Some Unpleasant Bargaining Arithmetics?

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- The agreement rule (e.g., unanimity, majority, super-majority) is a key component of multilateral bargaining environments.
- Large literature on comparing the performance of different voting rules in a variety of bargaining models.
- Emphasis has been primarily on the efficiency of equilibrium outcomes.

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- It is commonly believed that, contrary to majority rule, unanimity rule protects minorities from the possibility of expropriation and is therefore more equitable.
- We show that this is not necessarily the case in bargaining: unanimity rule can induce equilibrium outcomes that are more unequal (or less equitable) than equilibrium outcomes under majority rule.

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- The players have to collectively decide which project to implement (if any), and how to distribute the available surplus.

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  - If a proposal is submitted, all players then vote (sequentially) on whether or not to approve it.
  - If q players vote in favor, then the proposal is implemented and the game ends. Otherwise, a new player is selected and the process repeats itself (possibly *ad infinitum*).

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- $i \in \{1, ..., n\}$ specifies each player's ranking in the endowment distribution (with 1 denoting the least productive and n the most productive player).

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- Merlo and Wilson (1995, 1998) and Eraslan and Merlo (2002) with perfect correlation between the “cake” and the “proposer” processes.
- Deliberately “egalitarian” protocol, and no additional dimensions of heterogeneity other than in the players’ technology endowments.

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$$G = \frac{2\sum_{i=1}^{n} iy_i}{n\sum_{i=1}^{n} y_i} - \frac{n+1}{n}$$
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- Let G^U and G^M denote the Gini coefficients for the equilibrium payoff distribution under unanimity and majority rule.

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- For every $\epsilon > 0$, if $\delta > \frac{(1-\epsilon)n}{(1-\epsilon)n+\epsilon}$, in the unique SSP equilibrium of the unanimity game

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  - Since $\epsilon < \frac{\delta}{2+\delta\frac{n-1}{n}}$, we can find $\mu_1, \ldots, \mu_n$ satisfying these.

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- This "egalitarian" force generates regression toward the mean that equalizes expected equilibrium payoffs.

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- Payoffs are monotone: $v_i^q \leq v_{i+1}^q$.
- In the q-game, there is always agreement when player q proposes.

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$$v_i^U = \max\{0, \frac{1}{n(1-\delta)}(y_i - \frac{\delta}{n-\delta(\kappa^U - 1)} \sum_{j=\kappa^U}^{n} y_j)\}.$$

where

$$\kappa^U = \min\{i : y_i - \delta \sum_j v_j^U \geq 0\}.$$

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  - One equilibrium: $v_1^M = 0$, $v_2^M = v_3^M = 0.476$

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  - Player 1 needs the approval of either player 2 or 3 but not both: $\mu_2 = \mu_3 =$1/6 if $0.4 \geq \delta v_j^M$
  - If there is no agreement then players 2 and 3 both receive their continuation payoffs with probability $1/3$: $\mu_2 = \mu_3 = 1/3$ if $0.4 < \delta v_j^M$
  - One equilibrium: $v_1^M = 0$, $v_2^M = v_3^M = 0.476$
  - Another equilibrium: $v_1^M = 0.0533$, $v_2^M = v_3^M = 0.3733$

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- G^q is not monotonic in q $(G^1 = G)$.

Conjecture

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- There exists a $\bar{q} < n$ such that for any $q < \bar{q}$ and $q' > \bar{q}$, and for any equilibria of q-game and q'-game, we have $G^q \leq G \leq G^{q'}$.

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- In our environment, $G^M \leq G \leq G^U$. Ex post it depends.