.g. Haynes, Gurian and Nichols '97[25]) and the importance of accumulating campaign funds early in a contest (e.g. Goff '04[22]) has been widely studied, little attention has been paid to the timing of donations and the effect of campaign finance on the dynamics

[2] According to Mayer '96[32], "Bartels mentions campaign finance on exactly three pages, and then only in passing."

of primaries. A notable exception is McCarty and Rothenberg '00[34] who propose a model of the timing of donations and provide empirical support for their conclusions. Their focus, however, is on the bargaining between candidates and PACs rather than on the effect of donations on the dynamics of the election itself. Klumpp and Polborn ('06[30]) study a game-theoretic model of campaign spending and its effects on the dynamics of primary elections. They point out that fewer resources will be spent when the electoral calendar is sequential rather than simultaneous. However, they do not explicitly model donors, assuming instead that campaign funds are available but costly to candidates.

Early models of campaign finance took the relation between spending and votes as given[3]. More recent work has taken the position that campaign spending plays an informational role. One strand of the literature (e.g. Pratt '02[37], Roumanias '05[38]) has argued that campaign spending is indirectly informative, revealing private information held by donors. A second strand posits that campaigns are directly informative, revealing information about the candidates which cannot be falsified (e.g. Coate '04[13], Ashworth '06[4]). This is the position I take in this paper.

[3]See Morton and Cameron '92[35], Stratmann '05[40], and Ashworth '08[5] for excellent surveys.

2. Model

A political party must nominate a candidate to represent it in a general election. It does so by means of a primary election, decided by majority rule.

2.1. Candidates

There are two candidates running for the party's nomination: A and B. Candidates care only about winning the primary election. They differ in their electability $e \in \{h, l\}$ with $1 \geq h > l \geq 0$. Electability is a summary variable capturing charisma, political ability, and other characteristics which help a candidate win elections. It is further interpreted as the probability with which the candidate will win the general election if nominated. I will use the terms highly electable, high type, and h-type interchangeably.

There are two states of the world: A and B. In state A, candidate A has electability h while candidate B has electability l. In state B, the reverse is true.

While candidates known their own electability[4], voters and donors do not. Rather, I model them as Bayesian learners with prior beliefs over the state of the world $Pr(A) = p = \frac{1}{2}$.

[4] One could argue that candidates do not know any more than the public about their own appeal. I explore the consequences of making this alternative assumption in Section 5.3.

The limitation to two candidates may seem severe, especially in the context of U.S. presidential primaries where several serious candidates typically seek the nomination. I stick to this narrow focus primarily to keep the model tractable and for continuity with previous theoretical research on sequential elections (see Section 1.1). Nevertheless, there are two ways in which the model may be interpreted that make the assumption seem less stringent. First, one may consider the model as pitting the front-runner versus the field. Second, some researchers (e.g. Kessel '92[29]) divide the nomination process into stages. During the first, non-competitive candidates are winnowed out. During the second, the contest begins in earnest. This model may be interpreted as studying only the second phase of the primary.

2.2. Voters

There are 5 voters. When thinking of presidential primaries, I may take voter i to be a representative voter from state i, so that I am modeling a primary with 5 states. Let V be the set of voters with typical element $v \in V$.

All voters have identical preferences:

$$u = \begin{cases} 1 & \text{if the h-type wins the primary} \\ 0 & \text{otherwise} \end{cases}$$

This utility function can be interpreted as an expected utility function where the value to the voters of having their party win the general election is $\frac{1}{h-l}$: $u(e) = \frac{1}{h-l}e - \frac{l}{h-l}$, where e takes on the value (h or l) of the primary winner's electability. Thus, if the h-type wins the primary: $u(h) = \frac{h-l}{h-l} = 1$. If the l-type wins the primary: $u(l) = \frac{l-l}{h-l} = 0$.

2.3. Campaigns and Donors

Candidates may send an informative signal to voter j by running a campaign at cost c. For simplicity, I assume throughout that the cost of campaigning is constant across states. If a campaign is run, the voter receives a signal $s \in \{a, b\}$. For notational convenience, I say that a voter who does not receive a signal receives $s = \varnothing$. It is common in the literature (e.g. Feddersen and Pesendorfer '96) to call voters who receive $s \in \{a, b\}$ informed, and voters with $s = \varnothing$ uninformed.

If both candidates campaign actively, the signal has accuracy q, or more precisely, $Pr(a|A) = Pr(b|B) = q > \frac{1}{2}$.

If only one candidate campaigns, he is able to manipulate the information so that a signal favorable to him is sent. Thus, no information is revealed. As is shown in the proof of Theorem 1, a campaign by a single candidate will only be run on the equilibrium path if one of the candidates cannot afford to campaign.

A signal is the private information of the voter to which it is directed. This is meant to capture the effect of face-to-face impressions achieved through town hall meetings, rallies, TV commercials on local channels, etc.

There is one special interest group (SIG) who may choose to provide campaign funds to the candidates. A total donation to candidate i of d_i provides economic benefits of $b(d_i)$[5] if i wins the primary election *and* goes on to win the general election. If the party's candidate does not win the general election, the SIG gets a payoff of 0. Because h (l) represents the probability with which an h-type (l-type) will win the general election, the SIG's expected payoffs conditional on the primary winner's political talent, total donations d to that candidate, and donations $\hat{d}$ to the primary loser, are $hb(d) - d - \hat{d}$ ($lb(d) - d - \hat{d}$).

I assume that the SIG maximizes its expected payoffs. Thus, the SIG, like the voters, has an interest in having a highly electable nominee. However, while voters would prefer signals be sent to all states, the SIG faces a costly information acquisition problem and may decide to stop funding campaigns if it considers that the benefits of additional information are outweighed by its costs.

The assumption of a single SIG is a strong one, particularly in the context of American presidential primaries. In Section 5, I develop two alternative models of

[5]Some models of campaign finance explicitly model the bargaining between candidate and SIG which leads to the function b. I leave such an extension to future work.

multiple SIGs which lead to equivalent equilibrium behavior. The first identifies a cooperative equilibrium of the game with an arbitrary number K of identical SIGs. If all contribute c/K when collectively optimal, and the payoff functions are scaled down to $b(d)/K$, the strategic decision of a particular SIG is identical to that of a single SIG. The second takes seriously the role of individual small contributors by postulating a behavioral model of morally motivated campaign donations.

Let the net benefit of making donations d to an l-type nominee $lb(d) - d$ be a strictly concave function with an interior maximum $d^l > 5c$. This inequality guarantees that the SIG is willing, ex-ante, to fund all feasible campaigns for the winner. Given this condition, when the SIG pays out $2c$ to elicit an informative signal, only c is considered an informational cost, since the remaining c is an investment in political influence that would be made in any case. In other words, the SIG will make donations d^e to the party's nominee, but it will also have made donations $\hat{d}$ to the primary's loser. These donations to the loser pay off by enabling a competitive campaign which provided information to the voters. The assumption is also consistent with the observation in McCarty and Rothemberg ('00[34]) that most campaign donations are made after the primary season is over.

I assume that the electability of the primary winner is revealed to the SIG before it makes its final contribution decision at time 6. I make this assumption to keep the analysis as simple as possible. Without it, the SIG's payoffs would depend on the final vote count which determines the posterior probability that the nominee is an h-type, and therefore the size of the SIG's donation to the candidate.

Let d^h solve $\max hb(d) - d$. Then the SIG donates d^h if the primary winner is type h, and d^l otherwise. Therefore, the benefit to the SIG of having the h-type candidate win the primary is $\Delta b = \left(hb(d^h) - d^h\right) - \left(lb(d^l) - d^l\right) > 0$.

To summarize, the SIG's total payoffs, given that the primary winner's type is $e \in \{h, l\}$, and that it made donations $\hat{d}$ to the losing candidate, will be:

$$eb(d^e) - d^e - \hat{d}$$

2.4. Timing

There are 6 periods or dates, indexed by $t \in \{1, 2, 3, 4, 5, 6\}$. The primary election may take place in periods 1 through 5. The general election takes place at $t = 6$.

An electoral calendar $\Gamma : V \to \{1, 2, 3, 4, 5\}$ is a function specifying the time at which each voter will vote. The electoral calendar is set prior to the start of the game and is generally taken as exogenous, except in Section 4 where I consider the design of optimal electoral calendars. The electoral calendars most commonly studied in the literature are simultaneous (all voters vote at the same date) and sequential (one voter per date). In this paper, I consider all possible calendars.

Without loss of generality, I assume that states vote in order: $\Gamma(1) \leq \Gamma(2) \leq ... \leq \Gamma(5)$. Also, when some voters vote simultaneously so that the image of Γ contains fewer than five elements $|\Gamma(V)| < 5$, the voting takes place at $t = 1, ..., |\Gamma(V)|$, so dates without voting come at the end of the process.

Each period except 6, the timing of events is as follows:

1. The SIG decides how many campaigns to fund that period.
2. Candidates decide whether to spend the campaign funds. If they do, campaigns are run and signals sent to voters.
3. Voters cast their ballots.

As is discussed above, the following takes place at time 6:

1. The primary winner's type is revealed.
2. The SIG makes its final donations.
3. Nature decides the outcome of the general election.

4. The SIG's payoffs are realized.

To sum up, an *election* is an extensive form game with 5+3=8 players (5 voters, the SIG, and two candidates), where the order of moves and the information structure is as described above.

3. Equilibrium

Let H_t denote the set of all public history date-t histories h_t. Each h_t includes information on all prior votes, as well as on which voters received a signal, what donations each candidate received at each date, and whether and where those donations were spent by the candidates. Let $H = \cup_{t=1}^{6} H_t$.

A voting strategy for voter i is a function $\sigma_i : H_{\Gamma(i)} \times \{a, b, \varnothing\} \rightarrow \{A, B\}$. Thus, a voting strategy specifies who a voter will vote for given everything that has happened in the election and the voter's signal. Note that the amount of information observed by each voter is determined by the electoral calendar. For instance, in a sequential election voter 4 has access to h_4, but in a simultaneous election he can condition his vote only on his signal and h_1.

I define *simple voting* to be the following voting strategy:

$\sigma(a) = A$

$\sigma(b) = B$

$\sigma(\varnothing) = A$ if no uninformed voter has voted in the past.

$\sigma(\varnothing) = B$ if the previous uninformed voter voted A.

$\sigma(\varnothing) = A$ if the previous uninformed voter voted for B.

Simple voting has several attractive characteristics. First, it makes minimal requirements of voters' rationality and computational ability. Indeed, it is consistent with a wide range of theories of voter behavior, including expressive voting (Brennan and Lomasky '93[10]). Second, it is maximally informative, ensuring that the information contained in the campaigns is revealed to other voters and to the SIG and has a maximal effect on the outcome of the election. This last point is important in this model because the donation decision of the SIG depends on how much of a difference a campaign will make to the probability of a highly electable candidate being selected.

A funding strategy for the SIG is a function $d : H \to \mathbb{R}^2_+$ specifying how much the SIG will donate to each candidate as a function of past play. At date t, the SIG plays the date t strategy $d_t : H_t \to \mathbb{R}^2_+$.

That it is in the interest of uninformed voters to vote in such a way as to let the votes of informed voters determine the outcome of the election has been pointed out by Feddersen and Pesendorfer '06. Their behavior here is analogous to abstention in the Feddersen-Pesendorfer model.

I look at a class of perfect Bayesian equilibria involving simple voting. I call these *simple equilibria*:

1. Simple voting.

2. Candidates always campaign when financially feasible.

3. The SIG maximizes its expected payoffs at each stage. That is, d_t solves $\max E\left[eb(d^e) - d^e - \hat{d}|h_t\right]$.

Candidates will campaign whenever they have the funds to do so. This is enforced by an off-equilibrium belief, held by voters and the SIG, that failure to campaign is a sure sign of a low electability candidate. Because an l-type would have more to gain from impeding the flow of information, this restriction on beliefs is in the spirit of the Divinity condition on off-equilibrium beliefs (Banks and Sobel '87[6]).

If voters are voting simply and candidates campaign whenever feasible, the investment decision of the SIG is strategically equivalent to that of a statistician who must decide how many costly experiments to perform before making a binding decision. The following Theorem confirms that simple equilibria always exist.

Theorem 1. *Every election has a simple equilibrium.*

Proof. In the appendix. ■

The following Proposition uses the simple equilibria of Theorem 1 to confirm the intuition developed by Aldrich '80 [2] that momentum may arise because donors are more willing to fund candidates who have been successful in early contests. In this model, when the electoral calendar is sequential, when one of the candidates develops a large enough lead he will receive all subsequent votes. This happens because the SIG will stop funding the opposition, making further updating impossible, while the frontrunner continues to receive funds and campaign. A key difference from other game-theoretic models in which bandwagons arise such as Ali and Kartik[28] and Callander[12], however, is that once a bandwagon begins only one candidate continues to receive donations and spend money on campaigns, leading to uninformative signals. The bandwagon does not arise from learning by voters, but rather from learning by the donor. Nevertheless, the voting behavior I refer to as a bandwagon is the same as in these previous models.

Definition 2. *A bandwagon has formed if the leading candidate will receive all subsequent votes with probability 1.*

Proposition 3. *Suppose the electoral calendar is sequential. Then there exist campaign costs* $0 < c_{\text{seq}} < \bar{c}$ *such that, if* $c \in (c_{\text{seq}}, \bar{c})$*, a bandwagon forms with positive probability.*

Proof. In the appendix. ■

4. Optimal Electoral Calendars

Another point of interest is the design of an optimal primary schedule. It is important to keep in mind that we do not have uniqueness in terms of Nash equilibria. However, as I discuss in the preceding Section, I find the simple equilibria to be particularly convincing and, in what follows, I evaluate electoral calendars assuming that simple equilibria will be played in each case.

It is useful to introduce some easy to understand notation for particular electoral calendars. I use brackets {} to denote a calendar. The first number in brackets is the number of voters voting at date 1. The second number, separated from the first by a dash, denotes the number of voters voting at date 2. I repeat this process until all voters are accounted for. Thus, the sequential calendar is {1-1-1-1-1}, while a simultaneous election corresponds to the calendar {5}.

As a first step, I establish the optimal calendar for the SIG. That the SIG will prefer a sequential calendar is a result established in a different setting by DeGroot (1970)[16]. Intuitively, the sequential calendar lets the SIG condition their funding decision on the latest poll results, therefore giving the SIG a larger strategy set. The following Proposition describes the optimal primary schedule for the SIG.

Proposition 1. *(DeGroot '70) The donor-optimal primary schedule is sequential* $\{1\text{-}1\text{-}1\text{-}1\text{-}1\}$.

Although the SIG-optimal primary schedule is of a simple form, it does not coincide with presidential primary calendars in the United States. This may be because, in the United States, the primary schedule is determined by the National Committees of the parties[6]. The members of these committees have more in common with voters than with the SIG. That is, they do not bear the cost of the campaign. Thus, it is interesting to consider the primary schedule which maximizes the probability of selecting the highly electable candidate.

Definition 2. *Given* c*,* b*, and* q*, an electoral calendar is voter-optimal if it maximizes the ex-ante probability with which the h-type candidate wins the election.*

[6] In practice, the National Committee sets certain ground rules and state Committees individually decide when to hold their primary election. However, setting the rules goes a long way toward determining the final outcome.

Definition 3. *An electoral calendar is said to* dominate *another if the ex-ante probability of the h-type candidate winning the election is weakly higher for all c, b, and q.*

A calendar strictly dominates *another if it dominates it and the ex-ante probability of the h-type winning the election is strictly higher for some triple c, b, q.*

If the relation holds only for certain values of c, b, and q, I say that a calendar (strictly) dominates another over the relevant ranges of *c, b, q.*

There are many possible electoral calendars in a world with five voters (16 in fact, they are listed in the appendix). The following Lemma helps to narrow the field to some particularly interesting candidates: a pure sequential election ({1-1-1-1-1}) and a mixed calendar in which there is a block of three voters voting simultaneously at date 1 followed by the two remaining voters voting sequentially ({3-1-1})[7]. I call this second calendar a Super Tuesday calendar because of its structural similarity to presidential primary calendars in which large blocks of states vote on the same "Super Tuesday" early in the primary season.

[7] The calendars {3-1-1} and {1-2-1-1} are strategically equivalent so I could refer to either as a Super Tuesday calendar. Perhaps the second is more reminiscent of Super Tuesday since it allows for a single early vote, like Iowa and New Hampshire might be in the U.S. presidential primary, to happen before the block of voters are scheduled.

Lemma 4. *For any c, b, and q, one of the following electoral calendars is voter-optimal among all possible calendars: sequential {1-1-1-1-1} or Super Tuesday {3-1-1}.*

Proof. In the appendix. ■

The proof proceeds by listing all possible calendars in a five voter election. Then, I derive equivalence relationships which allow me to focus on a subset of calendars. For example, because of the symmetry of the election, which candidate takes a 1-0 lead after voter 1's informed vote does not matter for the SIG's funding strategy. That is, conditioning on the outcome of the first vote will have no effect on the SIG's funding strategy at that moment. Therefore, any calendar in which the first voter votes before all others is strategically equivalent to the calendar identical to it but in which voter 1 votes at the same time as voter 2. I then establish dominance relationships among the remaining calendars until I am left only with the candidates listed above. For example, a Super Tuesday calendar will always lead to the election of the high type candidate with at least as high a probability as a {4-1} calendar and, for some values of c, it will do so with strictly higher ex-ante probability.

The preceding Lemma sets the stage for the following result describing the voter-optimal electoral calendars. Given that I know that either a sequential or a

Super Tuesday calendar is optimal, it is much easier to make comparisons among them and arrive at a result describing when one dominates another. In some ways, the following Theorem is the central result of this paper. It is the first result in this literature in which a hybrid calendar (not strictly sequential or simultaneous) plays a major role. It also provides a game-theoretic, effectiveness-based explanation for the existence of Super Tuesdays.

Theorem 5. *For given b and q, there exist values $0 < c_{\text{seq}} < c_{st} < \hat{c}_{\text{seq}}$ such that:*

- *The Super Tuesday calendar dominates all other calendars, and strictly dominates the sequential calendar, when $c \in (c_{\text{seq}}, c_{st})$.*
- *The sequential calendar dominates all other calendars, and strictly dominates the Super Tuesday calendar, when $c \in (c_{st}, \hat{c}_{\text{seq}})$.*

Proof. In the appendix. ■

The simultaneous calendar, which has been widely studied and is usually used as a point of comparison to the sequential calendar, is dominated by the Super Tuesday calendar in this model. That is not to say, however, that it is never optimal. It is optimal whenever the costs of campaigning are low enough for the SIG to fund all five campaigns in a simultaneous election, or high enough so that at most one campaign is funded under any electoral calendar. The following Proposition clarifies.

Proposition 6. *For given b and q, there exist campaign costs $c_{sim} \in (0, c_{\text{seq}})$ and $\bar{c} > \hat{c}_{\text{seq}}$ such that the simultaneous electoral calendar is voter-optimal whenever $c < c_{sim}$ or $c > \bar{c}$.*

Proof. In the appendix. ■

5. Extensions and Alternative Modelling Approaches

Although it is common in the micro-founded literature on campaign finance to assume a small number of donors (Ashworth '06[4], Coate '04[13] to name just two examples), the assumption is rather restrictive. During the 1999-2000 U.S. election cycle 21 million individuals donated to the candidates' campaigns (Ansolabehere, de Figueiredo and Snyder '03[3]).

I argued above that the single SIG could be interpreted as an aggregate of the objective functions of many special interest groups or even of many voters for whom political donations are a particular type of consumption. In the following sections I substantiate these claims.

5.1. Multiple SIGs

Consider an arbitrary number K of identical special interest groups. SIG i receives benefits $\$\frac{b(d_i)}{K}$ from making total donations d_i to a candidate who goes on to win both the primary and the general election.

Proposition 1. *There exists an equilibrium of the game with K identical special interest groups in which the collective behavior of the SIGs is identical to that of a single large SIG.*

Proof. Consider the funding decision of SIG i when the other K-1 SIGs are following the funding strategy of the single SIG described in Theorem 1. SIG i will fund a campaign if $\frac{c/K}{\Delta b/K} = \frac{c}{\Delta b}$ is smaller than the increase in the probability of the h-type winning the nomination. That is, i's problem is identical to that of a single SIG. ■

5.2. Donations as Altruistic Behavior

Ansolabehere, de Figueiredo and Snyder '03[3] argue that small average donation sizes and the large number of donors to political campaigns make any theory of campaign finance in which donations are seen as investments which are expected to produce returns in the form of altered results or influence on policy-making implausible. They survey 40 articles which attempt to find a link between donations and voting records and find little evidence of a link. However, research looking at the behavior of donors (e.g. Brown et al. '95[11] and Gordon et al. '07[23]) finds that at least some groups of donors behave *as if* their donation were an investment with policy implications or could change the outcome of the election.

One way to reconcile these pieces of evidence is to propose a model of campaign donations as an altruistic act, motivated by its moral implications. One popular explanation of altruistic behavior holds that agents often use a simple version of Kant's categorical imperative to evaluate an action's moral salience (Harsanyi '80[24], Brekke et al. '03 [9]). In particular, an action is morally salient if, when adopted by everyone, it maximizes a social welfare function. This type of motivation, known as rule-utilitarian, has been used to explain altruistic behavior in recycling, community service, voting (Feddersen and Sandroni '06[19], Coate and Conlin '04[14]), information acquisition by voters (Feddersen and Sandroni '06[18]), and other prosocial behavior.

Consider a set M of identical moral agents of Lebesgue measure 1. Moral agents are a minority in each state, as they are outnumbered by selfish agents in set S of measure 2. Each agent must decide whether to donate to a candidate's campaign. Even though a single donation cannot influence the outcome of the election, the agents receive satisfaction from performing morally salient actions and receive ex-post utility $1_{(h)} + \sum_{t=1}^{5} \left(m1_{(M,t)} - d_t\right)$, where $1_{(h)}$ is an indicator function which takes the value 1 if the highly electable candidate wins the primary and 0 otherwise, $1_{(M,t)}$ is an indicator function which takes on the value 1 if the agent performed the morally salient action concerning the i'th campaign and 0 otherwise, and d_t denotes the donations made at date t.

An donation d_t is morally salient if it satisfies:

$$d_t = \arg\max_{d_t} E\left[\int_{M\cup S}(1_{(h)} - d1_{(j\in M)})dj|h_t\right] = \arg\max_{d_t} \left\{3P(h|h_t, d_t) - \sum_{i=1}^{t} d_i - E\left[\sum_{i=t+1}^{5} d_i|h_t, \right.\right.$$

Where $1_{(h)}$ is the utility of voters and $d = \sum_{i=1}^{5} d_i$ is the total donated by morally motivated agents. It is clear that the expectation of $1_{(h)}$ is $P_t(h|d)$, the probability that the high electability candidate will win the nomination given donations d_t at time t.

In my model, where total donations of $2c$ are necessary for candidates to continue informative campaigning, $d = 2c$ is the only relevant level since there is no loss from funding future state campaigns at a later date, making higher donations redundant and lower donations are merely wasteful[8]. Furthermore, because there

[8] Mixed strategies in which agents give c in expectation are also solutions to the maximization above, but can be ruled out by making the utility cost of money convex.

is a continuum of morally motivated agents, an individual's decision not to donate will not affect outcomes. Thus, I need not consider strategic deviations sacrificing m now in order to make future morally salient actions cheaper.

Thus, a candidate's campaign will continue to be funded as long as the satisfaction of undertaking the morally salient action, $m > 0$, is higher than the cost, and as long as it is socially optimal for the campaign to be funded:

$m > 2c$

$3\Delta P_{t,t+1}(h) > E\left[\sum_{i=t}^{5} d_i | h_t, d_t\right]$

where $\Delta P_{t,t+1}(h)$ denotes the increase in the probability of nominating candidate h given that at least one more campaign is funded. In contrast, a SIG will fund an additional campaign if $\Delta b \Delta P_{t,t+1}(h) > E\left[\sum_{i=t}^{5} \hat{d}_i | h_t, \hat{d}_t\right]$. Note that the SIG only considers donations to the eventual loser as informational costs, while morally motivated donors see all donations as informational expenses. Therefore, keeping a campaign going implies informational costs of c for the SIG and 2c for morally motivated donors. Nevertheless, the problem they are solving is isomorphic.

Proposition 2. *If $m > 2c$, and there are morally motivated voters, then donations to political campaigns will be made as if a single SIG with $\Delta b = \frac{3}{2}$ were funding the campaigns using the strategy of Theorem 1.*

5.3. Symmetric Information about Types

It is plausible to think that candidates are uncertain about their own electoral appeal and learn about their electability through the primary process along with the voters and the SIG.

If candidates are solely office motivated, however, a leading candidate will never agree to campaign even if he is well-funded. Consider a candidate with reputation better than a half. If he campaigns he will lose the nomination with positive probability even if he is, in fact, the h-type. If he refuses to campaign, however, no new information is revealed and the primary season will end with the same beliefs, so that the only rational way for voters to vote is for the leading candidate. Thus, we are left with no campaigning or donations if the prior $p \neq \frac{1}{2}$, and campaigns only in the first period if $p = \frac{1}{2}$.

5.3.1. Policy-Motivated Candidates

If candidates also care about whether a candidate from their party wins the general election, they may willingly choose to campaign in order to increase the probability that the most electable candidate from their party is selected. Define a candidate's utility as:

$$g(P, W) = \gamma P + (1 - \gamma) W$$

Where P is an indicator function for a candidate from his party winning the general election, W is an indicator function for whether the candidate in question has won the primary, and $\gamma \in [0, 1]$. $\gamma = 0$ corresponds to the case discussed above in which the candidate only cares about winning the primary. $\gamma = 1$ describes a candidate motivated only by policy.

Proposition 3. *Given any election, there exists $\bar{\gamma} > 0$ such that, if $\gamma > \bar{\gamma}$, then a candidate will always campaign when feasible.*

Proof. Suppose that a simple equilibrium is played. For this to be incentive compatible for the candidates it must be that at each possible t-history in which A has reputation $R > \frac{1}{2}$,

$$(1-\gamma) + \gamma[R + (1-R)l] < (1-\gamma)P_{t,t+1}(Aw) + \gamma[P_{t,t+1}(h) + (1 - P_{t,t+1}(h))l]$$

or $\gamma \Delta P_{t,t+1}(h)(1-l) > (1-\gamma)(1 - P_{t,t+1}(Aw))$

The left-hand-side is always positive. Because I are dealing with a finite game, there is an absolute maximum value which the right hand side can take. Thus, a $\bar{\gamma}_A > 0$ can be found for which the inequality above is satisfied at all t-histories in which A is leading as long as $\gamma > \bar{\gamma}$. A symmetric argument finds a $\bar{\gamma}_B$ which serves the same purpose for t-histories in which B is in the lead. Taking $\bar{\gamma} = \max\{\bar{\gamma}_A, \bar{\gamma}_B\}$ completes the proof. ■

6. Concluding Remarks

I have set out to illustrate how campaign finance can be the main driving force behind the dynamics of primary elections. I have done so by presenting a micro-founded, game-theoretic model of the interaction between voters, candidates, and donors. Underpinning my analysis is the conception of a primary election as a means by which like-minded members and adherents of a party acquire credible information about the electability of the candidates. The results in Section 3 show that it is possible for the process to unfold in such a way that all parties involved are willing to participate, information is revealed, and it is used effectively. I also show that bandwagons can arise as a consequence of learning by donors. This provides an alternative to previous theories of bandwagons based on learning by voters.

In Section 4, I present results characterizing the optimal electoral calendar for both donors and voters. Donors prefer to have a sequential primary so that funding decisions can be made gradually and the costs of funding the eventual loser to elicit information minimized. Voters coincide with this preference for sequential elections when campaign costs are relatively high. However, when campaign costs are in a given range, voters prefer electoral calendars in which a group of voters vote simultaneously early in the electoral calendar (at date 1 or 2). This type of calendar is reminiscent of those in recent U.S. presidential primaries which include "Super Tuesdays" in which several states hold their elections on the same day. These results are especially interesting in light of the ongoing controversy surrounding the scheduling of presidential primary elections in the United States. My analysis concludes that a frontloaded calendar, including a Super Tuesday, is optimal for voters and parties as long as competitive candidates are able to fully fund their campaigns in these early stages.

Given the simplicity of the model studied here, several generalizations seem likely to add richness to my conclusions. First, my focus on a contest between two candidates seems inadequate given the large number of candidates who generally contest presidential primaries in the United States, at least in the early stages of the process. Furthermore, campaign spending is, in reality, not a discrete variable. Different amounts of spending can lead to different results. Similarly, something may be learned from the margin of victory in a given district beyond what is revealed by a win or a loss. I am currently exploring these and other extensions.

7. Appendix

Proof of Theorem 1.

Proof of existence of simple equilibria with 5 voters.

I proceed by considering each voter's decision problem, from last to first. In evaluating the utility effects of deviations, I assume that the voter knows how earlier voters have voted. If this is not the case, and there are voters of a lower number voting at the same time, the expected benefit of a deviation will be a weighted average of those considered. Thus, if deviating is never worthwhile when the voter is able to condition on previous voters' votes, it is not profitable when the voter cannot condition on this information.

I begin by establishing some basic facts about the SIG's donation strategy which will help simplify the arguments in the proof of Theorem 1 other results. The following Claim states that whenever the SIG funds at least one, or at least two campaigns, it is optimal for it to fund voter 1 and 2's.

Claim 1. *Whenever it is optimal for the SIG to fund any campaigns, it is optimal for the SIG to fund voter 1's campaign. Furthermore, whenever it is optimal for the SIG to fund at least two campaigns, it is optimal for the SIG to fund voter 2's.*

Proof. If the SIG is going to fund at least one campaign, and because voters are identical except for the order they vote in, it loses nothing by having it be voter 1's. On the other hand, its choice set, conditional on whether voter 1's campaign was funded, is weakly larger.

Similarly, because the information revealed by voter 1's vote is not relevant to the SIG's decision to fund additional campaigns, the SIG gains nothing by waiting until this information is revealed. If the SIG funds voter 2's campaign, it is left with more alternatives when considering the campaigns of voters 3-5. ■

Definition 2. *A donation strategy d_t is relevant if there is an electoral calendar Γ and a history h_t such that d_t is a best response to simple voting for some triple c, b, q.*

Lemma 3. *In any relevant donation strategy, the maximum (over histories) total number of campaigns funded by the SIG is either odd or zero. Thus, under simple voting, uninformed votes will never decide the election, and it is a best response for them to vote simply given that other voters are also voting simply.*

Proof. Suppose that voters are voting simply. Consider first the SIG's decision whether to fund one or two campaigns. Funding one campaign leads to selecting an h-type with probability q. Funding two campaigns leads to an h-type nominee with probability $q\left(q+2\frac{1}{2}(1-q)\right)=q$. Intuitively, conditional on the first vote, adding an additional campaign can only tie the informative vote count or increase the frontrunners lead. At worst, the frontrunner's posterior will be $\frac{1}{2}$ for each candidate and does not change the optimal choice of candidate. The same logic applies to the difference between funding three and four campaigns. In that case, funding three campaigns selects the h-type with probability $q^2(q+3(1-q))=q^2(3-2q)$. Funding four leads to a probability of success of $q^2\left(q^2+4q(1-q)+6\frac{1}{2}(1-q)^2\right)=q^2(3-2q)$.

Therefore, the SIG will fund zero campaigns, one campaign, or fund until one candidate receives two out of three or three out of five informed votes. That is, an election will end with an even number of campaigns funded only if one candidate has a 2-0, 3-1, or 4-0 lead in informed votes. When there is an odd number of campaigns funded, whenever a voter does not receive an informative signal (i.e. at least one candidate does not have sufficient funds to campaign for that voter) there will be another voter in the same situation. Thus, according to simple voting, the first voter in question will vote A and the second B, allowing informed voters to determine the outcome of the election. Because informed voters vote their signal, this means that the candidate who finishes the election with the highest posterior probability of being an h-type will win. Therefore, voting simply is a best response for uninformed voters. After a 2-0 lead in informed votes, if no additional campaigns are run, simple voting specifies that the final vote count will be 4-1 or 3-2 with the frontrunner winning, so that simple voting is indeed a best response for uninformed voters. After a 3-1 or 4-0 informed vote lead, the election is decided and simple voting is also a best response for the remaining uninformed voters. ■

In what follows, I build on the preceding results and consider feasible deviations for each voter, given an any relevant continuation funding strategy for the SIG.

Fifth voter:

The fifth voter is either pivotal or irrelevant. Whenever a voter is pivotal, it is a strict best response for him to vote his signal. The probability of getting it right is $q > 1 - q$.

Fourth voter:

It is also true that the fourth voter will only be relevant if he is pivotal, i.e. if the vote is 1-2 or 2-1. Otherwise, the election is already decided.

Suppose the informative vote count is 2-1.

If voter 4 receives a signal a, voting his signal leads to a win by candidate A having received 3 positive signals. The worst outcome for A in terms of signals from this point on is 3-2, where A is the state of the world with prob. q>1-q.

If voter 4 receives a signal b, voting his signal will lead to the correct candidate being chosen with prob. q. Voting for A leads to A winning the election which will be the correct choice with prob. $\frac{1}{2} < q$.

If the vote is 1-2, the arguments are symmetric.

If the informative vote count is 1-1, 2-0, or 0-2 (i.e. one of the first three voters did not receive an informative signal), voter 4 will receive an informative signal only if it is the last one of the election by Lemma 3. Therefore, voter 4 is either pivotal (after 1-1) or irrelevant (after 2-0).

Third Voter:

Voter 3 can receive an informative signal when the informative voting has been 1-1, 2-0, or 0-2.

If the vote is 2-0 and the voter receives a signal a, state A will have received three positive signals and is the correct choice with probability at least q. Similarly, if the vote is 0-2 and voter 3 receives a b signal, it is a strict best response for him to vote for B and end the election.

That leaves scenarios in which the signal count, including 3's signal, is 2-1 or 1-2.

Suppose the signal count is 2-1.

The probability of a correct outcome if 3 votes his signal and all future campaigns are financed is:

$$q(1-(1-q)^2)+(1-q)q^2=-q^2\,(2q-3)$$

The probability of a correct outcome if 3 votes B and all future campaigns are financed is:

$$q^3+(1-q)(1-(1-q)^2)=q\,(2q^2-3q+2)$$

Clearly, $q(1-(1-q)^2)+(1-q)q^2>q^3+(1-q)(1-(1-q)^2)$ since

$$-q\,(2q-3)-(2q^2-3q+2)=-4q^2+6q-2$$

The first derivative of this difference is: $6-8q$

$-4q^2+6q-2$ has roots at 1 and .5, so the two expressions are equal at .5 and 1, while the derivative of the expression is positive at .5 and negative after 3/4, meaning that the expression is positive for all $q\in[.5,1]$.

If the campaign ends with 3's vote, he is pivotal and voting his signal is a strict best response by the arguments made above.

Second Voter:

Voter 2 necessarily inherits either a 1-0 or 0-1 vote count. Therefore, if he receives an informative signal, Claim 1 confirms that his signal count is either 2-0 or 1-1.

If the SIG will finance the third campaign regardless of 2's vote, a deviation by 2 is equivalent to a deviation by 3, which we have seen above is never profitable.

If the SIG will stop funding the trailer if the vote goes to 2-0, then one must verify directly that a deviation is not profitable.

Let the signal count be 2-0. If 2 votes his signal, A is elected which is the correct choice with probability $\frac{q^2}{q^2+(1-q)^2}$.

If 2 deviates making the vote count 1-1, the correct choice will be made with prob. at most:

$\frac{q^2}{q^2+(1-q)^2}\left(q^2\left(1+2(1-q)\right)\right)+\frac{(1-q)^2}{q^2+(1-q)^2}\left(q^2\left(1+2(1-q)\right)\right)=\left(q^2\left(1+2(1-q)\right)\right)=$
$-q^2\left(2q-3\right)$

$\frac{q^2}{q^2+(1-q)^2}+q^2\left(2q-3\right)=2q^2\left(2q-1\right)\frac{(q-1)^2}{2q^2-2q+1}>0$

Which is clearly less than $\frac{q^2}{q^2+(1-q)^2}$ since $2q<3$.

Now, let the signal count be 1-1. If 2 deviates, he will end the election with a choice that is correct with prob. .5. If he votes his signal, the campaign continues and, because other voters are voting informatively and the SIG is willing to fund campaigns so that a correct decision can be made with probability at least q, the correct outcome will be chosen with prob. at least q>.5.

First Voter:

If voter 1's vote will decide the election, it is a strict best response for him to vote his signal, as argued above.

If this is not the case, the campaign will continue regardless of his vote. Thus, a deviation by 1 will have the same effects as if voter 2 voted first (informatively) and then voter 1 deviated. We have seen in the step above that deviations by the second voter are not profitable.

For electoral calendars which are not strictly sequential, the calculus of deviations is very similar.

Lemma 4. *If no deviation by voters is profitable in a sequential election for any relevant funding strategy, no deviation is profitable for voters under any electoral calendar.*

Proof. When voting at the same date as other voters, the voter must consider a weighted average of the effect of his deviation conditional on the vote of those voters who are voting at the same time. Because, as I have shown, a deviation is never profitable, this is a weighted average of negative numbers and, thus, itself negative. Therefore, simple voting is always an equilibrium. ■

Finally, I must make clear that it candidates will always campaign when they have the funds to do so. This is easily enforced by off-equilibrium beliefs on the part of the voters that only an $l-$type would avoid campaigning. That makes not campaigning equivalent to losing the election if it is done when the election is still in play. These beliefs are not arbitrary. One may argue that it is more likely that an $l-$type candidate has more to gain (or hide) by not campaigning than an $h-$type, since he will go on to win the election with lower probability. Therefore, when one sees such a deviation from equilibrium play, it may be considered infinitely more likely to have come from an $l-$type. This is an application of the logic behind the Divinity refinement used in signalling games (Banks and Sobel '87[6]). If the election has been won, the winning candidate may continue to spend campaign funds as he does not value other uses for these funds.

Proof of Proposition 3

I begin by explaining the mechanics of a bandwagon in this model. Under simple equilibria, when the SIG stops making informational donations, it continues to make service-motivated donations to the frontrunner. The frontrunner, in turn, will campaign for the remaining voters. This means that all remaining voters will receive positive signals about the frontrunner. They will vote for him even though they know that the signals are not informative.

This particular series of events, specified by simple equilibrium, is not necessary for bandwagons to form. Rather, I focus on it because it allows a particularly simple specification of equilibrium strategies. If voters are aware that no more informative signals will be sent, it is a best response for them to vote for the frontrunner in order to assure his victory, as the frontrunner will finish the election with the highest posterior probability of being the h-type.

For the SIG to fund the first campaign, it must be that $c < \bar{c} = q - \frac{1}{2}$.

If the SIG stops funding after one informed vote has been cast, the frontrunner will win all remaining votes and thus a bandwagon will be trivially observed.

Suppose that one of the candidates has a 2-0 lead in informed votes in a sequential election. The SIG's posterior belief about the probability that the leading candidate is the correct choice is $\frac{q^2}{q^2+(1-q)^2}$. That is also the probability of the correct candidate winning the election if the SIG funds no further elections. If the SIG does continue to fund campaigns, it must be willing to do so until a candidate reaches 3 votes. This increases the probability of electing the correct candidate to:

$$\frac{q^2}{q^2+(1-q)^2}\left(q+(1-q)\,q+(1-q)^2\,q\right)+\frac{(1-q)^2}{q^2+(1-q)^2}q^3=\frac{q^3}{2q^2-2q+1}\left(2q^2-5q+4\right)$$

So that the increase in the probability of electing the correct candidate is:

$$\frac{(1-q)^2}{q^2+(1-q)^2}q^3-\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)^3=q^2\left(2q-1\right)\frac{(q-1)^2}{2q^2-2q+1}$$

This strategy brings with it an additional cost of:

$$c\left(\frac{q^2}{q^2+(1-q)^2}q+\frac{(1-q)^2}{q^2+(1-q)^2}\left(1-q\right)\right)+2c\left(\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)q+\frac{(1-q)^2}{q^2+(1-q)^2}q\left(1-q\right)\right)+$$
$$3c\left(\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)^2+\frac{(1-q)^2}{q^2+(1-q)^2}q^2\right)=\frac{c}{2q^2-2q+1}\left(2q^4-4q^3+3q^2-q+1\right)$$

Therefore, the SIG will fund voter 3 after a 2-0 start if:

$$c < B\frac{q^2(2q-1)\frac{(q-1)^2}{q^2+(1-q)^2}}{\frac{1}{q^2+(1-q)^2}(2q^4-4q^3+3q^2-q+1)}=Bq^2\left(2q-1\right)\frac{(q-1)^2}{2q^4-4q^3+3q^2-q+1}=c_{\text{seq}}$$

Therefore, the frontrunner will win all remaining votes in the primary after a 2-0 or 1-0 start whenever $c \in (c_{\text{seq}}, \bar{c})$.

Proof of Lemma 4.

I begin by listing all possible electoral calendars in a five voter election. I then prove through a series of Claims that we may focus on only three. Given Theorem 1, I assume throughout that voters vote simply. This allows me to focus on the SIG's funding decisions.

In what follows, I use the following notation: brackets signal that I am referring to a calendar, I separate the dates at which voting takes place with a dash and write the number of voters who vote at each date. Thus, the sequential electoral calendar is {1-1-1-1-1} while the simultaneous calendar is {5}. A question mark stands in for all possible variations of the missing values. For instance, {3-?}=[{3-1-1},{3-2}]. Numbers separated by a dash, but not in brackets denote a partial vote count. For example 2-1 means that one candidate has a 2 vote to 1 vote lead over his competitor.

1. Simultaneous {5}.

2. Sequential {1-1-1-1-1}.

3. Super Tuesday {3-1-1}.

4. {2-3}

5. {3-2}

6. {4-1}

7. {1-4}

8. {2-2-1}

9. {2-1-2}

10. {1-2-2}

11. {1-1-3}

12. {1-3-1}

13. {1-1-1-2}

14. {1-1-2-1}

15. {1-2-1-1}

16. {2-1-1-1}

Lemma 5. *One of the following electoral calendars is voter-optimal among all possible calendars: sequential, simultaneous, or Super Tuesday.*

In order to prove this Lemma, I establish a series of facts which will, together, make the result clear.

Claim 6. *Any calendar in which only voter 1 votes at date 1 is strategically equivalent to a calendar identical except that voter 1 votes at time 2 with the following block of voters.*

Proof. It is clear that the result of the first vote reveals no new information to the donor, that is, the donor knows that the election will either be 1-0 or 0-1 after one informative vote. Because of symmetry, these two situations are strategically equivalent. Therefore, the funding decision of the second voter's campaign will be the same regardless of whether the donor can condition on the outcome of the first vote. ■

This allows us to ignore calendars 7, 10, 11, 12, 13, 14, 15, and 16 and look only at 1, 5, 4, 6, 9, 8, 3, and 2 respectively (or vice-versa).

Claim 7. *Any calendar in which voters 1 through 3 vote at different dates than voters 4 and 5, who vote simultaneously (calendars 5, 9, 10, and 13), is weakly dominated by a calendar identical to it but in which voters 4 and 5 vote sequentially (calendars 3, 16, 15, and 2).*

Proof. If voters 1 through 3 are funded, the election will either be 2-1 or 3-0. Only the first case is relevant since the election is over if it is 3-0. If the last two voters vote simultaneously, the SIG will either fund both or neither since only two votes against the front-runner can change the result. Whenever c is such that the SIG funds both of the last two voters, it will also fund voter 4 in an election ending in {-1-1}, and fund voter 5 if needed. This is because the expected benefit of both strategies is the same, but the expected cost is strictly lower in the sequential case.

As observed above, if the SIG will fund any campaigns at all, it will fund voter 1's. Furthermore, if the SIG intends to fund more than one campaign, it should fund voter 2's since voter 1's vote does not provide new information useful for future funding decisions (the election will be 1-0 either way). If the SIG does not fund the third campaign, it may choose to fund one additional campaign only if the election is tied 1-1 (a 2-0 lead cannot be overcome by two votes, and 2-1 lead cannot be overcome by one vote).

Therefore, the only remaining question is whether voter 3's campaign funding could be adversely affected by having voters 4 and 5 vote sequentially rather than simultaneously. Suppose first that the election is tied after the first two voters have gone to the polls. The election will be 2-1 after an informative 3rd vote. If the

SIG does not fund voter 3's campaign, it will fund at most one more campaign, but in this case it may as well fund voter 3's. If the election is 2-0 after two votes, it is only optimal to fund further campaigns if the SIG is prepared to fund campaigns until a candidate reaches a 3 vote majority. This can be accomplished more cheaply if voters 4 and 5 vote sequentially. If voter 3 votes at the same time as voter 2, the calculus involves an odds weighted average of these two scenarios, so the conclusions continue to hold. Thus, the SIG's funding decisions are more likely to lead to a correct decision if the final two voters vote sequentially. ■

Claim 8. *The calendar {4-1} (no. 6) (and thus {1-3-1} (no. 12)) is dominated by Super Tuesday {3-1-1}.*

Proof. In all cases, the 5th voter campaign, when considered independently, will only be funded if the voter is strongly pivotal (i.e. if the vote total is tied).

If all four voters in the first block of {4-1} are funded, it must be that the donor would fund the fourth voter conditional on the election being 2-1 since it will either be 2-1 or 3-0, in which case the election is over. Therefore, if the fourth voter is funded in a {4-1}, it is also funded in a {3-1-1} when the election is still in play.

In a {4-1}, the SIG will never fund only 3 date-1 campaigns. Funding three voters in the first block of a {4-1} means that voter 5 will never be funded because voter 5 is funded only if the election is tied, which is impossible when an odd number of informative votes have been cast thus far. Moreover, if the SIG funds 2 date-1 campaigns, he can make the funding decision for the third (voter 5) after conditioning on the outcome of the first two (i.e. fund it only if the informative vote count is 1-1 and not 2-0). Therefore, it is strictly better for the SIG to fund 2 campaigns on date 1 and then fund voter 5 if the informative vote count is tied, thus giving the same probability of success at a strictly lower expected cost.

If only two campaigns are funded in the first block of {4-1}, at least two will be funded in a {3-1-1}. In both cases, only one additional campaign may be funded: if the election is 2-0 after the first block, the lead cannot be overcome, if it is 1-1 one additional vote will make it 2-1 and the last vote cannot overcome that lead. Therefore, if it is optimal to fund the two voters in the {4-1} it is also optimal to do so in {3-1-1}. ■

The following Claim shows that the sequential calendar dominates any calendar beginning {1-1-?} or {2-?}.

Claim 9. *Any calendar in which voters 1 and 2 vote at different dates than voters 3, 4 and 5 is dominated by a calendar identical except that voters 3, 4 and 5 vote sequentially.*

Proof. By Claim 1, voter 1's campaign will be funded whenever the comparison of these calendars is in question. Because the SIG can condition its choice on the outcome of 2's vote after it has taken place, if the SIG is going to fund more than one campaign, it is optimal for it to fund voter 2's. Therefore, I compare calendars conditional on two informative votes having been cast.

Suppose a candidate has a 2-0 lead after voters 1 and 2 have voted. Then, the SIG will only fund further campaigns if it is willing to fund campaigns until one candidate has received three favorable informed votes. This may be done at a lower expected cost when the final three voters vote sequentially because the SIG can choose to stop funding as soon as one candidate reaches 3 votes. Therefore, having the final three voters vote sequentially dominates all other arrangements of the last three voters conditional on the first two voters voting informatively for the same candidate.

Now suppose the election is tied after voters 1 and 2 have gone to the polls. The SIG will fund voter 3's campaign since the cost of previous campaigns is sunk and it was willing to fund the campaign of voter 1. One candidate will have a 2-1 lead after voter 3's vote. Because of the symmetry of the game, it does not matter which candidate it is for the SIG's funding decision and therefore a calendar in which voters 3 and 4 vote simultaneously is strategically identical to one in which they vote sequentially. By Claim 7, if the first three campaigns have been funded, the calendar with voters 4 and 5 voting sequentially dominates the one in which they vote simultaneously. ■

Application of these Claims leaves us with three contenders for the voter-optimal electoral calendar: sequential, simultaneous, and Super Tuesday. The simultaneous calendar is dominated by the Super Tuesday calendar. However, because of its special role in the literature, I will examine it more closely than other dominated calendars. I include the proof of this dominance relation in the following proof.

Proof of Theorem 5 and Proposition 6

Theorem 10. *There exist values* $c_{sim} < c_{\mathrm{seq}} < c_{st} < \hat{c}_{\mathrm{seq}} < \bar{c}$ *such that:*

- *The Super Tuesday calendar dominates all other calendars, and strictly dominates the sequential calendar, when* $c \in (c_{\mathrm{seq}}, c_{st})$.
- *The sequential calendar dominates all other calendars, and strictly dominates the Super Tuesday calendar, when* $c \in (c_{st}, \hat{c}_{\mathrm{seq}})$.
- *The simultaneous calendar weakly dominates all other calendars when* $c < c_{sim}$ *or* $c > \bar{c}$.

Proof. Funding one (or two) campaigns results in the h-type winning the nomination with probability q. The first voter will be funded under any electoral calendar if $c < \Delta b \left(q - \frac{1}{2}\right)$.

In a simultaneous election, funding three (or four) campaigns leads to selecting the correct candidate with probability:

$q^2 (q + 3 (1 - q)) = q^2 (3 - 2q)$

The increase in the probability of selecting the h-type resulting from funding three campaigns rather than one is:

$q^2 (3 - 2q) - q = -q (2q^2 - 3q + 1) = -2q^3 + 3q^2 - q > 0$

The additional cost, given that one campaign is being funded, is $2c$. Therefore, the SIG will fund three campaigns if:

$c < \Delta b \frac{1}{2} (-2q^3 + 3q^2 - q)$

Funding five campaigns leads to selecting the correct candidate with probability:

$q^3 \left(q^2 + 5q (1 - q) + 10 (1 - q)^2\right) = q^3 (10 - 15q + 6q^2)$

Subtracting the first expression from the second, we get the increase in probability of success from funding voters 4 and 5:

$q^3 (10 - 15q + 6q^2) - q^2 (3 - 2q) = 3q^2 (2q - 1) (q - 1)^2 = 6q^5 - 15q^4 + 12q^3 - 3q^2$

The difference in cost between the two funding strategies is $2c$, so the SIG will fund all five campaigns if:

$$c < \frac{1}{2}\Delta b \left(6q^5 - 15q^4 + 12q^3 - 3q^2\right) = c_{sim}$$

In a Super Tuesday election, funding all three date-1 campaigns leads to selecting the right candidate with probability:

$$q^3 \left[1 + 3(1-q) + 6(1-q)^2\right] = 10q^3 - 15q^4 + 6q^5$$

at an expected cost of:

$$3c + c\left(1 - q^3 - (1-q)^3\right) + c\left(6q^2(1-q)^2\right) = 3c\left(1 + q + q^2 - 4q^3 + 2q^4\right)$$

Funding only two campaigns in the first block leads to selecting the right candidate with probability:

$$q^2(1 + 2(1-q)) = q^2(3-2q)$$

at an expected cost of:

$$2c + c\left(1 - q^2 - (1-q)^2\right) = 2c(1 + q(1-q)) < 3c$$

Note that the SIG will be willing to fund this strategy for higher c than it is to fund three campaigns in a simultaneous election because the difference in cost from funding only one campaign to following this strategy is $c + 2cq(1-q) < 2c$, while the benefits of the change are the same.

The increase in the probability of nominating an h-type from funding voter 3's campaign is:

$$6q^5 - 15q^4 + 12q^3 - 3q^2$$

Subtracting the expected cost of the fund 2 strategy from that of the fund 3 I find the difference in expected cost:

$3c\left(1+q+q^2-4q^3+2q^4\right)-2c\left(1+q\left(1-q\right)\right)=c\left(6q^4-12q^3+5q^2+q+1\right)=$
$c\left(1+q\left(1-q\right)\left(1+6q\left(1-q\right)\right)\right)$

Therefore, the SIG will fund all 3 date-1 campaigns in a Super Tuesday election if:

$c<\Delta b\frac{6q^5-15q^4+12q^3-3q^2}{1+q(1-q)(1+6q(1-q))}=c_{st}$

Because $q\left(1-q\right)$ reaches a maximum for $q\in\left(0,1\right)$ at $\frac{1}{4}$, $q\left(1-q\right)\left(1+6q\left(1-q\right)\right)\leq \frac{5}{8}$ and therefore $c_{st}>c_{sim}$.

Funding only one campaign leads to selecting the h-type with probability q. The increase in the probability of success from this strategy to funding two date-1 voters in a Super Tuesday election is:

$q^2\left(3-2q\right)-q=-q\left(2q^2-3q+1\right)=-2q^3+3q^2-q>0$

The increase in cost from funding one campaign to funding two date-1 campaigns in a Super Tuesday election is:

$c+2cq\left(1-q\right)<2c$

Therefore, at least two date-1 campaigns will be funded in a Super Tuesday election if:

$c<\Delta b\left(\frac{-2q^3+3q^2-q}{c+2cq(1-q)}\right)=\bar{c}$

Suppose that one of the candidates has a 2-0 lead in informed votes in a sequential election. The SIG's posterior belief about the probability that the leading candidate is the correct choice is $\frac{q^2}{q^2+(1-q)^2}$. That is also the probability of the correct candidate winning the election if the SIG funds no further elections. If the SIG does continue to fund campaigns, it must be willing to do so until a candidate reaches 3 votes. This increases the probability of electing the correct candidate to:

$$\frac{q^2}{q^2+(1-q)^2}\left(q+(1-q)q+(1-q)^2 q\right)+\frac{(1-q)^2}{q^2+(1-q)^2}q^3=\frac{q^3}{2q^2-2q+1}\left(2q^2-5q+4\right)$$

So that the increase in the probability of electing the correct candidate is:

$$\frac{q^3}{2q^2-2q+1}\left(2q^2-5q+4\right)-\frac{q^2}{q^2+(1-q)^2}=q^2\left(2q-1\right)\frac{(q-1)^2}{2q^2-2q+1}$$

or,

$$\frac{(1-q)^2}{q^2+(1-q)^2}q^3-\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)^3=q^2\left(2q-1\right)\frac{(q-1)^2}{2q^2-2q+1}$$

This strategy brings with it an additional cost of:

$$c+c\left(\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)+\frac{(1-q)^2}{q^2+(1-q)^2}q\right)+c\left(\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)^2+\frac{(1-q)^2}{q^2+(1-q)^2}q^2\right)$$

$$=\frac{c}{q^2+(1-q)^2}\left(2q^4-4q^3+3q^2-q+1\right)$$

or,

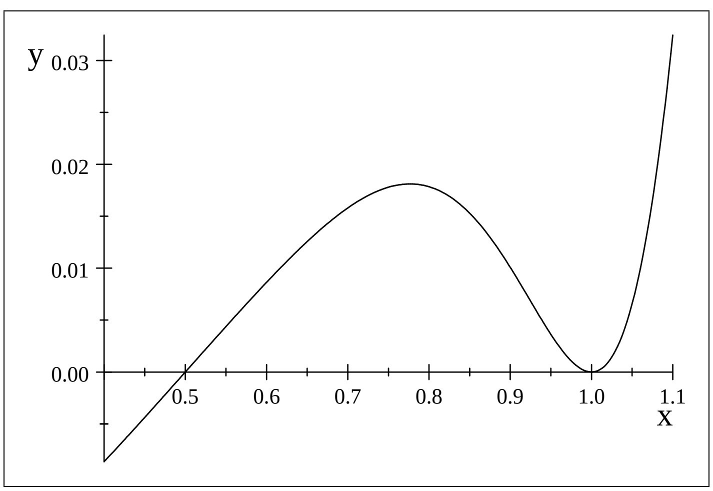
$$c\left(\frac{q^2}{q^2+(1-q)^2}q+\frac{(1-q)^2}{q^2+(1-q)^2}\left(1-q\right)\right)+2c\left(\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)q+\frac{(1-q)^2}{q^2+(1-q)^2}q\left(1-q\right)\right)+$$
$$3c\left(\frac{q^2}{q^2+(1-q)^2}\left(1-q\right)^2+\frac{(1-q)^2}{q^2+(1-q)^2}q^2\right)=\frac{c}{2q^2-2q+1}\left(2q^4-4q^3+3q^2-q+1\right)$$

Therefore, the SIG will fund voter 3 after a 2-0 start if:

$$c < \Delta b \frac{q^2(2q-1)\frac{(q-1)^2}{q^2+(1-q)^2}}{\frac{1}{q^2+(1-q)^2}(2q^4-4q^3+3q^2-q+1)} = \Delta b q^2 (2q-1) \frac{(q-1)^2}{2q^4-4q^3+3q^2-q+1} = c_{\text{seq}}$$

If $c_{\text{seq}} < c_{st}$, then there will be circumstances under which a Super Tuesday calendar outperforms a sequential calendar. This is because, when $c \in (c_{\text{seq}}, c_{st})$, the Super Tuesday calendar will continue to fund campaigns when they start 2-0 and go to 2-1, while with the sequential calendar funding would stop at 2-0. The Super Tuesday calendar takes advantage of the uncertainty about whether the election will start 2-0 or 1-1. Because the two calendars are identical after voter 3, this advantage is the only difference in this range. It is difficult to verify algebraically that $c_{\text{seq}} < c_{st}$, but straight forward to do so numerically as we need only check that the inequality holds for values of q in $\left(\frac{1}{2}, 1\right)$:

$$c_{st} - c_{\text{seq}} = \frac{6q^5-15q^4+12q^3-3q^2}{1+q(1-q)(1+6q(1-q))} - q^2 (2q-1) \frac{(q-1)^2}{2q^4-4q^3+3q^2-q+1} > 0$$



In a sequential election, if three campaigns have been financed leading to a

2-1 vote lead by one of the candidates, continuing to fund campaigns makes sense for the SIG only if it is willing to fund until one candidate has three votes. This leads to electing the correct candidate with probability $q^2(3-2q)$, while stopping funding now means the frontrunner will win the election, which is the correct choice with probability q. The increase in the probability of selecting the correct candidate is therefore:

$$q^2(3-2q)-q=-q\left(2q^2-3q+1\right)$$

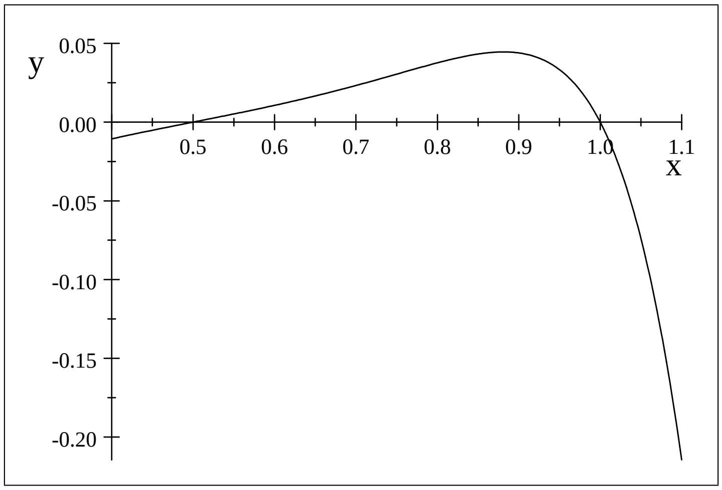
This strategy leads to additional expected costs of $c+c\left(2q\left(1-q\right)\right)=c\left(-2q^2+2q+1\right)$. Or if we derive it differently: $c\left(q^2+\left(1-q\right)^2\right)+4cq\left(1-q\right)=c\left(-2q^2+2q+1\right)$. Therefore, the SIG will continue this funding if:

$$c<\Delta b\frac{-q\left(2q^2-3q+1\right)}{-2q^2+2q+1}=\hat{c}_{\text{seq}}$$

On the other hand, if the SIG funds only two date-1 campaigns in a Super Tuesday election it will fund a third if the election is tied 1-1 in informed votes after the first block has voted, but will never fund more than that because a 2-1 lead which would ensue could never be overcome by a single informed vote. Therefore, there may be a range of costs, $c\in\left(c_{st},\hat{c}_{\text{seq}}\right)$, over which the sequential calendar strictly overperforms the Super Tuesday calendar in expected terms.

$$\hat{c}_{\text{seq}}-c_{st}=\frac{-q\left(2q^2-3q+1\right)}{-2q^2+2q+1}-\frac{6q^5-15q^4+12q^3-3q^2}{1+q(1-q)(1+6q(1-q))}>0$$

Again, I verify this inequality numerically.



y
0.05
0.00
-0.05
-0.10
-0.15
-0.20
0.5
0.6
0.7
0.8
0.9
1.0
1.1
x

■

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