

Dynamic Government Performance: Honeymoons and Crises of Confidence

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Abstract. We model the interplay between a government's performance, its expected lifetime, and the confidence it enjoys. Here, "confidence" can be broadly interpreted as the government's popularity, the size of its parliamentary majority, its reserve of talent, or other factors. Confidence evolves in response to performance, and if it evaporates then the government falls. We analyze how confidence influences ministers' behavior. A minister's tenure is determined by the performance of both himself and others. He chooses higher performance when the government is expected to last, which is so when others perform well. Multiple equilibria arise: in an optimistic equilibrium, high performance sustains a government indefinitely; in a pessimistic equilibrium, the government's expected demise is a self-fulfilling prophecy. When confidence evolves stochastically, however, there is a unique equilibrium in which a crisis of confidence begins if and only if negative shocks shift confidence below a critical threshold.

CONFIDENCE, PERFORMANCE, AND LONGEVITY

What determines a government's performance? How does its performance relate to its expected lifespan? And how does the government's perceived longevity influence the actions of its ministers? Whereas previous theoretical and empirical analyses have shed light on each of these questions in isolation, we offer an integrated theoretical account of the dynamic interplay between government survival and performance.

Central to our account is the feature that confidence in a government can help to sustain high performance, and so justify confidence; and, conversely, a crisis of confidence can prompt a fall in performance and so mark the premature end to an administration. This logic, once developed, suggests multiple self-fulfilling prophecies. Nevertheless, the presence of noise, so that a government is buffeted by unpredictable

events, can tie down a unique prediction: this comes in the form of a unique threshold level of confidence above which a government enjoys an ongoing honeymoon, and below which its confidence, performance, and longevity all collapse.

The purpose of our paper is to develop these claims within the context of a formal theoretical model. Our model brings together three components: a government's performance; the confidence enjoyed by it; and, finally, the administration's longevity.

In our model a government's performance is determined by the endogenously chosen actions of its executive members; the government performs well when its ministers do likewise. When choosing his actions a minister balances his own interests against those of the government. By underperforming he runs the risk of losing his position. Thus a minister performs well when his job is valuable to him. That job is valuable whenever he sees it as long lasting. However, his tenure is only partly under his own control; it is also determined by the perceived lifetime of his government.

This brings us to the second component of our theory: the perceived longevity of the government. When a government is thought to be secure in office, then its ministers can look forward to a long and valuable career. As noted above, this means that they prize their positions and so deliver high performance for the executive. Of course, if a minister believes his government to be on the verge of a collapse, then he has few reasons to devote great effort to his tasks. Summarizing, a belief in the security and longevity of an administration prompts high performance from its members.

The third component of our model provides the link between government survival and executive performance. At a basic level the successful re-election of an incumbent administration arguably depends upon retrospective evaluation by voters. Furthermore, in parliamentary democracies a government may be removed before the completion of its term, and so when confidence in the executive declines sufficiently there may well be a vote of confidence. We use, then, the assumption that sustained confidence in the executive is often necessary for a government's survival.

More abstractly, "confidence" is the government's stock of political capital. This can be interpreted in various ways: as the government's popularity; as the size of its working majority; or as the depth of the pool of talented ministrables used for effective governance. We model confidence as a dynamic process, responding to the ebb and flow of events as well as to executive performance. For example, a run of scandals can diminish a government's popularity, by-election defeats can erode its majority, and resignations can drain its pool of ministerial talent; on the other hand, policy successes, by-election wins, or the attraction of new talent can all bolster confidence.

Crucially, as confidence falls the government is exposed to a greater risk of early termination. Thus the link between performance and longevity comes via the buffer of an evolving state variable. Poor performance by a minister depletes this buffer, so imposing a negative externality on other executive members.

With our last link in place, a feedback loop emerges: a long-lived government induces high performance; this sustains confidence; finally, this confidence helps to maintain the longevity of the government and its ministers' careers. Equivalently, the players' actions (the ministers' performance choices) are strategic complements. This suggests the existence of multiple rational-expectations equilibria or, in lay terms, multiple self-fulfilling prophecies. If confidence evolves deterministically as a function of government performance then there is an optimistic equilibrium in which high performance sustains a government indefinitely. But there are also pessimistic equilibria, in which the government's expected demise is inevitable. Amongst robust equilibria, these prophecies take a simple form: ministers act optimistically if and only if confidence exceeds a critical threshold. However, the critical threshold which separates the zones of optimism and pessimism is arbitrary.

This might suggest that our formal modeling exercise yields no unique prediction and, furthermore, few new insights over those provided by the informal discussion offered here. Happily, however, this is not the case. Pushing further we address the problem of multiplicity by allowing confidence to evolve stochastically; that is, we incorporate the random uncontrolled events to which a government is exposed. The introduction of noise results, of course, in a more realistic model. However, rather than adding complexity to the results it instead pins down a unique equilibrium. Crucially, random events allow the confidence process to cross the threshold which separates the optimistic and pessimistic regimes. Anticipation of this (an agent in an optimistic world recognizes that a run of bad news might result in a switch of expectations) is enough to ensure that the equilibrium threshold is unique. Thus, there is a unique stock of confidence above which the government enjoys its initial honeymoon, and below which it suffers a crisis of confidence and collapse in performance. The bottom line is that, using our model, we are able to predict which prophecy will be fulfilled.

RELATED LITERATURE

Our theoretical model contributes by helping to answer three questions: what determines a government's performance? How does its performance relate to its lifespan? How does this lifespan influence the actions of its ministers?

At a basic level, government performance depends upon the qualities of office-holders and their incentives, and so a growing literature has addressed aspects of adverse selection and moral hazard. The former was analyzed by Dewan and Dowding (2005), Huber and Martinez-Gallardo (2008), and Berlinski, Dewan, and Dowding (2010) who related ministerial turnover to the arrival of new information to a leader; in contrast Dewan and Myatt (2007, 2010) and Indridason and Kam (2008) assessed how a leader provides incentives via a firing rule. Here we do not consider firing rules, but instead sharply focus upon our main new contributions: an analysis of the team dynamics of government performance; a focus on strategic complementarities that arise; and a novel resolution to a thorny equilibrium-selection problem.

An analysis of the relationship between performance and the government's expected tenure relates our work to the large literature on government survival. We offer two main contributions. Firstly, we bring to the table a new set of modeling techniques in order to analyze the dynamic process of government duration thereby addressing an important gap in the knowledge of these processes; Laver (2003, p. 28) revealed:

“Existing a priori models are essentially static and thus in some sense will always struggle as they attempt to explain the inherently dynamic processes of the life and death of governments. Yet serious dynamic modeling has made almost no inroads into mainstream political science, so these static models are currently all that is on offer.”

Secondly, we build micro-foundations for the “events” theory of government dissolution (Browne, Frendreis, and Gleiber, 1984, 1986) that relates a government's tenure to shocks that are beyond executive control. The standard view is that critical events affect the public support for different parties and so opens bargaining opportunities that, in turn, provide incentives for governing parties to seek early dissolution (Lupia and Strøm, 1995; Diermeier and Stevenson, 2000). In our rational-expectations framework, shocks that affect the government's expected lifespan influence ministerial career values, with a subsequent effect on performance that determines the actual lifespan. Even small shocks can affect the lifespan if they spark a crisis of confidence.

Our work also relates to a growing literature that connects policy performance to politicians' time horizons. In our model ministerial performance decreases in line with expectations over tenure. A recent study that identifies the relationship between expected tenure and performance is consistent with our model. Dal Bó and Rossi (2009) analyzed the performance of Argentinian senators whose term lengths were randomly assigned. They found that senators who were allocated four-year terms outperformed those allocated to two-year terms. More generally, Spiller and Tommasi (2007) and Mukherjee, Moore, and Bejar (2006) have shown that institutions that generate short time horizons can hinder policy performance.

Beyond these contributions, we offer new insights into a classic commons-exploitation problem. In our model, confidence is a stock of goodwill which helps a government to survive; it is a common-pool resource. Dynamic models of the common-pool-resource problem have been developed in economics by Tornell and Velasco (1992) amongst others. In an important and provocative contribution Kremer and Morcom (2000) modeled the harvesting of open-access resources. When such resources are storable (the example of elephant-sourced ivory motivates their paper) expectations over future prices determine current prices and so current depletion. If complete exhaustion of the resource is expected then future prices are higher; this bids up current prices; this in turn drives up current exploitation, and so fulfils the prophecy of extinction. On the other hand, there is also a sustainable equilibrium in which the resources survives and so prices and harvesting are both low. Related ideas have developed elsewhere in economics. For instance, Rowat and Dutta (2007) analyzed commons exploitation in the presence of capital markets. They again find multiple equilibria: either commons exploitation is low, and the commons regenerates in steady state; or, exploiters expect exhaustion and so rush to grab the remaining resource before it runs out, hence causing the exhaustion.

Where these economic models fail, however, is in resolving the problem of multiple equilibria. Our solution opens new possibilities for dynamic analyses of common-resource problems in economics and in political science. In particular, to give just one example, our model could be extended to assess the effect of institutional rules in budgeting as analyzed by Hallerberg and Marier (2004). Indeed whereas Mukherjee, Moore, and Bejar (2006) view the common-resource-pool problem and short-time horizons as rival explanations for fiscal ill-discipline in parliamentary democracies, our framework incorporates both of these elements.

A MODEL OF EXECUTIVE PERFORMANCE

In our model, a governing executive comprises a unit mass of ministers indexed by $i \in [0, 1]$. At each moment t in continuous time minister i chooses his performance $e_{it} \in [e_L, e_H]$. The simultaneous performance choices of ministers aggregate to form the average $e_t = \int_0^1 e_{it} di$ which constitutes the executive's overall performance. Each minister is individually negligible, and so he views the executive's performance as beyond his own control. This feature helps to simplify our exposition, but the continuous-mass specification is not crucial to our results; similar insights flow from a model within which an executive comprises a finite team of ministers.

A minister's performance can be thought of as the effort which he devotes to his portfolio of tasks (for instance, fulfilling the government's manifesto commitments) rather than to private interests (which might include the development of post-office opportunities, or even building a private support base within his party). Thus high performance is personally costly for him. We reflect this by specifying a continuous-time flow payoff $u(e_{it})$ which is enjoyed by a minister while serving in office, where $u(\cdot)$ is a strictly decreasing and continuous function. Hence, if a minister's tenure were assured and if his government were immortal then he would choose $e_{it} = e_L$. However, higher performance can (as we shall explain) lengthen his time in office.

A minister's time in office ends for one of two reasons. Firstly, he may be compelled to resign through an individual error, scandal, or other personal failure which is indicative of low performance; he may be forced to fall on his sword before the government serves out its term. Secondly, he may instead lose office together with the other members of the executive when the government is removed from power.

The first risk to which a minister is exposed is influenced by his performance: he faces a continuous-time hazard rate $\lambda(e_{it}) > 0$ of a forced resignation where $\lambda(\cdot)$ is a strictly decreasing and continuous function. Thus, if a minister diverts his effort away from his assigned tasks then he increases the probability of a premature end to his career. A disgraced minister receives a terminal payoff which we normalize to zero. Clearly, a minister who cares only about the longevity of his career would choose $e_{it} = e_H$.

The second risk operates at an aggregate level. The longevity of the government is influenced by the combined performance of the executive; the details of this process are described below. For an individual minister, the important feature is that if the government falls then he falls with it. A minister who loses his post in this way, rather than via an individual scandal, receives a terminal payoff of V^\dagger when the government dies. This payoff is positive, so that a minister does not actively wish to jump ship

rather than go down with the remaining crew. Thus there are benefits associated with avoiding the taint of scandal. For instance, such benefits may be pecuniary; post-ministerial earnings may be higher for those who see out the government term than for those who are forced to resign. We assume that the terminal payoff is not too large, so ensuring that a minister does not actively wish his government to fall.¹

We now turn to describe the mechanism via which the government's longevity is determined. The government is sustained in office by a stock of political capital which we call "confidence." We think of confidence as a time-varying variable with three features: firstly, it rises and falls in response to a government's performance; secondly, it is buffeted by shocks; and thirdly, as it falls a government is exposed to a greater risk of losing office. Thus confidence is a buffer stock of goodwill which stands between a government and defeat. This goodwill acts as a common pool resource which is eroded by individual failures and scandals, and replenished by policy successes.

Whereas our model is abstract, and so we leave open the precise definition of confidence, nevertheless we have in mind several interpretations. Perhaps the simplest is that confidence can be the popularity of a government. It might also be a government's working parliamentary majority which may be eroded by a backbench defection or augmented by a by-election win. As a further example we might also consider a situation (as in Dewan and Myatt, 2010) whereby the governing executive survives only while it has talent to staff ministerial positions: the stock of remaining confidence toward a government corresponds directly to the size of its talent pool.

Returning to the formal specification of our model, $x_t \in [x_L, x_H]$ is the confidence enjoyed by the government at time t . The first two features of confidence (it reacts to performance, and it is buffeted by chance events) are incorporated by supposing that confidence evolves in continuous time via a stochastic differential equation:

$$dx_t = r(e_t) dt + \sigma dz_t. \quad (1)$$

The flow change in confidence dx_t comprises two components: $r(e_t) dt$ is a deterministic component, where $r(e_t)$ is an increasing function; σdz_t is a random component, where dz_t is the increment of a standard Wiener process. We specify $r(e_H) > 0 > r(e_L)$ so that the expected movement of confidence can take either sign.

Continuous-time stochastic processes are rarely used in formal political-science models, and so we pause here to describe briefly a more familiar (but ultimately less tractable) discrete-time version. Our specification is the continuous-time analog of a

¹We assume that $V^\ddagger > V^\dagger > 0$, where $V^\ddagger \equiv \max_{[e_L, e_H]} [u(e)/\lambda(e)]$. These assumptions ensure that the career value of a minister declines as the expected life of his government shortens.

discrete-time process in which confidence evolves via

$$x_{t+1} = x_t + r(e_t) + \varepsilon_t \quad \text{where} \quad \varepsilon_t \sim N(0, \sigma^2). \quad (2)$$

Here $r(e_t)$ captures the deterministic step up (if performance is high) or step down (if performance is low) in the government's fortunes, whereas ε_t is a noise term, indexed by its variance σ^2 , which brings together the day-to-day random events and, perhaps, the whims of public opinion. If $\sigma = 0$ then such random events are absent and so each minister can predict perfectly the evolution of the confidence so long as he can predict the aggregate behavior of other ministers. For the continuous-time specification of (1), setting $\sigma^2 = 0$ generates the ordinary differential equation $dx_t/dt = r(e_t)$.

It remains to specify the link between confidence and the government's longevity. The upper bound to the range $[x_L, x_H]$ is a reflecting barrier: x_H is the maximum confidence which a government can enjoy. (For instance, a government cannot exceed a maximum approval rating, but it can still fall from grace.) The lower bound, however, is an critical absorbing barrier: if confidence falls to x_L then the government fails and the careers of its ministers are all terminated. For example, if a government loses parliamentary support then it can be replaced by a vote of no confidence.

A feature of our model is that confidence responds to team performance, and so acts of malfeasance or of poor performance by a minister not only jeopardize his own career, but also impose an externality on others by drawing on the confidence reservoir. Hence confidence—essentially, the stock of goodwill—is a commons resource which is open to the commons exploitation problems which are familiar throughout the social sciences. The actions of ministers also have a strategic effect on others, and it is these strategic effects which are central to much of our analysis.

MINISTERIAL PERFORMANCE

We now take the first step in our characterization of dynamic government performance. For now we consider a situation in which confidence evolves deterministically, so that $\sigma = 0$. Focusing on the play of pure strategies, this means that each minister can predict perfectly the evolution of confidence. What matters to him is the date of government termination: the time T_i (where we allow for $T_i = \infty$, so that a government is expected to live forever) at which minister $i \in [0, 1]$ expects confidence to reach x_L .

A minister balances the reduced flow payoff from an increase in his performance against the lessened risk of his forced resignation. The size of the latter effect depends upon the expected future value of his career to him, and so we write V_{it} for this career value. This is indexed by time, since it depends on the proximity to the

perceived end of the government's administration. Given his choice of performance at each time t , this career values satisfies the differential equation

$$[u(e_{it}) - \lambda(e_{it})V_{it}] dt + dV_{it} = 0. \quad (3)$$

This is an accounting identity: the first of the bracketed terms is the flow payoff from holding office; the second bracketed term is the flow expected loss from the possible loss of position; the final term is the expected loss or gain due to the passage of time. The performance e_{it} in (3) must be chosen optimally, and so solves

$$e_{it} = \arg \max_{e \in [e_L, e_H]} [u(e) - \lambda(e)V_{it}]. \quad (4)$$

The properties of the flow payoff $u(\cdot)$ and the resignation hazard $\lambda(\cdot)$ ensure that the optimally chosen performance is increasing in the minister's career value.²

These two equations a boundary condition pin down a complete solution for both e_{it} and V_{it} . The relevant boundary condition is that the minister's career value hits the terminal payoff V^\dagger as the government's term draws to a close. Formally, $V_{iT_i} = V^\dagger$ whenever T_i is finite; if a government lives forever then V_{it} is constant over time.

Turning to the details of this solution, we write V^\ddagger for the optimized career value of a minister when he expects the government to live indefinitely; this career value satisfies $V^\ddagger = \max_{[e_L, e_H]} [u(e)/\lambda(e)]$. We make the natural assumption that $V^\ddagger > V^\dagger$, so that a minister prefers to be a member of a fully secure administration than a member of an administration which is about to expire. It is also useful to write e^\ddagger and e^\dagger for the optimally chosen performances associated with the two career values V^\ddagger and V^\dagger , derived from (4); naturally, these performances satisfy $e^\ddagger \geq e^\dagger$.

The properties of the solution to the minister's problem are natural. His career value declines from a maximum of V^\ddagger , achieved as the remaining lifetime $T_i - t$ grows large, to a minimum of V^\dagger as $T_i - t$ vanishes. In tandem, his performance choice declines from a maximum of e^\ddagger to a minimum of e^\dagger .

We now consider the implications of performance choices for the evolution of confidence. Recall that $r(e_H) > 0 > r(e_L)$, so that confidence can either rise or fall. Once we consider optimally chosen performances, however, then at the aggregate level executive performance must satisfy $e_t \in [e^\dagger, e^\ddagger]$, so that the relevant performance range is possibly narrower than $[e_L, e_H]$. To keep things interesting, we consider situations in which e^\ddagger and e^\dagger differ, and where $r(e^\ddagger) > 0 > r(e^\dagger)$.³

²If there are multiple solutions to (4) then we set e_{it} equal to the largest solution.

³Technically, we require the optimal choice of performance to satisfy $r(e) > 0$ for career values close enough to V^\ddagger (but necessarily equal to it) and similarly $r(e) < 0$ for values close enough to V^\dagger .

To evaluate the properties of confidence, we now consider a situation in which (as in equilibrium) ministers share a common expectation about the government's lifetime.

Proposition 1. *Suppose that ministers share a common belief about the time T at which the government is expected to end. There is a unique length of time $\bar{\tau}$ such that confidence decreases over time if $T - t < \bar{\tau}$ and increases if $T - t > \bar{\tau}$. Therefore $\bar{\tau}$ measures the maximum length of the period of declining confidence.*

Time effects in government performance are common elements of political discourse: governments are deemed to enjoy “honeymoon” periods which may last which or be cut short; it is sometimes said that the “first one hundred days” of government are most productive. Yet there is little systematic analysis of such time effects. Plausible performance-related reasons for such effects include differences in personnel (perhaps more able ministers serve earlier) and differences in the incentives ministers face at different points in their term.⁴ Our analysis suggests that such a decline in executive performance can arise even if there are no changes to a minister's incentive scheme over time: all that is required is a common perception that the government will end in finite time.

Of course, our general notion of performance encapsulates key economic variables that may be manipulated by the government. In this sense we provide a plausible account linking time horizons of politicians to time variation in economic performance. Recent work by Mukherjee, Moore, and Bejar (2006) revealed a relationship between coalition governments, that have on average lower survival rates than single party governments, and fiscal spending. As has already been noted, the relationship between tenure and performance has previously been studied (Spiller and Tommasi, 2007) and documented in a natural experiment (Dal Bó and Rossi, 2009).

RATIONAL-EXPECTATIONS EQUILIBRIA

We have shown how commonly held beliefs about the government's lifespan determine the executive's performance. Importantly, the belief that the lifetime is large raises aggregate performance and helps confidence. This suggests that an optimistic belief that the government will survive, or a pessimistic belief that it faces imminent demise, can both form part of self-fulfilling prophecies. Here we explore this more formally, via consideration of rational-expectations equilibria in which the evolution of

⁴The latter possibility was explored by Dewan and Myatt (2010) who analyzed a situation where an executive's leader provides incentives by firing those who fail. Their analysis shows that, when the pool of ministerial talent is finite, the incentives to perform decline over the government's tenure.

confidence justifies ministers' expectations. For now we continue to restrict attention to a world in which confidence evolves deterministically, so that $\sigma^2 = 0$.

Formally, a rational-expectations equilibrium corresponds to a (possibly infinite) time T at which all ministers expect the government to end, which induces a path of evolving confidence which first reaches x_L (so ending the government's term) at time T .

One rational-expectations equilibrium involves an immortal government, so that $T = \infty$. Given the government's expected immortality, ministers' career values and performance choices will satisfy $V_{it} = V^\ddagger$ and $e_{it} = e_t = e^\ddagger$ for all i and t . By assumption, $r(e^\ddagger) > 0$ and so confidence will increase over time until it hits and remains at its maximum level x_H . From any starting point $x_0 > x_L$ this means that the government lasts forever, and so ministers' optimistic expectations are justified.

The remaining possibility is a mortal government which exhausts its stock of confidence at time T . The exhaustion occurs only during the low-performance era, and this era is limited: Proposition 1 reports that confidence falls only during the final $\bar{\tau}$ periods. This imposes an upper bound to the feasible stock of confidence that can be eliminated. We write \bar{x} for this maximum, which satisfies

$$\bar{x} = x_L - \int_{T-\bar{\tau}}^T r(e_t) dt, \quad (5)$$

where e_t is the aggregate performance choice associated with a termination date of T . If the initial stock of confidence (essentially, the government's popularity at the time of its election) is sufficiently high, so that $x_0 > \bar{x}$, then this stock cannot be fully depleted as part of a rational-expectations equilibrium. In this case, the unique rational-expectations equilibrium involves an immortal government.

In contrast, if initial confidence is lower, so that $x_0 < \bar{x}$, then mortal-government rational-expectations equilibria do arise. There are two such equilibria. One possibility is that $T < \bar{\tau}$. Drawing upon Proposition 1 once more, confidence declines throughout the lifetime of government. The exact value of T allows just enough time to eliminate the initial stock; it satisfies $x_0 = x_L - \int_0^T r(e_t) dt$. It is straightforward to confirm that this equation has a unique solution $T \in (0, \bar{\tau})$.

The remaining possibility is that $T > \bar{\tau}$. Again drawing upon Proposition 1, confidence must first rise, peak at time $t = T - \bar{\tau}$, and then fall. To satisfy the rational-expectations condition, confidence at its peak must equal \bar{x} . Hence, the length $T - \bar{\tau}$ of the high-performance regime must be just enough for confidence to grow from its initial value to the peak; formally, $\bar{x} = x_0 + \int_0^{T-\bar{\tau}} r(e_t) dt$.

We assemble the results emerging from this discussion into a formal proposition.

Proposition 2. *If initial confidence is high, so that $x_0 > \bar{x}$, then there is a unique rational-expectations equilibrium: the government is immortal, so that $T = \infty$, and confidence rises monotonically to its maximum. If initial confidence is lower, so that $x_0 < \bar{x}$, then there are two more equilibria: in one, the government's lifetime satisfies $T < \bar{\tau}$ and confidence declines monotonically; in the other, the lifetime satisfies $T > \bar{\tau}$ and confidence rises to a maximum of \bar{x} at time $t = T - \bar{\tau}$ before beginning its decline.*

The intuition for the first claim is straightforward. A government that begins office with a large majority and a strong lead in the polls is buoyed by confidence. Even a prolonged period of low performance would not eradicate its goodwill. Ministers are sufficiently assured of their tenure that they value their careers highly, and so they deliver the high performance that sustains an everlasting honeymoon. Turning to the second claim, if the government takes office with a thinner majority and a slender lead in the polls then it is less secure. Executive members know that the government will last only if their colleagues expect it to. Whether or not the government falls in finite time depends upon the coordination of ministers' expectations.

Note that when initial confidence falls below the safe-and-secure level of \bar{x} then we are unable to pin down a unique path of government performance. A question arises: which of the self-fulfilling prophecies will be fulfilled? It turns out that we can make significant progress here; once we re-introduce noise to the evolution-of-confidence process we will be able to make unambiguous predictions. But before doing so, we illustrate the results offered in Proposition 2 using a specific version of our model.

HIGH AND LOW PERFORMANCE

To illustrate our results we now consider a specification in which each minister faces a choice between high and low performance, so that $e_{it} \in \{e_L, e_H\}$; equivalently, he either devotes all of his available effort to the work of the executive, or alternatively diverts it all to his own private interests.⁵ For this binary-action specification it is helpful to simplify notation a little, and so we write $u_H \equiv u(e_H)$ and $u_L \equiv u(e_L)$ for the flow payoffs from high and low performance respectively, and similarly we write $\lambda_H \equiv \lambda(e_H)$, $\lambda_L \equiv \lambda(e_L)$, $r_H \equiv r(e_H)$, and $r_L \equiv r(e_L)$. The various restrictions on the model then boil down to $u_L > u_H > 0$, $\lambda_L > \lambda_H > 0$, and $r_H > 0 > r_L$.

⁵Technically, this restriction of the action space does not fit within our model's specification. Nevertheless, such a binary-action model is easily obtain by considering a specification in which both the payoff function $u(\cdot)$ and the resignation hazard rate $\lambda(\cdot)$ are both linear in performance; this ensures that a minister's optimal performance choice is at an extreme of the interval $[e_L, e_H]$.

Recall that e^\ddagger is the optimal performance choice of a minister in an immortal government, and e^\dagger is his choice when the government faces imminent defeat. For things to be interesting we need $e^\ddagger > e^\dagger$ so that perceived longevity does influence performance. In the binary-action world, this means that we need $e^\ddagger = e_H$ and $e^\dagger = e_L$.

To check that $e^\ddagger = e_H$, we consider the problem faced by a minister in a secure government. High performance generates an expected payoff of u_H/λ_H , whereas the low-performance payoff is u_L/λ_L . Hence we restrict to the case where $u_H/\lambda_H > u_L/\lambda_L$. To characterize the minister's performance choice more generally, we note that the optimality condition (4) generates high rather than low performance if and only if

$$u_H - \lambda_H V_{it} \geq u_L - \lambda_L V_{it} \quad \Leftrightarrow \quad V_{it} \geq V^* \quad \text{where} \quad V^* \equiv \frac{u_L - u_H}{\lambda_L - \lambda_H}. \quad (6)$$

V^* is easily interpreted. The numerator is the flow cost of choosing higher performance; the denominator is the effect on the arrival of resignations. The ratio of these terms is the effective cost of reducing resignation risk: only when his career value exceeds this does the minister find it worthwhile to perform well. To ensure that $e^\ddagger = e_L$ we simply check that a minister's terminal payoff in a failed government is low enough; this is the simple inequality $V^\dagger < V^*$.

To calculate the length $\bar{\tau}$ of the low-performance period of government, where $V_{it} < V^*$, we need to solve explicitly the differential equation (3) while imposing the boundary condition $V_{T_i} = V^\dagger$. During the twilight of a minister's career (when $t > T_i - \bar{\tau}$, so that $e_{it} = e_L$) the solution is straightforwardly obtained:

$$V_{it} = \frac{u_L}{\lambda_L} - \exp[-\lambda_L(T_i - t)] \left(\frac{u_L}{\lambda_L} - V^\dagger \right). \quad (7)$$

The first term on the right-hand-side, is the value of holding a position in an immortal government while delivering low performance. The second term $\exp[-\lambda_L(T - t)]$ is the probability that the minister survives until the end of the government's lifetime. If this happens, then the low-performance career value is swapped for the terminal payoff V^\dagger ; this generates the third and final term.

The solution for V_{it} in (7) has the properties we expect: the value of a minister's career is decreasing in t or, equivalently, increasing in the perceived remaining lifetime $T_i - t$. Hence, moving back through time V_{it} grows until, eventually, it reaches the critical value V^* .⁶ Thus, setting $t = T_i - \bar{\tau}$ and $V_{it} = V^*$ yields a unique solution for $\bar{\tau}$, which is reported in Proposition 3 below. This also determines the size of the maximum confidence stock which can be eliminated during the government's decline.

⁶Allowing $T_i - t \rightarrow \infty$ the solution for V_{it} converges to u_L/λ_L . Straightforward calculations confirm that $u_H/\lambda_H > u_L/\lambda_L$ implies that $u_L/\lambda_L > V^*$. Hence, if $T_i - t$ is sufficiently large then V_{it} exceeds V^* .

Proposition 3. *For the binary-action specification (where each minister chooses between high and low performance) the maximum length of the low-performance era is*

$$\bar{\tau} = \frac{1}{\lambda_L} \log \left[\frac{(u_L - \lambda_L V^\dagger)(\lambda_L - \lambda_H)}{\lambda_L u_H - \lambda_H u_L} \right]. \quad (8)$$

The maximum stock of confidence which can be depleted in this time is $\bar{x} = x_L + |r_L| \bar{\tau}$.

Recall that \bar{x} is a critical confidence level: if initial confidence exceeds this level (so that $x_0 > \bar{x}$) then there is a unique equilibrium in which the government is immortal.

Some comparative-static predictions which emerge are very natural. For instance, both \bar{x} and $\bar{\tau}$ are decreasing in u_H : making high performance more attractive shortens the period of declining confidence and lowers the bar to the unique-immortal-equilibrium regime. Note also that \bar{x} and $\bar{\tau}$ are both increasing in u_L , so that a greater payoff from low-performance lengthens the declining-confidence era. However, this is perhaps misleading, since increasing u_L while u_H is fixed mixes together two effects: it makes it more costly to supply high performance, so encouraging lower performance, but at the same time it increases the value of a minister's career, which encourages high performance. To isolate these two effects, it is useful to re-write $\bar{\tau}$ as:

$$\bar{\tau} = \frac{1}{\lambda_L} \log \left[\frac{(u_L/\lambda_L) - V^\dagger}{(u_L/\lambda_L) - V^*} \right] \quad \text{where} \quad V^* = \frac{u_L - u_H}{\lambda_L - \lambda_H}. \quad (9)$$

Fixing both the marginal cost $u_L - u_H$ and marginal effect $\lambda_L - \lambda_H$ of heightened performance, $\bar{\tau}$ and \bar{x} are decreasing in u_L . Again, this works through an income effect: making a minister's career more valuable encourages him to work harder to retain his job.

A further comparative-static result is that $\bar{\tau}$ and \bar{x} both fall as the terminal payoff V^\dagger grows. Thus, enhancing the value of a surviving minister (perhaps even via the receipt of "golden handshake" conditional on serving a full term) works in favour of increased performance. This might appear counter-intuitive: increasing post-career earnings should make maintaining a government career (relatively) less valuable and make ministers more willing to switch. However, what is going on here is that the overall benefit of a minister's career, relative to being fired from his position (rather than relative to his career as an ex-minister untainted by scandal) rises, and so the income effect induces higher performance. The gain in performance may even offset the more obvious costs, owing to a conflict of interest, that arise when allowing executive members to accept lucrative post-ministerial positions.⁷

⁷In the United Kingdom ministers must consult the committee for Standards in Public Life if they accept private-sector posts within two years of leaving the Government. At the time of writing there is concerted media pressure to restrict a ministers post-career earnings further.

CONFIDENCE THRESHOLDS

So far our analysis makes a unique prediction only when the initial stock of confidence is very high. For weaker initial conditions (that is, when $x_0 < \bar{x}$) there are three rational-expectations equilibria, and so different prophecies may be fulfilled; furthermore, the executive is potentially exposed to an arbitrary mood switch of expectations, so shifting play from an optimistic to a pessimistic path. Our task in this section is to take the first steps towards isolating a unique prediction.

As a first step, we argue strongly against the non-monotonic equilibrium in which confidence first rises and then fall. We offer three reasons for doing this.

Firstly, we observe that the fundamental situation faced by members of the executive is fully captured by the current stock of confidence. Arguably, behavior should depend only on fundamentals, and so ministers should always respond in the same way given any particular confidence level. Such behavior involves Markovian strategies. However, the non-monotonic equilibrium is non-Markovian: for each confidence level $x \in (x_0, \bar{x})$ there are two different points in time (one before $t = T - \bar{\tau}$ and one after) at which ministers choose different performance levels. In contrast, the other two (monotonic) equilibria are consistent with Markovian behavior.

Secondly, the non-monotonic equilibrium offers an unnatural comparative-static prediction: an increase in the initial stock of confidence shortens, rather than lengthens, the lifetime of the government. This is because an increase in x_0 reduces the length of time needed before confidence reaches its maximum of \bar{x} at time $t = T - \bar{\tau}$.⁸

Thirdly, the non-monotonic equilibrium is not robust to slight uncertainty about the stock of confidence, or to slight noise in its evolution. Consider what happens at time $t = T - \bar{\tau}$, when confidence is at its maximum level of $x_t = \bar{x}$. The equilibrium calls for ministers to adopt pessimistic expectations. However, even the slightest extra addition to goodwill would shift the executive into a regime satisfying $x_t > \bar{x}$, where the unique equilibrium involves an immortal administration. Relatedly, even a slight suspicion that this might happen would be enough to push performance up and so justify the suspicion. Summarizing, the equilibrium relies upon a knife-edge feature.

⁸Back-to-front comparative-static properties also emerge in related games. Consider, for instance, a textbook 2×2 “Stag Hunt” coordination game. The mixed-strategy equilibrium has unfortunate comparative-static properties: increasing the relative attractiveness of a pure strategy reduces the probability with which it is played in the mixed strategy. However, the mixed equilibrium is typically non-robust under a variety of equilibrium-selection criteria (for instance, risk dominance selects one of the pure equilibria) or reasonable strategy-revision processes (such a process moves away from the mixed equilibrium). The non-monotonic rational-expectations equilibrium in our model is analogous to the mixed-strategy Nash equilibrium of the Stag Hunt coordination game.

We proceed by focusing on the Markovian equilibria in which ministers' performance choices, career values, and the executive's lifetime all depend solely on the current stock of confidence. We write these three values as $e(x)$, $V(x)$, and $\tau(x)$. So, at time t minister i chooses performance $e_{it} = e(x_t)$, values his career at $V_{it} = V(x_t)$, and expects the government to last until $t + \tau(x_t)$. If the government is expected to be immortal ($\tau(x_t) = \infty$) then $e(x_t) = e^\dagger$ and $V(x_t) = V^\dagger$.

Some basic properties of a Markovian equilibrium are immediate. The executive's lifetime $\tau(x)$ must be (at least weakly) increasing in confidence: starting from some higher level $x' > x$, confidence must decline through the lower level x before the government term ends and so $\tau(x') \geq \tau(x)$. The government may be perceived as immortal, and so associated with any Markovian equilibrium is a unique "confidence threshold" $x^* \in [x_L, x_H]$ such that $\tau(x) = \infty$ (and hence $V(x) = V^\dagger$ and $e(x) = e^\dagger$) for all $x > x^*$, but $\tau(x)$ is finite when $x^* > x > x_L$. Furthermore, if confidence falls below the threshold then it must strictly fall over time; if it remained constant then (since behavior depends only on current confidence) it would remain constant forever, sustaining the government evermore. Summarizing, $\tau(\cdot)$ is a strictly increasing function below x^* , but becomes infinite (and so constant) above the confidence threshold.

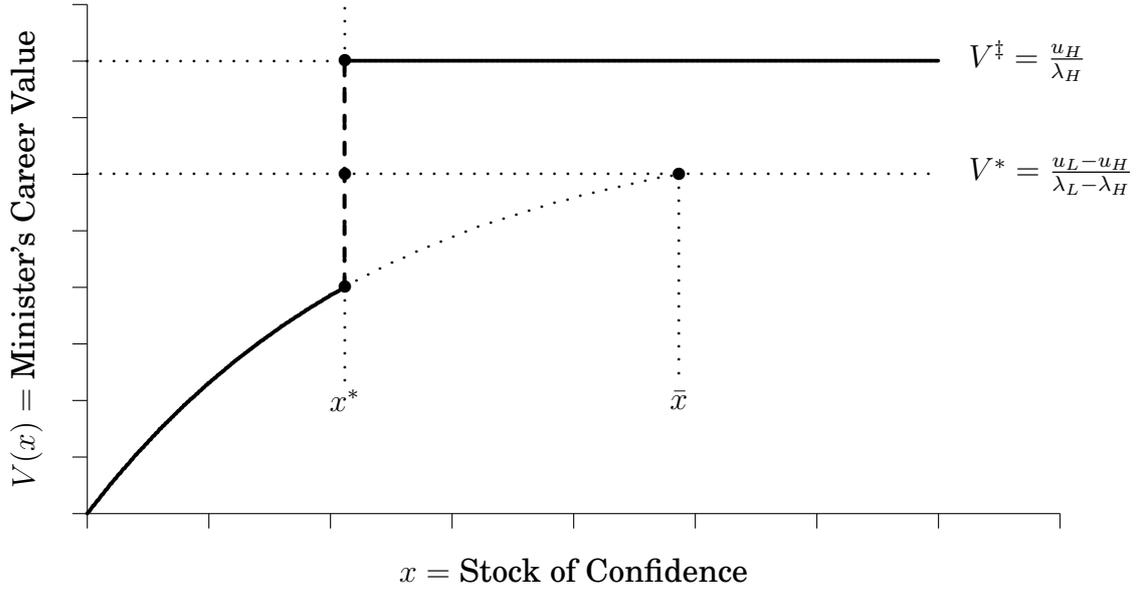
The properties of the government's perceived lifetime carry over to a minister's career value: $V(x)$ is a strictly increasing function for $x < x^*$, and (owing to the perceived immortality of the administration) satisfies $V(x) = V^\dagger$ for $x > x^*$. When confidence falls below the threshold, we can calculate $V(x)$ by assessing the value of a minister's career in the pessimistic equilibrium described in Proposition 2. Notice that such a pessimistic path can be consistent with rational expectations only when $x < \bar{x}$, which it turn implies that the confidence threshold must satisfy $x^* \leq \bar{x}$. In the pessimistic zone, it is also straightforward to characterize how a minister's career value changes as a function of changes in confidence. The differential equation (3) can be modified appropriately to this environment to yield

$$u(e(x)) - \lambda(e(x))V(x) + rV'(x) = 0, \quad (10)$$

which again stems from straightforward accounting. Within (10), the performance term $e(x)$ is obtained from the optimality condition (4), so that

$$e(x) = \arg \max_{e \in [e_L, e_H]} [u(e) - \lambda(e)V(x)]. \quad (11)$$

To obtain a complete solution for $V(x)$ for values of confidence below x^* we now need only a boundary condition. This is provided by the terminal-payoff condition at the expiry of the government's term, so that $V(x_L) = V^\dagger$.



Notes. This figure illustrates the value function $V(x)$ for a Markovian equilibrium in a binary-action world. The lower bound for confidence is $x_L = 0$, the terminal payoff for a minister is $V^\dagger = 0$. The other parameter choices at $u_L = 7$, $\lambda_L = 2$, and $r_L = -1$ for the low-performance regime; and $u_H = 4$, $\lambda_H = 1$, and $r_H = 1$ for the high-performance regime. Given that $V^\dagger = 0$, for the “pessimistic” regime where $x < x^*$, the value function (represented by the solid line in the figure) satisfies

$$V(x) = \frac{u_L}{\lambda_L} \left(1 - \exp \left[-\frac{\lambda_L x}{|r_L|} \right] \right).$$

This function hits V^* at a confidence level of \bar{x} , which in this case satisfies $\bar{x} = \log(7)/2$; there is a Markovian equilibrium for any x^* below this.

FIGURE 1. Value Functions and the Confidence Threshold

Proposition 4. *For each $x^* \in [x_L, \bar{x}]$ there is a Markovian equilibrium in which x^* acts as a critical confidence threshold. Beginning from above the threshold ($x > x^*$) performance is maximized, confidence rises to its maximum level, and the government lives forever. Beginning from below the threshold ($x < x^*$), however, confidence and executive performance both decline over time until the government falls in finite time.*

For the binary-action case it is straightforward to calculate fully the set of Markovian equilibria. Performance satisfies $e_t = e_H$ for $x_t > x^*$ and $e_t = e_L$ for $x_t < x^*$, while a minister’s career value is $V(x) = V^\dagger = u_H/\lambda_H$ for $x > x^*$, but otherwise

$$x < x^* \quad \Rightarrow \quad V(x) = \frac{u_L}{\lambda_L} - \exp \left[-\frac{\lambda_L(x - x_L)}{|r_L|} \right] \left(\frac{u_L}{\lambda_L} - V^\dagger \right). \quad (12)$$

(Such a value function is illustrated in Figure 1.) The properties of this equilibrium, and indeed of the general Markovian equilibria described in Proposition 4, are very

natural; put simply a single commonly understood barrier separates worlds of optimism ($x_t > x^*$) and pessimism ($x_t < x^*$). Nevertheless, there remain many equilibria; the critical threshold is not uniquely defined.

A formal resolution to the equilibrium-selection problem is provided in the next section when we consider the stochastic evolution of goodwill. Here, however, we offer a heuristic argument which suggests an equilibrium-selection criterion.

Consider a binary-action world, and for simplicity let $r_H = |r_L|$; thus the increase of goodwill in an optimistic world equals the decay in a pessimistic world. Furthermore, consider a Markovian equilibrium with a confidence threshold of x^* . Our aim will be to think more carefully about what happens as confidence passes that threshold.

Suppose that a minister sees a stock of goodwill equal to x^* , but remains slightly unsure of his observation. From his point of view it is equally likely that the government will move into the good regime, in which case his career is worth V^\dagger , or the bad regime, in which case his career is worth $V(x^*) \equiv \lim_{x \uparrow x^*} V(x)$, where $V(x)$ is obtained from (12). Hence (we emphasize that this is all heuristic) his expected career value is $(V^\dagger + V(x^*))/2$. Now, x^* is the critical threshold at which behavior switches and so he should be just indifferent between high and low performance. This means that his career should be worth V^* to him. This argument suggest that x^* must satisfy

$$V^* = \frac{V^\dagger + V(x^*)}{2} \quad \Leftrightarrow \quad x^* = x_L + \frac{|r_L|}{\lambda_L} \log \left[\frac{(u_L/\lambda_L) - V^\dagger}{(u_H/\lambda_H) + (u_L/\lambda_L) - 2V^*} \right]. \quad (13)$$

Thus the threshold between the two regimes lies at the point where the critical career valuation V^* which prompts confidence-increasing performance lies halfway between the career values in the optimistic and pessimistic regimes. (The threshold x^* illustrated in Figure 1 satisfies this property.)

Note that the solution for x^* given in (13) makes sense only if $x^* > x_L$. If the formulae yields $x^* < x_L$ then instead we expect a threshold of $x^* = x_L$, and so optimism is inevitable. Thus, in order for a collapse in confidence to be part of our chosen Markovian equilibrium then (following a further inspection of (13)) the terminal career payoff V^\dagger to be low enough; for $V^\dagger = 0$, we need $(u_H/\lambda_H) > 2V^*$.

We have offered here a rather informal argument which suggests that this particular threshold makes sense as the one which divides optimism from pessimism. The argument relies, however, on the idea that a minister is not precisely sure which world he is in. To make the argument rigorous, we need to introduce explicit uncertainty into the model. We do that by reverting to a world with stochastic goodwill.

THE STOCHASTIC EVOLUTION OF CONFIDENCE

When confidence evolves deterministically our tractable model produces illustrative, if somewhat stark, results relating to our three key elements: government performance, confidence, and longevity. Whereas in some environments confidence can sustain a high performing government in office indefinitely, in others confidence, performance, and expected longevity decline together.

The deterministic specification of our model lacks realism, however. Unforeseen shocks, policy disasters, and the whims of public opinion can destabilize even the best performing executive. Moreover, thus far we have been unable to provide unique predictions. In this section we show, somewhat remarkably, that the incorporation of the stochastic relationship between performance and tenure not only provides more realism to the modeling framework deployed, but also allows us to pin down a unique equilibrium and so offer crisp set of comparative statics.

We turn to our stochastic model specification in which confidence in the government is subject to the ebb and flow of a continuous stream of random shocks. Recall that evolving confidence satisfies the stochastic differential equation

$$dx_t = r(e_t) dt + \sigma dz_t, \quad (14)$$

where dz_t is the increment of a standard Wiener process.

To illustrate how this setting affects ministerial choices, consider a Markovian equilibrium in a noiseless world with a confidence threshold x^* (Proposition 4). In this context, suppose that $x_t = x^*$. As before, a minister faces uncertainty because poor individual performance may lead to his own premature demise. But now he is also uncertain about his government's longevity. This uncertainty stems from the noise σdz_t which means that the arrival of the next random shock could push confidence above x^* (in which case aggregate performance will be relatively high, confidence will rise in expectation, and so the lifetime of government is prolonged) or below x^* (in which case performance, confidence, and longevity all fall). More generally, even when confidence falls far below the threshold x^* , it remains possible that the government could enjoy a lucky streak of positive outcomes and so confidence could rise above the threshold; a minister in a declining government should incorporate this possibility. Similarly, a high-performing government enjoying maximum confidence can always suffer a run of bad luck. In the absence of noise, a single commonly understood barrier separated the worlds of optimism ($x_t > x^*$) from those of pessimism ($x_t < x^*$); when noise is present these two worlds may collide.

Turning to our formal analysis, we consider Markovian equilibria in which a minister's career value $V(x)$ and performance choice $e(x)$ both depend solely on the current stock of confidence. As before, performance is determined by the minister's desire to keep his job, and so satisfies the optimality condition $e(x) = \arg \max[u(e) - \lambda(e)V(x)]$ from (11). The extra step needed for the analysis of a stochastic-evolution world is a careful consideration of a minister's evolving career valuation $V(x)$.

As before, $V(x)$ is increasing in x . However, unlike the deterministic world $V(x)$ is smoothly and strictly increasing. To see why, note that in the deterministic world $V(x)$ is constant only when $V(x) = V^\ddagger$ for $x > x^*$; that is, when the government is immortal. Once random events are present, however, there is always positive probability that a sequence of negative shocks results in the government's defeat. Thus $V(x) < V^\ddagger$ for all x , and recalling the arguments used previously we can establish that career values must be strictly enhanced by greater confidence. The presence of uncertainty also rules out any discontinuities; the ebb and flow of random events smooths out the transition across any possible discontinuity. Thus, a sharp step up in a minister's career at an arbitrary threshold, as in Figure 1, cannot occur.

We turn now to describe the key differential equation which describes the link between confidence and the value of a ministerial position. As before, simple accounting ensures that

$$[u - \lambda V] dt + E[dV] = 0, \quad (15)$$

where it is understood that u and λ both depend on a minister's performance choice, and so on the current state of confidence via the perceived career value $V(x)$. The first and second bracketed terms are as before; they represent the flow payoff from holding office and the expected loss from a premature forced resignation. The third and final term is the expected change in the career value owing to the passage of time. When confidence evolves deterministically then this is constant. However, here we need to take expectations because the minister's career value can be influenced by random events. Adopting a second-order Taylor expansion and taking expectations (in more formal terms, this is the deployment of Itô's Lemma) we obtain

$$E[dV] = V'(x)r dt + \frac{\sigma^2 V''(x)}{2} dt, \quad (16)$$

where it is understood that r depends on aggregate performance and so, via $V(x)$, on the current stock of confidence enjoyed by the government.⁹ Bringing together (15)

⁹To understand (16), consider a small step in time Δt associated with a change in career value ΔV and a change in confidence Δx . Taking a second-order expansion, $\Delta V \approx V'(x)\Delta x + V''(x)(\Delta x)^2/2$. Treating r as a constant, $\Delta x \sim N(r\Delta t, \sigma^2\Delta t)$, and so $E[\Delta x] = r\Delta t$ and $E[(\Delta x)^2] = (r\Delta t)^2 + \sigma^2\Delta t$. Putting these things together, $E[\Delta V] = V'(x)r\Delta t + V''(x)[(r\Delta t)^2 + \sigma^2\Delta t]/2$. Allowing Δt to vanish, (16) emerges.

and (16), the value equation on the one hand, with that describing the evolution of the career value as confidence ebbs and flows on the other, we obtain

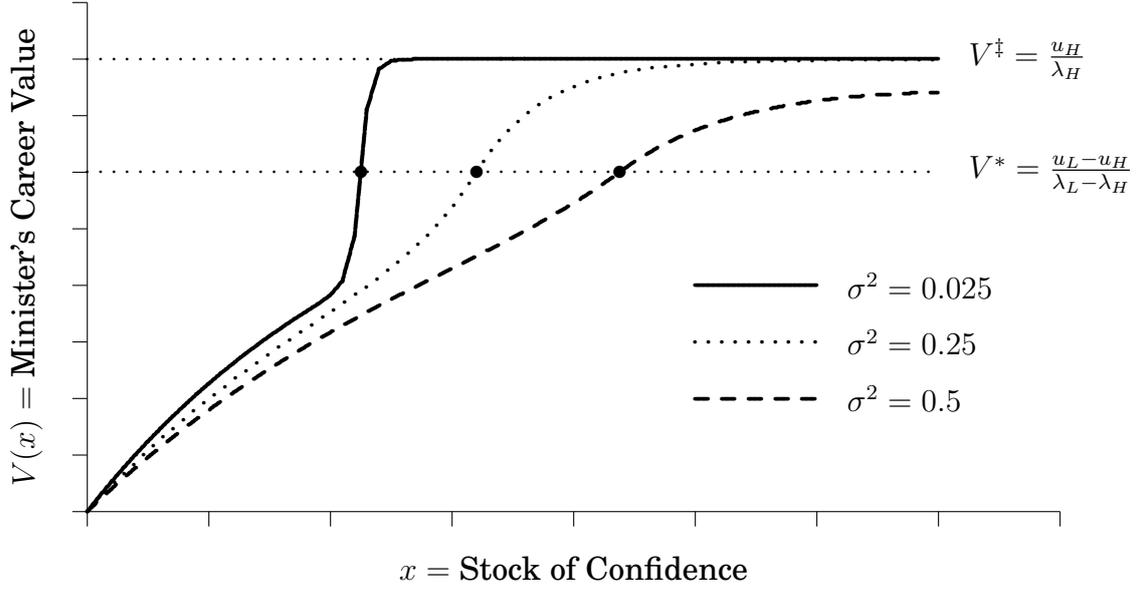
$$u - \lambda V(x) + rV'(x) + \frac{\sigma^2 V''(x)}{2} = 0. \quad (17)$$

Solving for a Markovian equilibrium boils down to solving for the right solution of this differential equation. Of course, such a second-order differential equation typically has a family of solutions, and so we need to look for two boundary conditions in order to pin down a unique value function, and so a unique equilibrium.

To obtain the necessary conditions, we examine the behavior of the value function at its boundaries. The first condition is that $V_L(0) = V^\dagger$: a minister who serves out his term obtains a “golden handshake” worth V^\dagger when the government dies with him in post. This boundary condition is something that we used to characterize the solution for $V(x)$ for confidence satisfy $x < x^*$ when we analyzed a world with deterministically evolving confidence. Here, however, this single boundary condition is not enough, owing to the second-order term in the differential equation (17). The second condition we need it obtained by looking to what happens when confidence reaches its maximum value at x_H . The condition we need is $V'(x_H) = 0$. This “smooth pasting” condition simply says that as confidence approaches its upper bound x_H it asymptotes; heuristically, this prevents a minister from expecting his career value to crash through its upper bound, and so ensures that $V(x_t)$ faces a reflecting barrier at $V(x_H)$.

With all of the ingredients in place, we are in a position to characterize fully a Markovian equilibrium. Here we restrict our attention to offering a full solution in a binary-action world. However, the approach we have taken here applies to a wider range of circumstances. With a binary-specification (either high or low performance) a threshold x^* separates (as before) the optimistic high-performance “honeymoon” world from the pessimistic low-performance “declining confidence” world. Within each of the two segments of the value function (that is, either side of x^*) the parameters u , λ , and r which are present in the differential equation (17) are all constant. When this is so, the equation is linear and so easy to solve. In our appendix we derive separate solutions and stitch them to together at the transition threshold x^* .¹⁰

¹⁰We find solutions $V_L(x)$ for $x < x^*$ and $V_H(x)$ for $x > x^*$, where these solutions satisfy the boundary conditions mentioned earlier. We then ensure that these solutions flow smoothly together at the critical confidence threshold x^* . Thus we need $V_L(x^*) = V_H(x^*) = V^*$ (so that two segments of the value function meet, and meet at the point where performance changes from low to high) and $V'_L(x^*) = V'_H(x^*)$ (so that the two segments paste smoothly together). The proof of Proposition 5 in the appendix provides further details.



Notes. This figure illustrates the value function $V(x)$ and critical confidence threshold arising from the unique Markovian equilibrium of a binary-action world with stochastic confidence. The parameter choices ($x_L = 0$, $x_H = 1.5$, $V^\dagger = 0$, $u_L = 7$, $\lambda_L = 2$, $r_L = -1$, $u_H = 4$, $\lambda_H = 1$, and $r_H = 1$) match those used in Figure 1. The value function is plotted for three different choices of noise. The bullet points “•” indicate the location of the threshold x^* for each of these two cases. As $\sigma^2 \rightarrow 0$ the value function converges to the value function illustrated in Figure 1.

FIGURE 2. Unique Value Function and Threshold with Stochastic Confidence

Proposition 5. Consider a binary-action world in which $\sigma^2 > 0$. Define:

$$x^* \equiv x_L + \frac{|r_L|}{\lambda_L} \log \frac{|r_L|[U_L - V^\dagger]}{r_H[U_H - V^*] + |r_L|[U_L - V^*]}$$

where $U_L \equiv \frac{u_L}{\lambda_L}$, $U_H \equiv \frac{u_H}{\lambda_H}$, and $V^* \equiv \frac{u_L - u_H}{\lambda_L - \lambda_H}$. (18)

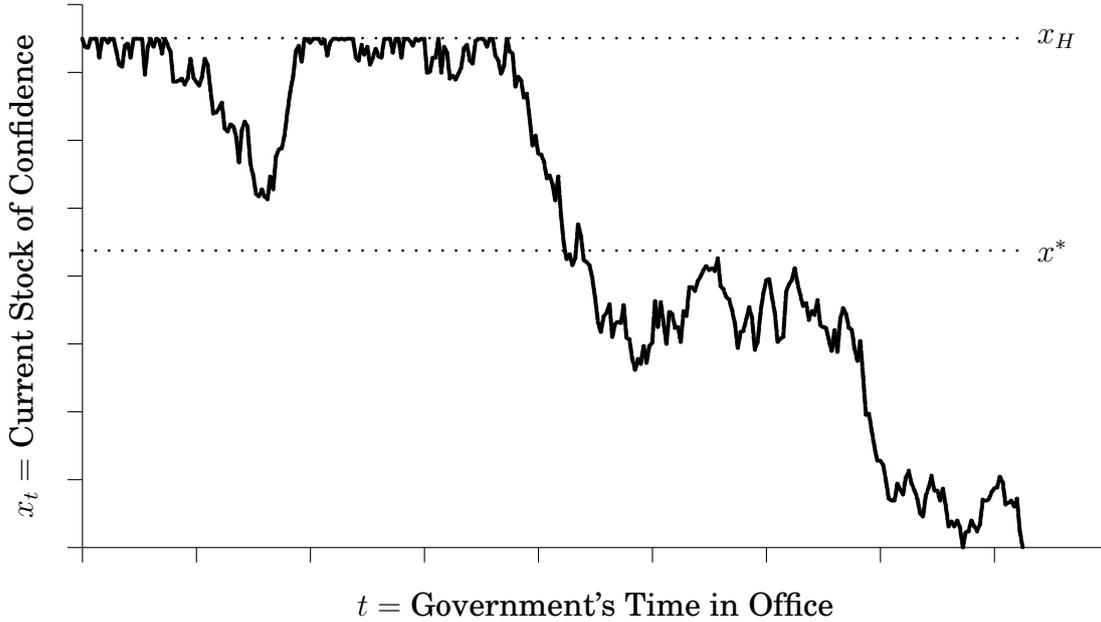
If $x_H > x^* > x_L$ and if σ^2 is sufficiently small, then there is a unique Markovian equilibrium in which ministers choose high performance if and only if confidence exceeds a unique critical threshold x^\diamond satisfying $x_H > x^\diamond > x_L$ and where $\lim_{\sigma^2 \rightarrow 0} x^\diamond = x^*$.

When x^* falls outside the stated range, then we obtain a regime in which ministers are unflinchingly optimistic (they deliver high performance for all confidence levels if $x^* < x_L$) or never have sufficient confidence to perform highly (if $x^* > x_H$). When $x_H > x^* > x_L$ then we have a unique equilibrium in which performance is high when confidence is above this threshold, and low when it falls below the threshold. Note that when $|r_L| = |r_H|$ then the solution for x^* is precisely that given by (13) which followed our earlier heuristic argument.

Here we have a unique equilibrium for a well defined x^* . This allows us to pin down our previous comparative static claims. In particular our claim concerning V^\dagger holds. An increase in the terminal career value, obtained when a minister sees out his term with his badge intact, reduces the critical threshold for good performance, and so a “golden handshake” enhances performance and lengthens the average tenure of ministers. Other comparative-static results follow by inspection. For instance (and quite naturally) the confidence threshold falls, and hence the size of the pessimistic world narrow, as we either increase r_H or reduce $|r_L|$. These parameters are the rates at which confidence rises and falls, respectively, in the high-performance and low-performance regimes.

Further comparative statics are suggested by inspection of Figure 2 which illustrates a minister’s equilibrium career value $V(x)$ for different values of the variance term σ^2 . We first note that as $\sigma^2 \rightarrow 0$ then, following the heuristic argument arguments made earlier, V^* is precisely midway between $V(x^*)$ and V^\ddagger . (More generally, the location of V^* between these two sides of the transition point depends on the relative size of r_H and $|r_L|$.) Around the critical threshold even a small increase in confidence can have a large effect on ministerial career values: once confidence surpasses the critical threshold, performance is high and small shocks are insufficient to stop ministers from attaining V^\ddagger ; similar observations apply below the threshold. As noise increases (via an increase in the variance term σ^2) this is no longer so. Any confidence boost that shifts performance (from low to high) faces a good chance of being reversed. So, even when confidence is at its highest level there remains a serious risk that a run of bad luck will lead to the end of the administration. What all this means is that an increase in noise reduces ministerial career values (as illustrated in the figure) and so acts as break on performance. A higher confidence threshold must then be breached before performance switches. Indeed, and following this logic, as σ^2 grows large then high performance can never be attained; in fact, the high-performance region disappears when σ^2 is sufficiently large.

Our key insight is that policy shocks and disasters are filtered via the three critical ingredients of our model: (i) they affect confidence in the executives’s capacity to govern effectively; (ii) they impact upon perceptions held by government ministers about security of their tenure and hence the current value of their political careers, and (iii) they affect executive performance that, in turn affects (i) and (ii). Via these mechanisms, a series of good shocks can bolster a government’s fortunes and lengthen its tenure, whilst a series of negative shocks can hasten its demise.



Notes. This figure illustrates a sample path for evolving confidence, generated using a discrete-time approximation to the continuous-time specification of our model. The parameter choices ($x_L = 0$, $x_H = 1.5$, $V^\dagger = 0$, $u_L = 7$, $\lambda_L = 2$, $r_L = -1$, $u_H = 4$, $\lambda_H = 1$, and $r_H = 1$) which match those used in Figures 1 and 2 are supplemented with a noise choice of $\sigma^2 = 0.5$. This generates an equilibrium threshold satisfying $x^* \approx 0.875$. The discrete-time approximation uses a time step of $\Delta t = 0.005$.

FIGURE 3. Sample Path of Evolving Confidence

We illustrate these effects in Figure 3. As the notes explain, this figure depicts a single simulation of evolving confidence for the model specification illustrated in earlier figures. Initially, the government begins in the high performance regime where confidence shifts stochastically from its ceiling level x_H to just below. About halfway through its (realized) term, a series of small negative shocks pushes confidence to below the threshold level. From there on in, low performance implies a downward trend until eventually confidence is extinguished. Changes to these parameter values will generate differences in the expected longevity of the government. The important point is that when confidence in the government is far above the threshold level then it is able to withstand shocks; close to the threshold, however, even the smallest tremor can send it into terminal decline.

This section is currently under revision.

CONCLUDING REMARKS

We incorporate random shocks, that may destabilize a government, in a dynamic agency perspective that links government performance, confidence in the executive, and the government's longevity. Our dynamic model relates to an existing literature described earlier. In those models, random shocks perturb the key parameters of a defined government formation game providing incentives for early dissolution of government. Our model has no such institutional detail, and as a consequence describes more general dynamics that relate to a wider set of cases including, though not limited to, multi-party parliamentary regimes. Our results are relevant to all democratic regimes in which length of government is not ascribed by the constitution. To some extent our results are relevant to nondemocracies also.

OMITTED PROOFS

Monotonicity of Ministerial Career Values. As the end of government draws near, a minister's career becomes less valuable and so his performance falls. To verify that this is so, we combine (3) and (4). Doing so, V_{it} satisfies

$$\frac{dV_{it}}{dt} = - \max_{e \in [e_L, e_H]} [u(e) - \lambda(e)V_{it}]. \quad (19)$$

The right-hand side is increasing in V_{it} . Hence, if the right-hand side is positive (respectively, negative) for some t , then V_{it} is increasing (respectively, decreasing) in t , which means that it must be increasing (respectively, decreasing) for all larger t ; this observation, once developed, establishes that V_{it} is monotonic in t . To check that V_{it} is decreasing, it is sufficient to compare V_{it} at $t = T_i$ (so that $V_{it} = V^\dagger$) and in the limit as $T_i - t \rightarrow \infty$, so that $V_{it} \rightarrow V^\ddagger$. By assumption $V^\ddagger > V^\dagger$, and so a minister's career value does fall over time; his optimally chosen performance is monotonically related to his career value and so his performance must also decline over time. \square

Proof of Propositions 1 and 2. These claims follow from arguments in the text. \square

Proof of Proposition 3. For the binary action case $e_{it} = e_L$ and $\lambda(e_{it}) = \lambda_L$ so long as $T_i - t < \bar{\tau}$. Hence, for such times the differential equation (19) reduces to

$$\frac{dV_{it}}{dt} = - [u_L - \lambda_L V_{it}]. \quad (20)$$

It is straightforward to confirm that the solution for V_{it} given in (7) satisfies this equation, and also satisfies V_{it} when $t = T_i$. To find $\bar{\tau}$ we simply set $T_i - t = \tau$ and $V_{it} = V^*$. Doing so and solving for τ yields (8) in the statement of the proposition.

Over the final $\bar{\tau}$ periods of the government's life, confidence depletes at rate $|r_L|$, which generates the final claim of the proposition. \square

Proof of Propostion 4. These proposition follows from the discussion in the text. \square

Proof of Proposition 5. $V(x)$ satisfies the second-order differential equation (17),

$$u - \lambda V(x) + rV'(x) + \frac{\sigma^2 V''(x)}{2} = 0, \quad (21)$$

where in general the terms u , λ , and r all depend on performance $e(x)$ where $e(x) = \arg \max[u(e) - \lambda(e)V(x)]$. Naturally, without further structure we are unable to described an explicit general solution to this differential equation. However, for the binary-action case the terms u , λ , and r are all locally independent of x away from the threshold x^* . Hence, for $x \neq x^*$ the differential equation (21) is linear, and so is easily solved. When $x > x^*$, this general solution is

$$V_H(x) = \alpha_H^+ e^{-\beta_H^+ x} + \alpha_H^- e^{-\beta_H^- x} + \gamma_H \text{ where } \beta_H^\pm = \frac{r_H \pm \sqrt{r_H^2 + 2\sigma^2 \lambda_H}}{\sigma^2} \text{ and } \gamma = \frac{u_H}{\lambda_H}, \quad (22)$$

and where the α_H^\pm coefficients remain to be determined. Similarly, for $x < x^*$,

$$V_L(x) = \alpha_L^+ e^{-\beta_L^+ x} + \alpha_L^- e^{-\beta_L^- x} + \gamma_L \text{ where } \beta_L^\pm = \frac{r_L \pm \sqrt{r_L^2 + 2\sigma^2 \lambda_L}}{\sigma^2} \text{ and } \gamma = \frac{u_L}{\lambda_L}. \quad (23)$$

These two components to the value function solution must join smoothly at the critical confidence threshold x^* . Moreover, at the threshold the career value of a minister must make him indifferent between high and low performance. Hence

$$V_H(x^*) = V_L(x^*) = V^* \quad \text{and} \quad V'_H(x^*) = V'_L(x^*). \quad (24)$$

The overall solution must also satisfy the two boundary conditions discussed in the text. Using the notation introduced here, these conditions are

$$V_L(x_L) = V^\dagger \quad \text{and} \quad V'_H(x_H) = 0. \quad (25)$$

(24) and (25) yield five equations which determine five unknowns: the free parameters α_L^\pm and α_H^\pm from the value function solutions, and the confidence threshold x^* .

It is straightforward to solve for the four free parameters α_L^\pm and α_H^\pm in terms of x^* by considering separately the two segments of the value function.

Consider the pessimistic segment. The conditions $V_L(x_L) = V^\dagger$ and $V_L(x^*) = V^*$ are

$$V^\dagger - \gamma_L = \alpha_L^+ e^{-\beta_L^+ x_L} + \alpha_L^- e^{-\beta_L^- x_L} \quad \text{and} \quad V^* - \gamma_L = \alpha_L^+ e^{-\beta_L^+ x^*} + \alpha_L^- e^{-\beta_L^- x^*}. \quad (26)$$

These equations are linear in α_L^+ and α_L^- , and solve straightforwardly:

$$\alpha_L^- = \frac{(\gamma_L - V^\dagger)e^{-\beta_L^+(x^*-x_L)}e^{\beta_L^-x_L} - (\gamma_L - V^*)e^{\beta_L^-x_L}}{e^{-\beta_L^-(x^*-x_L)} - e^{-\beta_L^+(x^*-x_L)}} \quad \text{and} \quad (27)$$

$$\alpha_L^+ = \frac{(\gamma_L - V^*)e^{\beta_L^+x_L} - (\gamma_L - V^\dagger)e^{-\beta_L^-(x^*-x_L)}e^{\beta_L^+x_L}}{e^{-\beta_L^-(x^*-x_L)} - e^{-\beta_L^+(x^*-x_L)}}. \quad (28)$$

Note that $V_L'(x) = -\beta_L^+\alpha_L^+e^{-\beta_L^+x} - \beta_L^-\alpha_L^-e^{-\beta_L^-x}$, and so

$$\begin{aligned} V_L'(x^*) &= \frac{(\beta_L^+ - \beta_L^-)(\gamma_L - V^\dagger)e^{-(\beta_L^+ + \beta_L^-)(x^*-x_L)}}{e^{-\beta_L^-(x^*-x_L)} - e^{-\beta_L^+(x^*-x_L)}} \\ &\quad + \frac{(\gamma_L - V^*) \left[\beta_L^- e^{-\beta_L^-(x^*-x_L)} - \beta_L^+ e^{-\beta_L^+(x^*-x_L)} \right]}{e^{-\beta_L^-(x^*-x_L)} - e^{-\beta_L^+(x^*-x_L)}} \end{aligned} \quad (29)$$

Now consider the optimistic segment. The conditions $V_H(x^*)$ and $V_H'(x_H) = 0$ are

$$V^* - \gamma_H = \alpha_H^+ e^{-\beta_H^+ x^*} + \alpha_H^- e^{-\beta_H^- x^*} \quad \text{and} \quad 0 = \beta_H^+ \alpha_H^+ e^{-\beta_H^+ x_H} + \beta_H^- \alpha_H^- e^{-\beta_H^- x_H}, \quad (30)$$

and these solve to give us

$$\alpha_H^- = -\frac{\beta_H^+(\gamma_H - V^*)e^{\beta_H^- x_H}}{\beta_H^+ e^{-\beta_H^-(x^*-x_H)} - \beta_H^- e^{-\beta_H^+(x^*-x_H)}} \quad \text{and} \quad (31)$$

$$\alpha_H^+ = \frac{\beta_H^-(\gamma_H - V^*)e^{\beta_H^+ x_H}}{\beta_H^+ e^{-\beta_H^-(x^*-x_H)} - \beta_H^- e^{-\beta_H^+(x^*-x_H)}}. \quad (32)$$

We can plug these solutions into $V_H'(x) = -\beta_H^+\alpha_H^+e^{-\beta_H^+x} - \beta_H^-\alpha_H^-e^{-\beta_H^-x}$ and obtain

$$V_H'(x^*) = \frac{\beta_H^-\beta_H^+(\gamma_H - V^*) \left[e^{-\beta_H^-(x^*-x_H)} - e^{-\beta_H^+(x^*-x_H)} \right]}{\beta_H^+ e^{-\beta_H^-(x^*-x_H)} - \beta_H^- e^{-\beta_H^+(x^*-x_H)}}. \quad (33)$$

So far we have obtained solutions for the value function in both the pessimistic and optimistic worlds as a function of the critical confidence threshold x^* . These solutions satisfy the relevant boundary conditions, and meet at $V_L(x^*) = V_H(x^*) = V^*$. The one remaining condition is that $V_L'(x^*) = V_H'(x^*)$, so that the two different parts of the value function meet smoothly at the confidence threshold. We can use this condition to find x^* , by equating the expressions in (29) and (33).

Clearly, we cannot obtain a general explicit solution for x^* . However, when noise is low (so that σ^2 is small) the expressions for $V_H'(x^*)$ and $V_L'(x^*)$ simplify dramatically. To proceed, first let us note the limiting values of the β_L^\pm and β_H^\pm parameters as $\sigma^2 \rightarrow 0$:

$$\beta_H^- \rightarrow -\frac{\lambda_H}{r_H} \quad \beta_H^+ \rightarrow \left[\frac{2r_H}{\sigma^2} \right] \rightarrow \infty \quad \beta_L^+ \rightarrow \frac{\lambda_L}{|r_L|} \quad \beta_L^- \rightarrow \left[\frac{-2|r_L|}{\sigma^2} \right] \rightarrow -\infty. \quad (34)$$

(The limiting properties of β_H^- and β_L^+ are obtained via an application of l'Hôpital's rule.) Inspecting the expression (33) for $V_H'(x^*)$, for $x^* < x_H$ notice that $e^{-\beta_H^+(x^*-x_H)} \rightarrow \infty$ while $e^{-\beta_H^-(x^*-x_H)}$ remains finite, and so the former term in both the numerator and denominator of $V_H'(x^*)$ dominates when σ^2 is small. This means that, for small σ^2 , we have $V_H'(x^*) \approx \beta_H^+(\gamma_H - V^*)$. A little more formally,

$$x^* < x_H \quad \Rightarrow \quad \lim_{\sigma^2 \rightarrow 0} \frac{V_H'(x^*)}{\sigma^2} = 2r_H(\gamma_H - V^*). \quad (35)$$

Similarly, for $V_L'(x^*)$ in (29) and for $x^* > x_L$ terms involving $e^{-\beta_L^-(x^*-x_L)}$ dominate. Taking limits, we find that

$$x^* > x_L \quad \Rightarrow \quad \lim_{\sigma^2 \rightarrow 0} \frac{V_L'(x^*)}{\sigma^2} = 2|r_L|(\gamma_L - V^*)e^{-(\lambda_L/|r_L|)(x^*-x_L)} - 2|r_L|(\gamma_L - V^*). \quad (36)$$

Equating the expressions in (35) and (36) yields the solution for x^* given in (18). \square

REFERENCES

- BERLINSKI, S., T. DEWAN, AND K. DOWDING (2010): "The Length of Ministerial Tenure in the UK, 1945-97," *Journal of Politics*, 74(2).
- BROWNE, E. C., J. P. FRENDRIS, AND D. W. GLEIBER (1984): "An 'Events' Approach to the Problem of Cabinet Stability," *Comparative Political Studies*, 17(2), 167–197.
- (1986): "The Process of Cabinet Dissolution: An Exponential Model of Duration and Stability in Western Democracies," *American Journal of Political Science*, 30(3), 628–650.
- DAL BÓ, E., AND M. A. ROSSI (2009): "Term Length and Political Performance," unpublished paper.
- DEWAN, T., AND K. DOWDING (2005): "The Corrective Effect of Ministerial Resignations," *American Journal of Political Science*, 49(1).
- DEWAN, T., AND D. P. MYATT (2007): "Scandal, Protection, and Recovery in the Cabinet," *American Political Science Review*, 101(1), 63–77.
- (2010): "The Declining Talent Pool of Government," forthcoming in the *American Journal of Political Science*.
- DIERMEIER, D., AND R. T. STEVENSON (2000): "Cabinet Terminations and Critical Events," *American Political Science Review*, 94(3), 627–640.
- GAUDET, G., M. MOREAUX, AND S. SALANT (2002): "Private Storage of Common Property," *Journal of Environmental Economics and Management*, 43, 283–302.
- HALLERBERG, M., AND P. MARIER (2004): "Executive Authority, the Personal Vote, and Budget Discipline in Latin American and Caribbean Countries," *American Journal of Political Science*, 48(3), 571–587.

- HUBER, J. D., AND C. MARTINEZ-GALLARDO (2008): "Replacing Cabinet Ministers: Patterns of Ministerial Stability in Parliamentary Democracies," *American Political Science Review*, 103(2), 169–180.
- INDRIDASON, I., AND C. KAM (2008): "Cabinet Reshuffles and Ministerial Drift," *British Journal of Political Science*, 38(4), 621–656.
- KREMER, M., AND C. MORCOM (2000): "Elephants," *American Economic Review*, 90, 213–234.
- LAVER, M. (2003): "Government Termination," *Annual Review of Political Science*, 6, 23–40.
- LUPIA, A., AND K. STRØM (1995): "Coalition Termination and the Strategic Timing of Legislative Elections," *American Political Science Review*, 89(3), 648–685.
- MUKHERJEE, B., W. H. MOORE, AND S. BEJAR (2006): "Time Horizons Matter: The Hazard Rate of Coalition Governments and the Size of Government," unpublished paper.
- ROWAT, C., AND J. DUTTA (2007): "The Commons with Capital Markets," *Economic Theory*, 31(1), 225–254.
- SPILLER, P. T., AND M. TOMMASI (2007): *The Institutional Foundations of Public Policy: The Case of Argentina*. Cambridge University Press, Cambridge, UK.
- TORNELL, A., AND A. VELASCO (1992): "The Tragedy of the Commons and Economic Growth: Why does Capital Flow from Poor to Rich Countries," *Journal of Political Economy*, 100(6), 1208–1231.