

# Ideology and Competence in Alternative Electoral Systems.\*

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## Abstract

We develop a model of elections in proportional (PR) and majoritarian (FPTP) electoral systems. The model allows for an endogenous number of candidates, differentiation of candidates in a private value dimension, or ideology, and in a common value dimension, which we interpret broadly as quality. Voters are fully rational and strategic. We show that the quality of candidates is always at least as high in majoritarian electoral systems as in proportional electoral systems, and that in all equilibria in which candidates are differentiated, the number of candidates is at least as large in PR as in FPTP (where exactly two candidates run). Moreover, we provide conditions under which the rankings are strict. The diversity of ideological positions represented in elections can be larger in proportional or majoritarian elections. In the most efficient equilibrium, however, FPTP implies perfect convergence of candidates at the median. Since a convergent equilibrium can never occur in PR, and voters are risk averse, the most efficient equilibrium in FPTP dominates the most efficient equilibrium in PR in terms of voters' welfare.

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# 1 Introduction

Electoral systems translate votes cast in elections to the number of seats won by each party in the national legislature. By affecting how voters' preferences are ultimately mapped into policy outcomes, they are one of the fundamental institutions in representative democracies. They also influence indirectly - through the choices they induce in voters and politicians - most key features of modern political systems: from the number of alternatives faced by voters and the diversity of ideological positions they represent, to the quality of the candidates, of their staff and of their platforms. These outcomes are themselves naturally intertwined: the less diverse are the policy positions represented by candidates running for office, the larger the number of voters that will be swayed by a quality differential between these alternatives; the more diverse the ideological positions represented by candidates running for office, the larger the incentive for a new candidate to run representing an intermediate ideological alternative.

The contribution of this paper is to tackle jointly the effect of alternative electoral systems on the number of candidates running for office and the quality and ideological diversity of their platforms. To do so, we develop a model of electoral competition that integrates three different approaches in formal models of elections. We allow free entry of candidates, differentiation in a private value dimension, or *ideology*, and differentiation along a common value dimension, which we interpret throughout as *quality*. The main result of the paper is that the quality of candidates is always at least as high in First Past the Post (FPTP) electoral systems than in Proportional Representation (PR) electoral systems, and that in all equilibria in which candidates are differentiated, the number of candidates is at least as large in PR as in FPTP (where exactly two candidates run). We show, moreover, that under mild conditions these rankings are in fact strict; i.e., for a "large" set of parameters, PR leads to more candidates, with strictly less quality on average, than FPTP. The diversity and polarization of the ideological positions represented in the election can in general be larger or smaller in PR than in FPTP. In the most efficient equilibrium, however, FPTP implies perfect convergence of candidates at the median. Since a convergent equilibrium can never occur in PR, and voters are risk averse, the most efficient equilibrium in FPTP dominates the most efficient equilibrium in PR in terms of voters' welfare.

In the model, each potential candidate is endowed with an ideological position that he can credibly implement if he chooses to run and gets elected. While in principle all

positions in the ideological space can be represented in an election, running for office is costly, and therefore in equilibrium only some positions are championed. With the field of competitors given, candidates running for office can then invest resources to improve the quality of the alternative they represent. This differentiation along a bounded common-values dimension can in fact have several interpretations. To fix ideas, we invite the reader to think of candidates as investing valuable resources (money, time, or effort) to increase the probability that their staff is competent or non-corrupt. We assume that in deciding whether to run for office or not, each potential candidate only cares about the spoils he can appropriate from being in office, and that voters are risk averse and fully rational, and therefore vote strategically.

The incentives of voters and politicians are shaped by the electoral system under consideration. Here we focus on proportional and majoritarian electoral systems. The rationale for this is twofold. First, PR and FPTP are two of the most commonly employed electoral systems in modern democracies around the world.<sup>1</sup> Second, proportional and majoritarian systems represent *ideal* entities at the opposite side of the spectrum of what is possibly the main attribute of electoral systems: how each system maps votes into seats. In its purest form, PR implies that the share of votes obtained by each party in the election translates to an equal share of seats for the party in the legislature. FPTP, instead, gives all the control to the candidate obtaining a plurality of votes.<sup>2</sup>

If we extend the comparison to the broader link between votes and *policy outcomes*, the basic difference in how these alternative electoral systems transform votes (in the electorate) to patterns of representation (in government) induces a second level in which proportional and majoritarian systems typically differ. As Lizzeri and Persico (2001) note: “Proportional systems are usually associated with many parties having an influence on policymaking, through the process of post-election bargaining [...] Majoritarian systems are thought to favor the party with the highest share of the vote, in the sense that more power of policy setting is conferred to that party [...]” Together, these two channels generate

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<sup>1</sup>Around one fourth of countries employing FPTP electoral systems, and around one third of all countries employing PR systems. These proportions change from 23.6 percent to 32.4 per cent for FPTP, and from 35 percent to 30.9 percent for PR when the universe is the class of established democracies, see IDEA (2005).

<sup>2</sup>This is, of course, a very stylized representation of a complex and diverse array of electoral institutions. As Cox (1997) argues, however, “much of the variance in two of the major variables that electoral systems are thought to influence - namely, the level of disproportionality between each party’s vote and seat shares, and the frequency with which a single party is able to win a majority of seats in the national legislature - is explained by this distinction.”

two main differences between proportional and majoritarian systems. First, proportional and majoritarian systems differ in how they ultimately reward vote shares: vote shares are rewarded more disproportionately in majoritarian systems, and more proportionally in PR systems. Second, given a voting outcome in the electorate, the process of post-election bargaining in PR introduces more uncertainty for voters in the selection of the policy outcome. To capture these two fundamental differences in a stylized manner within our model, we assume that in FPTP the candidate with a plurality of votes appropriates all rents from office and implements the policy he represents, while in PR systems the policy outcome is the result of a lottery between the policies represented by the candidates participating in the election, with weights equal to their vote shares (or seat share in the assembly).

To prove our main result, we begin by characterizing equilibria of the model for FPTP. We show that in any FPTP electoral equilibrium in which candidates represent different policies, only two top quality candidates compete for office, and that these candidates are symmetrically located around the median ideological position in the electorate. The existence of a two candidate equilibrium along the lines described above relies heavily on strategic voting. For suppose that even after entry of a third candidate, all voters vote for their preferred candidate among the two equilibrium candidates. Then any deviation from sincere voting would only cause this voter's least preferred equilibrium candidate to win the race for sure. The Duvergerian result that exactly two differentiated candidates will run for office in FPTP, on the other hand, follows from our assumption that voters are risk averse.

We then show that in PR elections admit multi-candidate symmetric equilibria in which the average quality of candidates is non-maximal, as long as candidates are sufficiently differentiated in the ideological dimension. The reason for this follows from the tension that emerges in our model between differentiation in policies and quality. The closer candidates are in terms of their ideological position, the larger is the number of voters that can be attracted by a given increase in quality by one of the candidates. This implies in turn that candidates will be more aggressive in the game of quality competition the closer they are to one another, eventually competing away their rents. Candidates that are sufficiently differentiated in the ideological dimension, instead, are not close substitutes for voters. Hence, quality competition is relaxed and candidates running for office can choose non-maximal quality while still getting a positive share of office rents in equilibrium. The limit

to the degree of horizontal differentiation among candidates is entry: candidates cannot be too differentiated in PR elections, for otherwise - given that strategic voting is sincere - this will induce entry.

Together, these results imply that the number of differentiated candidates in majoritarian electoral systems is lower, and their quality higher, than in PR electoral systems.

In our stylized PR elections, each candidate running for office captures a proportion of office rents equal to his share of votes in the election. In several polities, however, it might be more appropriate to assume that the majority party can obtain an additional reward over and above its share of votes in the election. To capture this feature, we also consider a modified version of the model in which the candidate with the largest number of votes obtains a majority premium in both the probability of implementing her policy and in the proportion of office rents he appropriates after the election. This new environment (PR-Plus) can then be thought of as an intermediate electoral system between PR and FPTP. We show that in large elections in PR-Plus there exists an electoral equilibrium with two top quality candidates, symmetrically located around the median voter. We also show that in large elections, for any majority premium electoral equilibria are either of this kind, or such that a single candidate obtains a plurality of votes. Thus, in large electorates, equilibrium behavior in PR-Plus resembles that in FPTP. Under PR-Plus, however, the set of two-candidate equilibria has to be pruned to rule out complete convergence, and under some conditions also extreme polarization. Moreover, these results do not imply a complete discontinuity with the PR environment: for a fixed size of the electorate, in fact, if the majority premium is sufficiently small, PR-Plus elections admit equilibria with more than two candidates choosing non-maximal quality, as in the case of pure PR.

## 2 Related Literature

Our paper is related to three strands of literature. The first strand focuses on the relation between electoral systems and the number of political parties. This literature provides several formalizations of the well-known *Duvergerian* predictions, namely that the number of parties tend to be larger in proportional electoral systems than in majoritarian systems (where only two parties should run). This literature, contrary to our paper, considers only “horizontal” or ideological differentiation between parties. Examples are Osborne and Slivinski (1996) for a comparison between plurality and plurality with runoff under

sincere voting, and Morelli (2004) for a comparison between majoritarian and proportional electoral systems under strategic voting. These papers show that the comparison crucially depends on the initial distribution of preferences among voters. See also Cox (1997) for an empirical discussion of the Duvergerian predictions.<sup>3</sup>

A second strand of literature analyzes how variations in the electoral system affect policy outcomes. Myerson (1993) focuses entirely on how the nature of electoral competition affects promises of redistribution by candidates in the election. Building on this work, Lizzeri and Persico (2001) consider redistribution and provision of public goods in PR and FPTP electoral systems. In both of these papers, the emphasis is not on differentiation (in ideology or quality) but rather on the vote-buying strategies of the candidates. Austen-Smith and Banks (1988) and Baron and Diermeier (2001) consider models of elections *and* legislative outcomes in PR, where rational voters anticipate the effect of their vote on the bargaining game between parties in the elected legislature, for a given number of parties. Finally, several recent papers consider alternative electoral systems with strategic voters when the relevant policy outcome is not bargaining over a fixed prize, but instead taxation and redistribution (e.g., Austen-Smith (2000), Persson, Roland, and Tabellini (2003)), or corruption (Persson, Tabellini, and Trebbi (2006)).

Finally, our paper is also related to the large literature that, following Stokes (1963)'s original critique to the Downsian model, incorporates competition in *valence* issues, typically within FPTP, and with a given number of candidates (two). For recent papers see Ashworth and Bueno de Mesquita (2007), Callander (2008), Carrillo and Castanheira (2006), Eyster and Kittsteiner (2007), Herrera, Levine, and Martinelli (2005), and Meirowitz (2007).<sup>4</sup> Of these, the closest paper to ours is Ashworth and Bueno de Mesquita (2007). Following the insight of d'Aspremont, Gabszewicz, and Thisse (1979) in the context of firms, Ashworth and Bueno de Mesquita obtain that in FPTP elections with two candidates, candidates have an incentive to "diverge" in order to soften valence competition. It is useful to clarify, however, that while the same underlying mechanism linking policy and quality differentiation is present in our model, the centrifugal force does not exist here,

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<sup>3</sup> For papers that study entry in FPTP elections under the assumption of sincere voting see, e.g., Palfrey (1984), and Greenberg and Shepsle (1987). For papers that study entry in FPTP under strategic voting see, e.g., Feddersen, Sened, and Wright (1990), and Besley and Coate (1997). For models of differentiation and entry in industrial organization, see d'Aspremont, Gabszewicz, and Thisse (1979), Shaked and Sutton (1982, 1987), and Perloff and Salop (1985).

<sup>4</sup> See also Groseclose (2001), Aragonés and Palfrey (2002), Schofield (2004), and Kartik and McAfee (2007) for models where one candidate has an exogenous valence advantage.

since in our case an endogenous field of candidates is endowed with fixed policy positions and voters are strategic.

### 3 The Model

Let  $X = [0, 1]$  be the ideology space. In any  $x \in X$  there is a finite set of potential candidates, each of whom will perfectly represent policy  $x$  if elected. There are three stages in the game. In the first stage, all potential candidates simultaneously decide whether or not to run for office. Potential candidates only care about the spoils they can appropriate from being in office, and must pay a fixed cost  $F$  to run for office. We denote the set of candidates running for office at the end of the first stage by  $\mathcal{K} = \{1, \dots, K\}$ . In a second stage, all candidates running for office simultaneously invest in quality  $\theta_k \in [0, 1]$ . Candidates can acquire quality  $\theta_k$  at a cost  $C(\theta_k)$ ,  $C(\cdot)$  increasing and convex. We let  $C(1) \equiv \bar{c}$  and to allow competitive elections we assume that  $F + \bar{c} \leq \frac{1}{2}$ . In the third stage,  $N$  fully strategic voters vote in an election, where we think as  $N$  being a large finite number. A voter  $i$  with ideal point  $z^i \in X$  ranks candidates according to the utility function  $u(\cdot; z^i)$ , which assigns to candidate  $k$  with characteristics  $(\theta_k, x_k)$  the payoff  $u(\theta_k, x_k; z^i) \equiv 2\alpha v(\theta_k) - (x_k - z^i)^2$ , with  $v$  increasing and concave. The ideal point of each voter is drawn at the beginning of the third stage from a uniform distribution with support in  $[0, 1]$ .

The electoral system determines the mapping from voting profiles to policy outcomes and the allocation of rents. In FPTP the candidate with a plurality of votes appropriates all rents from office and implements the policy he represents. In PR systems, instead, the policy outcome is a lottery between the ideologies championed by the candidates participating in the election, with weights equal to their vote shares in the election (or seat share in the assembly). The (expected) share of rents captured by each candidate is also proportional to his vote share in the election. Let  $s_k$  denote the proportion of votes for party  $k$ , and  $m_k$  the proportion of rents captured by party  $k$ . Also, let  $\theta_{\mathcal{K}} \equiv \{\theta_k\}_{k \in \mathcal{K}}$ , and  $x_{\mathcal{K}} \equiv \{x_k\}_{k \in \mathcal{K}}$  denote the quality and policy positions of the candidates running for office. The expected payoff of a candidate  $k$  running for office in electoral system  $j$  can then be written as

$$\Pi_k^j(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = m_k^j(\theta_{\mathcal{K}}, x_{\mathcal{K}}) - C(\theta_k) - F. \quad (1)$$

We assume that  $m_k^{PR}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = s_k(\theta_{\mathcal{K}}, x_{\mathcal{K}})$ , and that under FPTP ties are broken by the

toss of a fair coin, so that letting  $H \equiv \{h \in \mathcal{K} : s_k = s_h\}$ ,

$$m_k^{FP}(\theta_{\mathcal{K}}, x_{\mathcal{K}}) = \begin{cases} \frac{1}{|H|} & \text{if } s_k \geq \max_{j \neq k} \{s_j\} \\ 0 & \text{o.w.} \end{cases}$$

A strategy for candidate  $k$  is a running decision  $e_k \in \{0, 1\}$ , and a quality decision  $\theta_k(\mathcal{K}, x_{\mathcal{K}}) \in [0, 1]$ . A strategy for voter  $i$  is a function  $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) \in \mathcal{K}$ , where  $\sigma_i(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}}) = k$  indicates the choice of voting for candidate  $k$ , and  $\sigma = \{\sigma_1(\cdot), \dots, \sigma_N(\cdot)\}$  denotes a voting strategy profile. A *short run electoral equilibrium* is a set of candidates running for office  $\mathcal{K}^*$ , policy positions  $x_{\mathcal{K}^*}$ , quality choices  $\theta_{\mathcal{K}^*}$ , and a voting profile  $\sigma^*$  such that: (i)  $\theta_k^*$  is optimal for  $k$  given  $\{\theta_{\mathcal{K}^* \setminus k}^*(\mathcal{K}^*, x_{\mathcal{K}^*}^*), x_{\mathcal{K}^*}^*, \sigma(\mathcal{K}^*, x_{\mathcal{K}^*}^*, \theta_{\mathcal{K}^* \setminus k}^*, \theta_k^*)\}$ ; i.e.,  $\theta_{\mathcal{K}^*}^*$  is a (pure Nash) equilibrium of the continuation game  $\Gamma_{\mathcal{K}^*}$ , and (ii) if  $k \in \mathcal{K}^*$ , then  $\Pi_k(\mathcal{K}^*, x_{\mathcal{K}^*}^*, \theta_{\mathcal{K}^*}^*, \sigma^*(\mathcal{K}^*, x_{\mathcal{K}^*}^*, \theta_{\mathcal{K}^*}^*)) \geq 0$  (no exit). An *electoral equilibrium* is a Subgame Perfect Nash Equilibrium in pure strategies of the game of electoral competition; i.e., a short run electoral equilibrium with the additional condition of non-profitable entry: (iii) if  $k \notin \mathcal{K}^*$ , then  $\Pi_k(\mathcal{K}^* \cup k, x_k, x_{\mathcal{K}^*}^*, \theta_k^*, \theta_{\mathcal{K}^*}^*, \sigma^*(\mathcal{K}^* \cup k, x_k, x_{\mathcal{K}^*}^*, \theta_k^*, \theta_{\mathcal{K}^*}^*)) < 0$  in an equilibrium of the continuation game.

An *outcome* of the game is a set of candidates running for office  $\mathcal{K}$ , policy positions  $x_{\mathcal{K}}$ , and quality choices  $\theta_{\mathcal{K}}$ . A *polity* is a triplet  $(\alpha, \bar{c}, F) \in \mathfrak{R}_+^3$ . We say that the model *admits* an electoral equilibrium with outcome  $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$  if there exist a set of polities  $P \subseteq \mathfrak{R}_+^3$  with positive measure such that if a polity  $p \in P$  then there exists an electoral equilibrium with outcome  $(\mathcal{K}, x_{\mathcal{K}}, \theta_{\mathcal{K}})$ .

## 4 Main Results

In this section we state our main result regarding the comparison between alternative electoral systems (Theorem 1). In particular, we show that average quality among all candidates running for office is always (weakly) higher under majoritarian elections than under proportional elections, and that in all equilibria in which candidates are differentiated, the number of candidates running for office is (weakly) smaller under majoritarian electoral systems. We show, moreover, that under mild conditions, these rankings are in fact strict; i.e, for a relatively large set of parameters, PR leads to more candidates, with strictly less quality on average, than FPTP.

We begin our analysis by considering the case of majoritarian or First-Past-the-Post (FPTP) electoral systems. We show that majoritarian elections have a powerful impact

on behavior, in the sense that in equilibrium the number of candidates running for office and their choice of quality is uniquely determined. Moreover, although there are multiple equilibria (in fact a continuum), they can be ranked according to a utilitarian social welfare function. The following result characterizes equilibria in FPTP elections.

**Proposition 1** *Consider elections in FPTP electoral systems. An electoral equilibrium always exists. In any equilibrium in which candidates represent different ideological positions: (i) exactly two candidates compete for office, (ii) candidates are symmetrically located around the median in the policy space (i.e.,  $x_1 = 1 - x_2$ ), and (iii) they choose maximal quality (i.e.,  $\theta_1^* = \theta_2^* = 1$ ).*

Proposition 1 shows that in our environment, Duverger’s law holds in almost all electoral equilibria. Although many candidates can run for office, majoritarian elections trim down competition between differentiated candidates to two top quality candidates. The degree of ideological differentiation between candidates, however, is not pinned down by equilibrium: FPTP elections admit both equilibria in which candidates are perfect centrists, and maximally polarized. For some parameter values, there also exists an equilibrium in which more than two *perfectly centrist* candidates run for office. These candidates, however, are identical, in that they represent the same ideological position.

To see the intuition for the result, note that in FPTP elections all candidates running for office must tie in equilibrium. In fact, given the winner-takes-all nature of FPTP elections, candidates will choose to run for office only if they have a positive chance of winning. From the fact that in any equilibrium all candidates must tie, it follows that (a) voters must vote sincerely, and that (b) candidates must be choosing maximal quality. Given that voters are uniformly distributed in  $[0, 1]$ , these facts also imply that (c) in any equilibrium the set of candidates running for office must be located symmetrically with respect to the median ideological position. To see that there cannot be an electoral equilibrium with  $K > 2$  differentiated candidates running for office, note that if this were the case, (a) and (b) imply that by deviating and voting for any candidate  $j$  other than her preferred candidate, a voter could get candidate  $j$  elected with probability one. Revealed preference from equilibrium therefore implies that this voter must prefer the lottery among all  $K^*$  candidates running for office to having  $j$  elected for sure. However, voters’ risk-aversion and (c) imply that any voter must prefer a centrist candidate (i.e., located at the median) to the equilibrium lottery. As a result any voter must also prefer a centrist candidate to

any other candidate which is not her most preferred choice, and in particular a candidate with an ideological position which is between the median and her most preferred ideological position. But this leads to a violation of single-peakedness, which is not consistent with the assumption of a strictly concave utility function.<sup>5</sup> As a result, the only possible equilibrium must have exactly two symmetrically located candidates choosing maximal quality. In the proof we show that such an equilibrium exists, and in fact that there is a continuum of two-candidates symmetric equilibria, with candidates choosing maximal quality.

Two remarks are in order. First, the reason why any two-candidates symmetric configuration can be supported in equilibrium follows from two assumptions: (i) strategic voting, and (ii) the fact that each potential candidate, when deciding whether to run or not, only cares about the spoils he can appropriate from being in office. To see how strategic voting operates in this context, suppose that all voters vote for their preferred candidate among the two equilibrium candidates, even after entry of a third candidate. Then any deviation from sincere voting by any voter would only cause her least preferred equilibrium candidate to win the race for sure, and therefore entry is never profitable in any two-candidate race. Assumption (ii) rules out situations where potential candidates may choose not to run for office simply because another candidate championing a “close” ideological position is already running. Note that without this assumption perfect convergence in FPTP elections cannot be supported in equilibrium (see the “citizen-candidate” models of Osborne and Slivinski (1996) and Besley and Coate (1997)). Stated differently, assuming policy-motivated candidates immediately implies some policy divergence in equilibrium. We show, however, that even assuming (ii), convergence cannot be supported in equilibrium in PR elections. Second, note that two-candidates symmetric equilibria can be easily ranked in terms of aggregate voters’ welfare. In particular the equilibrium that displays perfect convergence, i.e., where  $x_1 = x_2 = \frac{1}{2}$  and  $\theta_1 = \theta_2 = 1$ , is the most efficient in the sense that it maximizes aggregate welfare.<sup>6</sup>

Our examination of electoral competition in FPTP yielded sharp predictions, restricting competition for office to two top quality candidates across almost all equilibria (with the

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<sup>5</sup>Feddersen, Sened, and Wright (1990) use a similar argument in the context of pure private value model.

<sup>6</sup>To see this notice that when  $x_1 = 1 - x_2 = \frac{1}{2} - y$  for  $y \in [0, \frac{1}{2}]$  and  $\theta_1 = \theta_2 = 1$ , aggregate welfare equals

$$\int_0^1 \left( \frac{1}{2} u \left( 1, \frac{1}{2} + y; z_i \right) + \frac{1}{2} u \left( 1, \frac{1}{2} - y; z_i \right) \right) dz_i = \alpha v(1) - \left( y^2 + \frac{1}{12} \right),$$

which is decreasing in  $y$  and maximized at  $y = 0$  or  $x_1 = x_2 = \frac{1}{2}$ .

exception of the convergent equilibria, in which more than two candidates can run). We consider next the case of proportional (PR) electoral systems. The characterization of equilibria in PR leads to dramatically different results with respect to the FTPT case. Not only we can now support multi-candidate equilibria with non-maximal quality choice, but complete convergence (in the sense of all candidates running for office representing the same ideological position) cannot be supported in equilibrium. Before proving these results we state a useful lemma.

**Lemma 1** *In any electoral equilibrium in PR elections with  $K \geq 2$  candidates running for office, voters vote sincerely.*

By showing that in PR elections sophisticated voting is in fact equivalent to sincere voting, this lemma greatly simplifies the characterization of equilibria. To get an intuition for this result, recall that in PR each candidate running for office is elected and implements his ideology with a probability proportional to the share of votes received in the election. As a consequence, when a voter votes for a certain candidate, she is affecting the lottery among all candidates running for office by increasing the weight on that particular candidate's position. But this implies that voting for a candidate which is not the most preferred one is always a strictly dominated strategy. In fact, by switching her vote to his most preferred candidate, a voter only affects the lottery's weights of exactly two candidates. But we know that with two alternatives strategic voting and sincere voting coincide.

We are now ready to state our second major result, which characterizes equilibrium outcomes in PR elections. Let  $\Gamma^{FP}$  and  $\Gamma^{PR}$  denote electoral equilibria in FPTP and PR systems respectively, and let  $\bar{\theta}(\Gamma^j)$  denote the average quality among all candidates running for office in  $\Gamma^j$ , for  $j \in \{FP, PR\}$ . Then, we have the following result:

**Proposition 2** *Consider elections in PR*

- (1) *The model admits an electoral equilibrium in PR,  $\Gamma^{PR}$ , such that  $\bar{\theta}(\Gamma^{PR}) < 1$ . Moreover, there exists a decreasing sequence of polities  $\mathcal{P} \equiv \{\alpha_K, \bar{c}_K, F_K\}_{K \geq 3}$  such that for any polity  $(\alpha_K, \bar{c}_K, F_K) \in \mathcal{P}$  we can support an electoral equilibrium  $\Gamma_K^{PR}$  with  $K \geq 3$  candidates running for office and  $\bar{\theta}(\Gamma_K^{PR}) < 1$ .*
- (2) *The model does not admit an electoral equilibrium in PR in which all candidates represent the same policy.*

One important lesson that emerges from Proposition 2 is that in PR elections, candidates running for office must be sufficiently differentiated in the ideological dimension. To see this, consider first the case of perfect convergence at the median (i.e., only centrist candidates run for office). This outcome - which is indeed attainable in equilibrium in FPTP - cannot be supported in PR. The reason for this is that if all candidates running for office are centrist, it is always possible for a candidate with ideology close to the median to run capturing almost half of the votes. Since the converging candidates were making non-negative rents in the proposed equilibrium, the entrant's expected payoff from running must be positive as well, and there is no way to deter his entry. The driving force behind this result is that strategic voting in PR is sincere, on and off the equilibrium path; i.e., given any field of candidates running for office, voting for the preferred (closest) candidate in the set is strictly dominant for any voter.

The previous argument can be extended to show that, in any symmetric equilibrium in PR, candidates must be sufficiently "spaced" with respect to their ideological positions, and as a consequence they will not be perfect substitutes for voters. The reason for this follows from the tension that emerges in our model between differentiation in policies and quality. The closer candidates are in terms of their ideological position, the larger is the number of voters that can be attracted by a given increase in quality by one of the candidates. This implies in turn that candidates will be more aggressive in the game of quality competition the closer they are to one another, eventually competing away their rents. Candidates that are sufficiently differentiated in the ideological dimension, instead, are not close substitutes for voters. Hence, quality competition is relaxed and candidates running for office can choose non-maximal quality while still getting a positive share of office rents in equilibrium. The limit to the degree of horizontal differentiation among candidates is entry: candidates cannot be too differentiated in PR elections, for otherwise - given that strategic voting is sincere - this will induce entry. We exploit the above properties in the proof of Proposition 2 to construct a particular class of equilibria - that we call Location Symmetric Equilibria (LSE) - in which candidates are equally spaced in the ideological dimension. This suffices to show that the model admits electoral equilibria in PR with more than two candidates in which the average quality of candidates is non-maximal. We also show, moreover, that we can support equilibria of this class with an increasingly larger number of parties given sufficiently lower costs of running for office and attaining high quality, and a sufficiently smaller responsiveness of voters to quality - equivalently, a sufficiently larger "ideological

focus” of voters (Stokes (1963)).

Combining the results of Proposition 2 together with our earlier results in Proposition 1, we can conclude that average quality is always weakly higher, and the number of differentiated candidates weakly smaller under FPTP systems than under PR systems. Moreover, we can find a full measure set of parameters for which the inequalities are strict. The next theorem, which follows as an immediate corollary of Propositions 2 and 1 summarizes the comparison.

**Theorem 1**

- (1) *In any electoral equilibrium under FPTP in which candidates are differentiated, the average quality of candidates running for office is weakly higher and their number weakly smaller than under any electoral equilibrium under PR.*
- (2) *There exists a decreasing sequence of polities  $\mathcal{P} \equiv \{\alpha_K, C_K(B), F_K\}_{K \geq 3}$  such that, for any polity  $(\alpha_K, C_K(B), F_K) \in \mathcal{P}$ , we can support under PR an electoral equilibrium with  $K$  candidates running for office,  $\Gamma_K^{PR}$ , where  $\bar{\theta}(\Gamma^{FP}) > \bar{\theta}(\Gamma_K^{PR})$  for any electoral equilibrium  $\Gamma^{FP}$  under FPTP.*

We conclude this section by suggesting a possible welfare comparison between electoral systems. Given the multiplicity of equilibria under both systems, we confine our comparison to be between the most efficient equilibrium in terms of aggregate voters’ welfare in FPTP, which we label  $\tilde{\Gamma}^{FP}$ , and the most efficient equilibrium in PR,  $\tilde{\Gamma}^{PR}$ . Then we have:

**Proposition 3**  *$\tilde{\Gamma}^{FP}$  dominates  $\tilde{\Gamma}^{PR}$  in terms of aggregate voters’ welfare.*

Note that if we consider the class of LSE under PR, the welfare comparison comes as an immediate corollary of our previous results. In fact, we already know that it is not possible to have convergence in PR elections and, given the same level of quality, concavity of voters’ preferences implies that any voter strictly prefers the *expected candidate* with ideological position corresponding to the expected value of the equilibrium lottery to the lottery itself. The result follows from noticing that in any LSE the expected candidate is in fact centrist, and in the most efficient equilibrium in FPTP all candidates running for office are centrists. However, the result of Proposition 3 holds more generally for any electoral equilibrium in PR. To see this, note that first that for *any* equilibrium in PR,

any voter prefers the expected candidate of the equilibrium lottery to the lottery itself. If this expected candidate is centrist, we are done. Otherwise, by concavity of voters' preferences, a centrist candidate will always be preferred by a majority of voters to the expected candidate.

## 5 Majority Premium

We have assumed up to now that in PR elections each candidate running for office captures a proportion of office rents equal to his share of votes in the election. In various polities, however, it might be reasonable to expect that the majority party can obtain an additional reward over and above its share of votes in the election. In several parliamentary democracies adopting some form of PR, for instance, the *formateur* is typically the head of the majority party.<sup>7</sup> To capture this feature, we consider next a modified version of the problem, in which the majority candidate is elected and captures all the rents from office with a probability more than proportional to his vote share. In particular, we assume that the candidate with the largest number of votes obtains a majority premium  $\gamma \in (0, 1)$  in both the probability with which his policy is implemented and in the proportion of office rents he attains after the election. We call this new environment PR-plus (PRP). In PRP, letting as before  $H \equiv \{h \in \mathcal{K} : s_k = s_h\}$ ,  $k$ 's proportion of office's rents after the election,  $m_k$ , is given by

$$m_k = \begin{cases} s_k(1 - \gamma) + \frac{\gamma}{|H|} & \text{if } s_k \geq \max_{j \neq k} \{s_j\}, \\ s_k(1 - \gamma) & \text{o.w.} \end{cases} \quad (2)$$

PRP can then be thought of as an intermediate electoral system between PR ( $\gamma = 0$ ), and FPTP ( $\gamma = 1$ ). The next proposition characterizes PRP elections in large electorates. We show that in large electorates there exists an electoral equilibrium with two top quality candidates, symmetrically located around the median voter, provided that the candidates are not too polarized. We also show that for any majority premium  $\gamma$ , in large elections electoral equilibria are either of this kind, or such that a single candidate appropriates the majority premium with certainty.

### Proposition 4

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<sup>7</sup>In Greece, for example, the fact that the *formateur* has to be the head of the majority is mandated by law.

- (1) *There exists  $\bar{n}$  such that for all  $n \geq \bar{n}$ , there is an electoral equilibrium in which two top quality candidates, symmetrically located around the median voter (i.e.,  $x_1 = 1 - x_2 < 1/2$ ), compete for office.*
- (2) *Fix any sequence of equilibria  $\{\tilde{\sigma}_n\}_{n_0}^\infty$ . There exists  $\bar{n}$  such that if  $n \geq \bar{n}$ , then in  $\tilde{\sigma}_n$ , either two top quality, symmetrically located candidates run for office, or a single candidate appropriates the majority premium with certainty.*

The main intuition for the existence of equilibria with two top quality candidates is that for any majority premium  $\gamma$ , the strategic problem of individual voters in PRP resembles - for sufficiently large electorates - the analogous problem in FPTP. As a result, we can support an equilibrium with two candidates, 1 and 2, by having voters coordinate in voting for their preferred choice among these candidates, even after entry of a third candidate  $\ell$ . To see this, consider without loss of generality a voter  $i$  with preferences  $\ell \succ_i 1 \succ_i 2$  (note that we only need strategic voting among voters whose preferred candidate in  $\{1, 2, \ell\}$  is the entrant,  $\ell$ ). Voter  $i$  faces the following tradeoff. On the one hand, by switching to vote sincerely in favor of the entrant, the voter is transferring  $1/n$  probability mass from her second best candidate ( $k = 1$ ) to her most preferred candidate ( $\ell$ ). On the other hand, she is also inducing a jump of  $\gamma/2$  in the probability that the policy of her least favorite candidate in  $\{1, 2, \ell\}$  emerges as the policy outcome, to be “financed” by a parallel decrease in the probability of her second best candidate’s policy being chosen. For large  $n$ , the second effect dominates, and  $i$  has incentives to vote strategically.<sup>8</sup>

This result, however, should not be taken to imply a complete discontinuity with the PR environment. Note that for fixed  $n$ , and given a strategy profile for all other voters, the incentive to vote strategically in the way described above increases monotonically in the majority premium  $\gamma$ , and in the polarization of candidates 1 and 2: for any strategy profile of the remaining voters, if  $i$  has an incentive to vote strategically given some  $\gamma$ , then  $i$  also has an incentive to vote strategically given  $\gamma' > \gamma$ . Similarly, if  $i$  has an incentive to vote strategically for some given degree of ideological differentiation between candidates 1 and 2, then  $i$  also has an incentive to vote strategically for a larger payoff differential among equilibrium candidates. In fact, it is easy to see that if candidates running for office are not differentiated at all, then there cannot be strategic voting of this type, as

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<sup>8</sup>The intuition for the second part of the proposition follows along the same lines, and is only slightly more involved.

in this case supporting the preferred candidate  $\ell$  comes at not cost. But this implies that there cannot be electoral equilibria with perfect convergence in PRP. On the other hand, in general candidates cannot be too polarized either, for otherwise a deviation by one of the candidates to lower quality, forgoing the majority premium, can be profitable for sufficiently small  $\gamma$ . All in all, while equilibrium behavior of voters and candidates in PRP can resemble behavior in FPTP, the set of equilibria of this class has to be pruned to rule out complete convergence and under some conditions also extreme polarization.

At this point, a natural question to ask is whether equilibria with more than two candidates running for office choosing non-maximal quality - which we have shown can be supported in equilibrium in PR - can survive in the case of PRP elections. The answer is yes, provided that the size of the majority premium is not too big. To see this, note first that whenever a candidate is ahead by at least two votes in a PRP election, strategic voting must be sincere, since in this case any individual deviation in the voting strategy cannot affect the identity of the majority candidate. With this result in mind, consider a location symmetric equilibrium in PR ( $\gamma = 0$ ) such that three candidates run for office choosing non-maximal quality, and the centrist candidate obtains the sincere vote of slightly more than a third of the electorate. Consider now the case of a positive but small majority premium  $\gamma$ , fixing all other parameters of the model. From our previous remark, sincere voting remains a best response when other voters vote sincerely. Moreover, with small enough  $\gamma$ , winning a plurality of the vote is not worth a deviation from the optimal quality choice in the pure PR environment. Finally, note that if the entry of a fourth candidate was not profitable in the case of  $\gamma = 0$ , this has to be true also in the case of a small majority premium. In fact, it is enough for this that when  $\gamma = 0$ , the equilibrium candidates' rents in the continuation game following entry are strictly positive, but we know that this will be the case generically.

## 6 Conclusion

By affecting how voters' preferences are ultimately mapped into policy outcomes, electoral systems are one of the fundamental institutions in representative democracies. They also influence indirectly, through the choices they induce in voters and politicians, most key features of modern political systems. The approach and contribution of this paper is to tackle jointly the effect of alternative electoral systems on the number of candidates

running for office and the quality and ideological diversity of their platforms. To do so, we develop a model of electoral competition that integrates three different approaches in formal models of elections. We allow free entry of candidates, we allow differentiation in a private value dimension, or *ideology*, and we allow differentiation along a common value dimension, which we interpret throughout as *quality*. The main result of the paper is that the quality of candidates is always at least as high in First Past the Post (FPTP) electoral systems than in Proportional Representation (PR) electoral systems, and that consistent with the Duvergerian predictions, the number of candidates representing different ideological positions is always at least as large in PR as in FPTP (where exactly two candidates run). We show, moreover, that under mild conditions, these rankings are in fact strict; i.e, that for a relatively large set of parameters, PR leads to more parties, with strictly less quality on average, than FPTP. The diversity and polarization of the ideological positions represented in the election can in general be larger or smaller in PR than in FPTP. In the most efficient equilibrium, however, FPTP implies perfect convergence of candidates to the median. Since a convergent equilibrium can never occur in PR, and voters are risk averse, the most efficient equilibrium in FPTP dominates the most efficient equilibrium in PR in terms of voters' welfare.

## 7 Appendix

**Proof of Proposition 1.** Note first that in any equilibrium all candidates that are running for office must tie, since otherwise there would be at least one candidate who would lose for sure and - given the fixed cost of running for office  $F > 0$  - would prefer not to run. Since candidates are tying, in equilibrium voters must vote sincerely. If this were not the case, there would exist some voter who is not voting for her most preferred candidate in equilibrium but who could have this candidate winning with probability one by deviating to voting sincerely. Third, note that in any equilibrium it must be that  $\theta_k^* = 1$  for all  $k \in \mathcal{K}^*$ . In fact, since all candidates that are running for office must tie in equilibrium, if  $\theta_h^* < 1$  for some  $h \in \mathcal{K}^*$ , candidate  $h$  can profitably deviate by choosing  $\tilde{\theta}_h = \theta_h^* + \nu$ , for some sufficiently small  $\nu > 0$  (winning the election with probability one). The previous results and the assumption that voters' preferences are uniformly distributed in  $X$  imply that in any equilibrium the set of candidates running for office must be located symmetrically with respect to  $\frac{1}{2}$ . We have then established that in any equilibrium (i) running candidates must tie, (ii) voting is sincere, and (iii)  $\theta_k^* = 1$  for all  $k \in \mathcal{K}^*$ , and that (iv) running candidates must be symmetrically located.

We show next that there cannot be an electoral equilibrium with  $K > 2$  running candidates representing different ideological positions. If this were the case, (i) and (iii) imply that by deviating and voting for any candidate  $j$  other than her preferred candidate, a voter could get candidate  $j$  elected with probability one. But then equilibrium implies that this voter must prefer the lottery among all  $\mathcal{K}^*$  running candidates to having  $j$  elected for sure. This implies, in particular, that

$$\frac{1}{|\mathcal{K}^*|} \sum_{k \in \mathcal{K}^*} u(1, x_k^*; z^i) \geq u(1, x_{K-1}^*; z^i) \quad (3)$$

for all voters such that  $z^i > \frac{x_{K-1}^* + x_K^*}{2}$ , i.e., all voters whose most preferred winning candidate is  $k = K$  and next most preferred winning candidate is  $k = K - 1$ . On the other hand, strict concavity of  $u(\cdot; z^i)$  with respect to policy and (i), (iii), and (iv) imply that for all  $z^i$

$$u(1, \frac{1}{2}; z^i) > \frac{1}{|\mathcal{K}^*|} \sum_{k \in \mathcal{K}^*} u(1, x_k^*; z^i). \quad (4)$$

Combining (3) and (4), we obtain

$$u(1, \frac{1}{2}; z^i) > u(1, x_{K-1}^*; z^i)$$

for all voters such that  $z^i > \frac{x_{K-1}^* + x_K^*}{2}$ . But  $K > 2$  and (iv) imply that  $\frac{1}{2} \leq x_{K-1}^*$ . Hence  $u(\cdot; z^i)$  cannot be single-peaked for all voters such that  $z^i > \frac{x_{K-1}^* + x_K^*}{2}$ , contradicting the strict concavity of  $u(\cdot; z^i)$ .<sup>9</sup>

Finally, note that  $\bar{c} + F \leq \frac{1}{2}$  implies that a unique candidate equilibrium cannot be supported, since otherwise a second candidate, symmetrically located with respect to the median, will always find it profitable to run. As a result, the only possible equilibrium must have exactly two symmetrically located candidates choosing maximal quality. We are only left to show that such an equilibrium exists. So consider a strategy profile in which two top quality candidates, 1 and 2, symmetrically located around the median voter (i.e.,  $x_1 = 1 - x_2 < 1/2$ ), compete for office. Voters vote sincerely among these two candidates on the equilibrium path. If, off the equilibrium path, a third candidate  $\ell$  enters the electoral competition, then we require that voters vote sincerely among candidates in  $\{1, 2\}$  for all  $(\theta_1, \theta_2, \theta_3)$  for which  $\max\{\theta_1, \theta_2\} = 1$ .<sup>10</sup> We show that this strategy profile is an electoral equilibrium. First note that on the equilibrium path, voters are best responding, since with two candidates strategic voting is sincere. Next note that given that  $\bar{c} + F \leq \frac{1}{2}$ , equilibrium rents of the two running candidates are always non-negative. Since candidates are choosing maximal quality in equilibrium,  $\theta_1^* = \theta_2^* = 1$ , the only possible deviation in the quality game is downwards. But any such deviation would entail sure loss, and is thus not profitable. Suppose now that a third candidate  $\ell$  such that  $x_\ell \in [0, 1]$  decides to enter. Recall that voters vote sincerely among candidates in  $\{1, 2\}$  for all  $(\theta_1, \theta_2, \theta_3)$  for which  $\max\{\theta_1, \theta_2\} = 1$ . But given these strategies, there is no voter which can benefit from a deviation. In fact, since candidates 1 and 2 are tying, any deviation from sincere voting between candidate 1 and candidate 2 in order to support the entrant will determine a victory of the least preferred candidate instead of having a lottery between  $k = 1$  and  $k = 2$ . But then the strategy profile  $(x_1^*, \theta_1 = 1)$ ,  $(x_2^* = 1 - x_1^*, \theta_2 = 1)$ ,  $(x_3, \theta_3 = 0)$ , together with the same strategy profile for voters is an equilibrium in the continuation, and entry is not profitable. ■

**Proof of lemma 1.** Suppose voter  $i$ 's preferred candidate is  $k^*(i) \in \mathcal{K}$ , and that

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<sup>9</sup>It should be noted that property (iv), which follows from the assumption that voters' preferences are uniformly distributed, is in fact not needed to show that an equilibrium with more than two candidates cannot exist. Indeed, the argument can be slightly modified in order to account for a general continuous distribution  $F$  of voters' preferences.

<sup>10</sup>It is not necessary to specify the strategy profile any further.

$\tilde{k} \in \mathcal{K}$  and  $\tilde{k} \neq k^*(i)$ . Let  $t_k(\sigma_{-i}^v)$  denote the number of votes for candidate  $k$  given a voting strategy profile  $\sigma_{-i}^v$  for all voters other than  $i$ . The payoff for  $i$  of voting for  $\tilde{k}$  given  $\sigma_{-i}^v$  is

$$U(\tilde{k}; \sigma_{-i}^v) = \sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{[t_{\tilde{k}}(\sigma_{-i}^v) + 1]}{N} u(x_{\tilde{k}}; z^i) + \frac{t_{k^*(i)}(\sigma_{-i}^v)}{N} u(x_{k^*(i)}; z^i).$$

Similarly, the payoff for  $i$  of voting for  $k^*(i)$  given  $\sigma_{-i}^v$  is

$$U(k^*(i); \sigma_{-i}^v) = \sum_{k \neq \tilde{k}, k^*(i) \in \mathcal{K}} \frac{t_k(\sigma_{-i}^v)}{N} u(x_k; z^i) + \frac{t_{\tilde{k}}(\sigma_{-i}^v)}{N} u(x_{\tilde{k}}; z^i) + \frac{[t_{k^*(i)}(\sigma_{-i}^v) + 1]}{N} u(x_{k^*(i)}; z^i).$$

Thus

$$U(k^*(i); \sigma_{-i}^v) - U(\tilde{k}; \sigma_{-i}^v) = \frac{1}{N} [u(x_{k^*(i)}; z^i) - u(x_{\tilde{k}}; z^i)],$$

which is positive by definition of  $k^*(i)$ . Since  $\sigma_{-i}^v$  was arbitrary, this shows that voting sincerely strictly dominates voting for any other available candidate and is thus a dominant strategy for voter  $i$ . It follows that in all Nash equilibria in the voting stage voters vote sincerely among running candidates. ■

### Proof of Proposition 2.

Proof of Part 1. Step 1. The first step towards proving the result is to provide conditions for the existence of electoral equilibria of a simple class, which we call location symmetric equilibria (LSE). In equilibria of this class, all candidates running for office are located at the same distance to their closest neighbors; i.e.,  $x_{k+1} - x_k = \Delta$  for all  $k = 1, \dots, K - 1$ ,  $x_1 = 1 - x_K = \Delta_0$ , and all interior candidates  $k = 2, \dots, K - 1$  choose the same level of quality. Within this class, the relevant competitors for any candidate  $k$ 's decision problem are  $k$ 's neighbors,  $k + 1$  and  $k - 1$ . This is enough to show that payoff functions are twice differentiable in the relevant set (non-differentiability can only arise for quality choices that are not optimal), and that whenever rents cover variable costs, first order conditions in the quality subgame completely characterize best response correspondences (see Iaryczower and Mattozzi (2008) for more details).

With this in mind, consider then two candidates  $k$  and  $j > k$  with policy positions  $x_k$  and  $x_j > x_k$ , and choosing quality levels  $\theta_k$  and  $\theta_j$ , and let  $\tilde{x}_{k,j} \in \mathcal{R}$  denote the (unique) value of  $x$  for which  $u(\theta_k, x_k; x) = u(\theta_j, x_j; x)$ , so that  $u(\theta_k, x_k; z^i) > u(\theta_j, x_j; z^i)$  if and only if  $z^i > \tilde{x}_{k,j}$ . This always exists, and is given by

$$\tilde{x}_{k,j} = \frac{x_k + x_j}{2} + \alpha \frac{[v(\theta_k) - v(\theta_j)]}{|x_j - x_k|}. \quad (5)$$

Note then that provided that all cutpoints  $\tilde{x}_{k,k+1}$  are in  $(0, 1)$  for  $k = 1, \dots, K-1$ ,  $k$ 's vote share given  $\{(\theta_\ell, x_\ell)\}_{\ell \in \mathcal{K}}$  is  $s_k(\theta_k; \theta_{-k}, x) = \tilde{x}_{k,k+1} - \tilde{x}_{k-1,k}$ , and therefore from (1) for PR, the payoff for an interior candidate  $k = 2, \dots, K-1$  is

$$\Pi_k(\theta_{\mathcal{K}}, x_{\mathcal{K}}, \mathcal{K}) = \Delta + \alpha \left[ \frac{v(\theta_k) - v(\theta_{k+1})}{\Delta} + \frac{v(\theta_k) - v(\theta_{k-1})}{\Delta} \right] - C(\theta_k) - F.$$

Defining  $\Psi(\theta) \equiv \frac{v'(\theta_k)}{C'(\theta_k)}$ , and noting that this is a decreasing function,  $k$ 's best response is then

$$\theta_k^* = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) & \text{if } \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \leq 1, \\ 1 & \text{if } \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) > 1. \end{cases} \quad (6)$$

Similarly, for an extremal candidate (say  $k = 1$ ),

$$\Pi_1(\theta_{\mathcal{K}}, x_{\mathcal{K}}, \mathcal{K}) = \Delta_0 + \frac{\Delta}{2} + \alpha \left[ \frac{v(\theta_1) - v(\theta_2)}{\Delta} \right] - C(\theta_1) - F,$$

and  $k = 1$ 's best response is then

$$\theta_1^* = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) & \text{if } \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) \leq 1, \\ 1 & \text{if } \Psi^{-1}\left(\frac{\Delta}{\alpha}\right) > 1. \end{cases} \quad (7)$$

We can now state and prove two intermediate results. For convenience, we define  $L \equiv \max\{2\bar{c}, \bar{c} + F, \frac{1-2F}{K-1}\}$  and  $U \equiv \min\{2(\bar{c} + F), \frac{1}{K}\}$ . We will show that

(a) *If  $\max\{\alpha\Psi(1), L\} < \min\{U, 2\alpha\Psi(1)\}$ , then there exists a LSE with  $K$  candidates running for office in which all interior candidates choose maximal quality, and extremal candidates choose non-maximal quality.*

(b) *If  $\max\{2\alpha\Psi(1), L\} < U$ , then there exists a LSE with  $K$  candidates running for office and choosing a non-maximal quality.*

Results (a) and (b) then imply in particular that if  $\max\{\alpha\Psi(1), L\} < U$ , there exists a LSE,  $\Gamma^{PR}$ , in which  $\bar{\theta}(\Gamma^{PR}) < 1$ .

Proof of (a). Suppose that in equilibrium  $\theta_k^* = 1$  for  $k = 2, \dots, K-1$ . For  $\theta_k = 1$  to be optimal for  $k$  given  $\theta_{-k}$  it must be that the marginal vote share given that rivals are also choosing maximal quality is higher than the marginal cost at  $\theta_k = 1$ ; i.e.,  $\frac{2\alpha}{\Delta}v'(1) \geq C'(1)$ , or equivalently  $\Delta \leq 2\alpha\Psi(1)$ . For non-negative rents we must have  $\Pi_k^* = \Delta - (\bar{c} + F) \geq 0$ ; i.e.,

$\Delta \geq \bar{c} + F$ . For extremal candidates to choose non-maximal quality, i.e.,  $\theta_1^* = \theta_K^* = \Psi^{-1}(\frac{\Delta}{\alpha})$ , it must be that  $\Delta > \alpha\Psi(1)$ . For non-negative rents we need

$$\Pi_1^* = \Pi_K^* = \Delta_0 + \frac{\Delta}{2} - \frac{\alpha}{\Delta}[v(1) - v(\theta_1^*)] - C(\theta_1^*) - F \geq 0.$$

Since  $\theta_1^*$  maximizes  $\Pi_1(\theta_1)$ , then  $\Pi_1^* \equiv \Pi_1(\theta_1^*) \geq \Pi_1(\theta_1)$  for all  $\theta_1 \neq \theta_1^*$ , and thus it is enough to show that  $\Pi_1(1) \geq 0$ . But this is  $\Delta_0 + \frac{\Delta}{2} \geq \bar{c} + F$ , which holds whenever  $\Delta_0 \geq \frac{\Delta}{2}$  since  $\Delta \geq \bar{c} + F$ . Now consider entry by  $j$  at  $x_j \in (x_k, x_{k+1})$  for  $k = 1, \dots, K-1$ . Note that we can always sustain in the continuation game an equilibrium such that  $\hat{\theta}_j = \hat{\theta}_k = 1$  for all  $k = 1, \dots, K$ , provided that  $\frac{\Delta}{2} \geq \bar{c}$  which, given the previous conditions, also implies that  $\Delta_0 \geq \bar{c}$ . Since for equilibrium this type of entry must be non-profitable, it must be that  $\hat{\Pi}_j = \frac{\Delta}{2} - \bar{c} - F < 0$ , or equivalently  $\Delta < 2[\bar{c} + F]$ . For no entry at the extremes (i.e., in  $[0, x_1)$  and  $(x_K, 1]$ ), a sufficient condition is that  $\Delta_0 \leq F$  and  $2\bar{c} \leq \Delta$  (this last inequality guarantees that extreme candidates will not drop out in the continuation game). Substituting  $\Delta_0 = \Delta_K = \frac{1-(K-1)\Delta}{2}$ , we have that a sufficient condition for existence of a LSE with  $K$  candidates running for office in which all interior candidates choose maximal quality and extremal candidates choose non-maximal quality is  $\Delta \in (\max\{L, \alpha\Psi(1)\}, \min\{U, 2\alpha\Psi(1)\})$ .

Proof of (b). Consider first the interior candidates  $k = 2, \dots, K-1$ . If  $\theta_j^* = \theta_r^* < 1$  for all  $j, r \neq k$ , then  $k$ 's marginal vote share is differentiable, and  $k$ 's FOC is given by  $\frac{2\alpha}{\Delta}v'(\theta_k^*) = C'(\theta_k^*)$ . Therefore,

$$\theta_k^* = \theta^* = \Psi^{-1}\left(\frac{\Delta}{2\alpha}\right) \text{ for all } k = 2, \dots, K-1.$$

Moreover, since  $\theta^* < 1$ , it must be that  $\Delta > 2\alpha\Psi(1)$ . Non-negative rents for interior candidates requires that  $\Pi_k^* = \Delta - C(\theta^*) - F \geq 0$ , or equivalently  $\theta^* \leq C^{-1}(\Delta - F)$ . Substituting  $\theta^*$  we get  $\Delta \geq 2\alpha\Psi(C^{-1}(\Delta - F))$ . Note that  $2\alpha\Psi(1) \geq 2\alpha\Psi(C^{-1}(\Delta - F))$  if and only if  $\Delta \geq \bar{c} + F$ . Then, as long as in equilibrium  $\Delta \geq \bar{c} + F$  (i.e.,  $\Pi_k(1) \geq 0$  for  $k = 2, \dots, K-1$ ),  $\Delta \geq 2\alpha\Psi(1)$  implies  $\Delta \geq 2\alpha\Psi(C^{-1}(\Delta - F))$ ; i.e., if interior candidates are choosing (the same) non-maximal quality, they obtain non-negative rents. It will be sufficient for our result to look for equilibria in which  $\Delta \geq \bar{c} + F$ , and therefore we require that

$$\max\{\bar{c} + F, 2\alpha\Psi(1)\} < \Delta. \tag{8}$$

Next, we consider the possibility of entry. First, we require that all candidates that all equilibrium candidates have an incentive not to drop from the competition in any continuation game. For this it is sufficient that  $\max\{\Delta_0, \frac{\Delta}{2}\} \geq \bar{c}$ . Since  $2\Delta_0 + (K-2)\Delta = 1$ , then  $\Delta_0 = \frac{1-(K-2)\Delta}{2}$ , and the previous condition can be written as

$$2\bar{c} \leq \Delta \leq \frac{1-2\bar{c}}{K-2}. \quad (9)$$

Suppose now that  $j$  enters at  $x_j \in (x_k, x_{k+1})$  for  $k = 1, \dots, K-1$ , and define  $\delta_j^r \equiv \frac{x_{k+1}-x_j}{\Delta}$ . Suppose first that in the continuation  $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = 1$ . Then it must be that

$$\alpha v'(1) \left[ \frac{1}{\delta_j^r \Delta} + \frac{1}{\Delta} \right] \geq C'(1),$$

$$\alpha v'(1) \left[ \frac{1}{(1-\delta_j^r)\Delta} + \frac{1}{\Delta} \right] \geq C'(1).$$

Then if  $\delta_j^r \geq \frac{1}{2}$  ( $j$  enters in  $(x_k, x_{k+1})$  closer to  $x_k$  than to  $x_{k+1}$ ) the first two inequalities above hold if and only if  $\Delta \leq \alpha\Psi(1) \left[ 1 + \frac{1}{\delta_j^r} \right]$ , or  $\delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}$ . Thus, the continuation strategy profile is a Nash equilibrium for  $\frac{1}{2} \leq \delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}$ , which is feasible if and only if  $\Delta \leq 3\alpha\Psi(1)$ . When instead  $\delta_j^r \leq \frac{1}{2}$  ( $j$  enters closer to  $x_k$ ) then we need  $\Delta \leq \alpha\Psi(1) \left[ 1 + \frac{1}{(1-\delta_j^r)} \right]$ , or  $\delta_j^r \geq \frac{\Delta - 2\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}$ . Thus, the continuation strategy profile is a Nash equilibrium for  $\frac{\Delta - 2\alpha\Psi(1)}{\Delta - \alpha\Psi(1)} \leq \delta_j^r \leq \frac{1}{2}$ , which is feasible if and only if  $\Delta \leq 3\alpha\Psi(1)$ . Therefore, the strategy profile  $\hat{\theta}_k = \hat{\theta}_{k+1} = \hat{\theta}_j = 1$  is a Nash equilibrium in the continuation for entrants such that

$$\frac{\Delta - 2\alpha\Psi(1)}{\Delta - \alpha\Psi(1)} \leq \delta_j^r \leq \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}, \quad (10)$$

where  $2\alpha\Psi(1) < \Delta \leq 3\alpha\Psi(1)$ . Since the entrant in this case obtains  $\hat{\Pi}_j = \frac{\Delta}{2} - [\bar{c} + F]$ , then as long as in equilibrium

$$\Delta < 2[\bar{c} + F], \quad (11)$$

entry in an “interior” region as in (10) is not profitable. And it should be clear that this rules out “interior” entrants only, since  $2\alpha\Psi(1) < \Delta$  from (8) implies with (10) that  $\delta_j^r \in (0, 1)$ .

Consider then  $\delta_j^r > \frac{\alpha\Psi(1)}{\Delta - \alpha\Psi(1)}$  ( $j$  enters close to  $x_k$ ; the other case is symmetric). Consider the continuation  $\hat{\theta}_k = \hat{\theta}_j = 1$ ,  $\hat{\theta}_{k+1} = \Psi^{-1}\left(\frac{\delta_j^r}{1+\delta_j^r} \frac{\Delta}{\alpha}\right) < 1$ . This is clearly an equilibrium in the

continuation ( $j$  and  $k$  have even a greater incentive to choose 1 than in the previous case since they are now closer substitutes). For entry not to be profitable, we need

$$\hat{\Pi}_j = \frac{\Delta}{2} + \frac{\alpha}{\delta_j^r \Delta} [v(1) - v(\hat{\theta}_{k+1})] - [\bar{c} + F] < 0,$$

and a sufficient condition for the above inequality to be true is

$$\Delta \leq 2F. \quad (12)$$

To see this, suppose that the division of the electorate between  $k$  and  $j$  were fixed, with cutpoint  $\tilde{x}_{kj} = \frac{x_k + x_j}{2}$ . Then  $j$  would optimally choose  $\tilde{\theta}_j = \Psi^{-1}(\frac{\delta_j^r \Delta}{\alpha}) < \hat{\theta}_{k+1}$ , and we have that

$$\hat{\Pi}_j \leq \frac{\Delta}{2} - \frac{\alpha}{\delta_j^r \Delta} [v(\hat{\theta}_{k+1}) - v(\tilde{\theta}_j)] - [C(\tilde{\theta}_j) + F] < \frac{\Delta}{2} - [C(\tilde{\theta}_j) + F].$$

Consider next optimality and non-negative rents for extremal candidates, and no-entry conditions at the extremes. Note first that given that interior candidates are choosing non-maximal quality, then optimal quality by extremal candidates must be non-maximal as well. In particular, it must be that  $\theta_1^* = \theta_K^* = \Psi^{-1}(\frac{\Delta}{\alpha})$ . For no entry at the extremes it is sufficient as before that  $\Delta_0 < F$ , and since  $\Delta_0 = \frac{1-(K-1)\Delta}{2}$  this can be written as

$$\frac{1-2F}{K-1} < \Delta. \quad (13)$$

For non-negative rents we need  $\Pi_1^* = \Delta_0 + \frac{\Delta}{2} - \frac{\alpha}{\Delta} [v(\theta^*) - v(\theta_1^*)] - C(\theta_1^*) - F \geq 0$ . Since  $\Pi_1^*$  is maximized at  $\theta_1^*$ , then  $\Pi_1(\theta_1^*) \geq \Pi_1(\theta_1)$  for all  $\theta_1 \neq \theta_1^*$  and, as a result, it suffices to show that  $\Pi_1(\theta^*) > 0$ , or equivalently,  $\frac{(K-2)}{2}\Delta + [C(\theta^*) + F] \leq \frac{1}{2}$ . But since in equilibrium  $\Delta \geq C(\theta^*) + F$ , then it is sufficient that

$$\Delta \leq \frac{1}{K}. \quad (14)$$

We have then shown that the strategy profile specified above is an electoral equilibrium (in which all candidates choose non-maximal quality) if  $\Delta$  satisfies conditions (8) - (14). Now, (8) and (12) imply that for this to be feasible it is necessary that  $\bar{c} < F$  (\*). From (\*),  $\bar{c} + F < \Delta$  in (8) and (14) imply (9), and (11) implies (12). The relevant conditions on the degree of policy differentiation,  $\Delta$ , can then be written as  $\max\{2\alpha\Psi(1), L\} \leq \Delta < U$ , as we wanted to show.

Proof of Part 1. Step 2. Now let  $L_K \equiv \max\{2\bar{c}_K, \bar{c}_K + F_K, \frac{1-2F_K}{K-1}\}$  and  $U_K \equiv \min\{2(\bar{c}_K + F_K), \frac{1}{K}\}$ . Result (a) above shows that if  $\max\{\alpha_K \Psi(1), L_K\} < \min\{U_K, 2\alpha_K \Psi(1)\}$ , then there exists a LSE in which extremal candidates choose interior quality. Result (b) above shows that if  $\max\{2\alpha_K \Psi(1), L_K\} < U_K$ , then there exists a LSE in which all candidates running for office choose interior quality. Thus, if  $\max\{\alpha_K \Psi(1), L_K\} < U_K$ , there exists a LSE,  $\Gamma_K^{PR}$ , in which  $\bar{\theta}(\Gamma_K^{PR}) < 1$ . After some algebra,<sup>11</sup> we note that  $\max\{\alpha_K \Psi(1), L_K\} < U_K$  is implied by the following three inequalities

$$\bar{c}_K < F_K, \quad (15)$$

$$\frac{1}{2K} \leq F_K \leq \frac{1}{K} - \bar{c}_K, \quad (16)$$

and

$$\alpha_K < \frac{1}{\Psi(1)K}. \quad (17)$$

Now let  $\bar{c}_K = \frac{1}{2K}(1 - \varepsilon)$  for  $\varepsilon \in (0, 1)$ . Then there exists  $F_K$  satisfying (16). Choose  $F_K = \frac{(1+\varepsilon)\gamma}{2} \frac{1}{K}$  for  $\frac{1}{1+\varepsilon} < \gamma < 1$ . Finally, let  $\alpha_K = \frac{\eta}{\Psi(1)K}$  for some  $\eta \in (0, 1)$ . This triple  $(\bar{c}_K, F_K, \alpha_K)$  satisfy (15), (16) and (17) for any  $K \geq 3$ . This concludes the proof of part 1 of the Theorem.

Proof of Part 2. Consider first the case of  $K = 2$ . Note that since identically located candidates are perfect substitutes, in equilibrium quality must be maximal. Otherwise candidate  $k$  can increase rents discretely (in fact capturing all votes) by increasing  $\theta_k$  (and costs) only marginally. The rents of candidates are non-negative if and only if  $\frac{1}{2} - \bar{c} \geq F$ . To show that an equilibrium cannot exist it is enough to show that there exists a small positive  $\nu$  such that entry of a third candidate at  $x' = \frac{1}{2} - \nu$  is always profitable. Note that if a third candidate  $j$  enters at  $x'$  with  $\theta_j = 1$  either  $\hat{\theta}_k = 1$  for  $k = 1, 2$ , or  $\hat{\theta}_k = 1$  and  $\hat{\theta}_{-k} = 0$ ,  $k = 1, 2$  ( $\frac{1}{2} - \bar{c} \geq F$  implies that the case  $\hat{\theta}_k = 0$ ,  $k = 1, 2$  can never happen). If  $\frac{1}{2}(1 - \frac{x'+1/2}{2}) - \bar{c} = \frac{3-2x'}{8} - \bar{c} \geq 0$ , we have that in the continuation game  $\hat{\theta}_k = 1$ ,  $k = 1, 2$ , and to deter entry at  $\tilde{x}$  we need  $\frac{x'+\frac{1}{2}}{2} - \bar{c} < F$ . When  $\nu \rightarrow 0$  the two last inequalities become  $\frac{1}{2} - \bar{c} \in [\frac{1}{4}, F]$ . Together with the above condition for non-negative rents for candidates, the

<sup>11</sup>The condition  $\max\{\alpha_K \Psi(1), L_K\} < U_K$  embodies six relevant inequalities: (a)  $\alpha_K \Psi(1) < 2[\bar{c}_K + F_K]$ , (b)  $\alpha_K \Psi(1) < 1/K$ , (c)  $2\bar{c}_K < 1/K$ , (d)  $(\bar{c}_K + F_K) < 1/K$ , (e)  $\frac{1-2F_K}{K-1} < 1/K$  and (f)  $\frac{1-2F_K}{K-1} < 2[\bar{c}_K + F_K]$ . Note that (e) can be written as  $F_K > \frac{1}{2K}$ , and (f) as  $F_K > \frac{1}{2K} - \frac{K-1}{K}\bar{c}_K$ . Thus (e) implies (f). Moreover, from this it follows that  $\frac{1}{K} < 2[\bar{c}_K + F_K]$ , and that therefore (b) implies (a). Finally, given (15), (d) implies (c). Inequalities (d) and (f) give (16).

last expression implies that a two candidate equilibrium with perfectly centrist candidates exists if and only if  $F \geq \frac{1}{4}$  and  $\frac{1}{2} - \bar{c} = F$ . If instead  $\frac{3-2\tilde{x}}{8} - \bar{c} < 0$ , we have that in the continuation game one of the two running candidates will drop, i.e.,  $\hat{\theta}_k = 1$ , and  $\hat{\theta}_{-k} = 0$ ,  $k = 1, 2$ . Since to deter entry at  $\tilde{x}$  it must be that  $\frac{x'+\frac{1}{2}}{2} - \bar{c} < F$ , in this case when  $\nu \rightarrow 0$  we need  $\frac{1}{2} - \bar{c} \leq \min\{\frac{1}{4}, F\}$ . Once again combining the last expression with the above condition for non-negative rents for candidates we get that a two candidate equilibrium with perfectly centrist candidates exists if and only if  $F \leq \frac{1}{4}$  and  $\frac{1}{2} - \bar{c} = F$ . If  $K > 2$  we need  $\frac{x'+\frac{1}{2}}{2} - \bar{c} < F$  and  $\frac{1}{K} - \bar{c} \geq F$ , which leads to a contradiction when  $\nu \rightarrow 0$ . ■

#### Proof of Proposition 4.

(1) For given  $n$ , consider a strategy profile in which two top quality candidates, 1 and 2, symmetrically located around the median voter (i.e.,  $x_1 = 1 - x_2 < 1/2$ ), compete for office. Voters vote sincerely among these two candidates on the equilibrium path. If, off the equilibrium path, a third candidate  $\ell$  enters the electoral competition, then we require that voters vote sincerely among candidates in  $\{1, 2\}$  for all  $(\theta_1, \theta_2, \theta_3)$  for which  $\max\{\theta_1, \theta_2\} = 1$ .<sup>12</sup> We show that a strategy profile of this class, with  $\Delta \equiv x_2 - x_1$  sufficiently small, is an electoral equilibrium for large  $N$ . First note that on the equilibrium path, voters are best responding, since with two candidates strategic voting is sincere. Next note that given that  $\bar{c} + F \leq \frac{1}{2}$ , equilibrium rents of the two running candidates are always non-negative. Since candidates are choosing maximal quality in equilibrium,  $\theta_1^* = \theta_2^* = 1$ , the only possible deviation in the quality game is downwards. So suppose that candidate 1 deviates to some  $\theta_1 < 1$ . Note that since candidates were tying in equilibrium, and that voters must vote sincerely, this deviation entails the loss of the majority premium  $\gamma$  for sure. Given  $\theta_2^* = 1$ , and  $\theta_1 < 1$ , the payoffs of candidate 1,  $\Pi_1 = (1 - \gamma)\tilde{x}_{12}(\theta_1, 1) - C(\theta_1)$  are continuous and differentiable (as before,  $\tilde{x}_{12}(\theta_1, \theta_2)$  represents the voter who is indifferent between candidates 1 and 2 given  $\theta_1, \theta_2$ ). Extending the choice set to include  $\theta_1 = 1$ , but assuming away the possibility of obtaining the majority premium  $\gamma$ , the most profitable “deviation” is then to play

$$\hat{\theta}_1 = \begin{cases} \Psi^{-1}\left(\frac{\Delta}{\alpha(1-\gamma)}\right) & \text{if } \Delta > \alpha(1-\gamma)\Psi(1), \\ 1 & \text{if } \Delta \leq \alpha(1-\gamma)\Psi(1). \end{cases} \quad (18)$$

It follows that if  $\Delta \leq \alpha(1-\gamma)\Psi(1)$ , 1 prefers not to deviate. To deter this deviation, therefore, it suffices to consider strategy profiles such that  $\Delta \leq \alpha(1-\gamma)\Psi(1)$ . Suppose

<sup>12</sup>It is not necessary to specify the strategy profile any further.

now that a third candidate  $\ell$  such that  $x_\ell \in [0, 1]$  decides to enter. Recall that voters vote sincerely among candidates in  $\{1, 2\}$  for all  $(\theta_1, \theta_2, \theta_3)$  for which  $\max\{\theta_1, \theta_2\} = 1$ . But given these strategies, there is no voter which can benefit from a deviation, provided that  $N$  is large enough. To see this, suppose without loss of generality that voter  $i$  prefers candidate 1 to candidate 2, and note that  $i$ 's equilibrium payoff, voting for  $k = 1$ , is

$$U(1; \sigma_{-i}^v) = \left( \frac{1}{2}(1 - \gamma) + \frac{\gamma}{2} \right) [u(x_1; z_i) + u(x_2; z_i)]$$

Deviating and voting for an entrant  $\ell$ ,  $i$  obtains

$$U(\ell; \sigma_{-i}^v) = \frac{N-2}{2N}(1-\gamma)u(x_1; z_i) + \left( \frac{1}{2}(1-\gamma) + \gamma \right) u(x_2; z_i) + \frac{1}{N}(1-\gamma)u(x_\ell; z_i)$$

For equilibrium, it is necessary that  $U(\ell; \sigma_{-i}^v) - U(1; \sigma_{-i}^v) < 0$ , which is always true if  $u(x_\ell; z_i) < u(x_1; z_i)$ . If instead  $u(x_\ell; z_i) > u(x_1; z_i)$ , this occurs if and only if

$$\frac{1-\gamma}{\gamma} < \frac{N [u(x_1; z_i) - u(x_2; z_i)]}{2 [u(x_\ell; z_i) - u(x_1; z_i)]},$$

but this is satisfied for large enough  $N$ , since  $x_1 \neq x_2$ . This concludes the proof of part (i).

(2) Suppose, contrary to the statement of the theorem, that there does not exist such  $\bar{n}$ . Then for any  $n$  there exists  $n' > n$  such that  $K \geq 3$  candidates tie for the win in  $\tilde{\sigma}_n$ . We show that this is not possible. First, note that if a set of candidates  $W \subseteq \mathcal{K}$  tie for the win, then all voters voting for candidates in  $W \subseteq \mathcal{K}$  vote for their preferred candidate within  $W$  (for otherwise a voter could induce a strictly preferred lottery over outcomes by voting for her preferred candidate in  $W$ ). But then  $\theta_k = 1$  for all  $k \in W$ , for otherwise there exists a candidate  $\ell \in W$  with  $\theta_\ell < 1$ , who would gain from deviating to  $\theta'_\ell = \theta_\ell + \eta$  for sufficiently small  $\eta > 0$ . So suppose first that in equilibrium all  $K > 2$  candidates in  $\mathcal{K}$  tie, with  $\theta_k = 1$  for all  $k$ , and let  $k^*(i)$  denote  $i$ 's preferred candidate in  $\mathcal{K}$ . It is immediate here that all voting is sincere, for otherwise any voter not voting sincerely would induce a strictly preferred lottery over outcomes by voting for their preferred candidate  $k^*(i)$ . Since all candidates are tying choosing maximal quality and voting is sincere, candidates must be equally spaced. Next, note that equilibrium implies that all voters  $i \in N$  must prefer the equal probability lottery among all  $k \in \mathcal{K}$  induced in equilibrium to the lottery that is implied after a deviation to any candidate  $\ell \neq k^*(i)$ . Now, if for any  $n$  there exists

$n' > n$  such that this strategy profile is an equilibrium, it must be that all voters  $i \in N$  must prefer the equal probability lottery among all  $k \in W$  induced in equilibrium to the degenerate lottery in which they get any candidate  $\ell \neq k^*(i)$  for sure. To see this, note that  $i$ 's equilibrium payoff, voting for  $k^*(i)$ , is

$$U(k^*(i); \sigma_{-i}^v) = \sum_{k \in \mathcal{K}} \left[ \frac{1}{K} \frac{N-1}{N} (1-\gamma) + \frac{\gamma}{K} \right] u(x_k; z_i) + \frac{1}{N} (1-\gamma) u(x_{k^*(i)}; z_i).$$

Deviating and voting for  $\ell \neq k^*(i)$ ,  $i$  obtains

$$U(\ell; \sigma_{-i}^v) = \sum_{k \in \mathcal{K}} \left[ \frac{1}{K} \frac{N-1}{N} (1-\gamma) \right] u(x_k; z_i) + \left[ \frac{1}{N} (1-\gamma) + \gamma \right] u(x_\ell; z_i).$$

The deviation gain  $U(\ell; \sigma_{-i}^v) - U(k^*(i); \sigma_{-i}^v) < 0$  implies then that

$$u(x_\ell; z_i) - \frac{1}{K} \sum_{k \in \mathcal{K}} u(x_k; z_i) < \frac{1}{N} \frac{(1-\gamma)}{\gamma} [u(x_{k^*(i)}; z_i) - u(x_\ell; z_i)],$$

but since for any  $n$  there exists  $n' > n$  such that this strategy profile is an equilibrium, it must be that  $u(x_\ell; z_i) < \frac{1}{K} \sum_{k \in \mathcal{K}} u(x_k; z_i)$ , for otherwise, we can always find an  $n'$  that would reverse this inequality. Thus, if there does not exist a largest finite  $n$  for which all  $K > 2$  candidates in  $\mathcal{K}$  can tie in equilibrium, it must be that all voters  $i \in N$  must prefer the equal probability lottery among all  $k \in W$  induced in equilibrium to the degenerate lottery in which they get any candidate  $\ell \neq k^*(i)$  for sure. But then the same argument as in Theorem 1 shows that this can not be an equilibrium.

Next suppose that  $2 \leq |W| < K$  candidates tie for the win in equilibrium, and let  $W$  denote the set of winning candidates and  $L$  the set of losing candidates. This can't be an equilibrium either for sufficiently large  $N$ , since otherwise a voter  $i$  voting for one of the losing candidates  $\ell_0 \in L$  could gain by breaking the tie among the candidates in  $W$  in favor of her favorite candidate among  $W$ ,  $w_0$ . To see this, denote the fraction of votes obtained by candidate in  $W$  by  $\omega$ , and note that  $i$ 's equilibrium payoff, voting for  $\ell_0 \in L$ , is

$$U(\ell_0; \sigma_{-i}^v) = \sum_{w \in W} \left[ \omega(1-\gamma) + \frac{\gamma}{|W|} \right] u(x_w; z_i) + \sum_{\ell \in L} \frac{t_\ell}{N} (1-\gamma) u(x_\ell; z_i)$$

The expected payoff of deviating and voting for  $w_0 \in W$  is instead

$$U(w_0; \sigma_{-i}^v) = \sum_{w \in W} \omega(1 - \gamma)u(x_w; z^i) + \left[ \frac{1}{N}(1 - \gamma) + \gamma \right] u(x_{w_0}; z^i) + \sum_{\ell \neq \ell_0 \in L} \frac{t_\ell}{N}(1 - \gamma)u(x_\ell; z^i) + \frac{(t_{\ell_0} - 1)}{N}(1 - \gamma)u(x_{\ell_0}; z^i)$$

But then  $U(w_0; \sigma_{-i}^v) - U(\ell_0; \sigma_{-i}^v) > 0$  if and only if

$$\frac{\gamma}{1 - \gamma} > \frac{1}{N} \frac{[u(x_{\ell_0}; z^i) - u(x_{w_0}; z^i)]}{\left[ u(x_{w_0}; z^i) - \frac{1}{|W|} \sum_{w \in W} u(x_w; z^i) \right]},$$

which holds for sufficiently large  $n$ . ■

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