

Theory and Proofs for “The Personality of Politicians”*

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Abstract

We develop a citizen-candidate model of redistribution between both rich and poor citizens, and between legislative districts, where citizens that are more productive in the private sector are also more effective at directing transfers to their district if elected to a public office. When competition between legislators to secure funds for their districts is weak, tax rates are set by the median voter of the median district. However, when competition between legislators becomes fierce, all districts prefer legislators who are successful in the private sector. As these citizen-candidates will be richer than the median voter of the median district, this will lead, in equilibrium, to lower redistribution between rich and poor.

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1 Introduction

We study redistribution in a model where redistribution occurs not just between rich and poor, but also between legislative districts. This model features citizen-candidates (Osborne and Slivinski, 1996; Besley and Coate, 1997) where citizens who are more productive in private employment are also more able to garner redistribution to their district.

We find that when *legislative discipline* exists—that is, institutional details or party structure limits the degree of competition between legislators—then fairly standard results obtain: tax rates will be those preferred by the median voter of the median district, and legislators will be heterogeneous. However, when legislative discipline is poor, then voters in every district will want to elect a legislator who is capable of grabbing as large a share of tax revenue as possible. However, as the legislators capable of doing so are also successful in the private sector, they will favor lower tax rates. Thus, although in equilibrium there will be no redistribution between districts, the threat of redistribution between districts leads to lower tax rates. These results hold even if parties are mainly policy motivated, and are able to choose legislative candidates.

The intuition behind our result comes from the fact that, when competition between legislators for resources is intense, an individual legislator will have a large effect on the amount of resources a district receives from the government, but only a small effect on broad policy, such as the tax rate. Thus, even if the median voter in a particular district prefers a high tax rate, she is willing to elect a legislator who prefers a low tax rate for his superior ability to corral government resources for the district. As long as parties place some value on winning, they will provide candidates that are able to win.

We build on this intuition and show that as legislatures become larger, higher levels of legislative discipline may still result in low tax rates, even when the median voter of the median district prefers high tax rates. Moreover, a legislative wage, in our citizen-candidate framework, will create differences in the preferences of legislators and those they represent: this will be particularly acute for poor citizens and legislators. This will lead to lower tax rates both by shifting the preferences of legislators, and by changing the structure of

the equilibrium as poor and rich legislators become more similar in their preferences over taxation.

Our model links legislative discipline to tax policy, the composition of a legislature, and the number of bills introduced in the legislature. Our results may be applicable to explaining differences in the size of government and legislative composition between Europe, where legislative discipline is generally thought to be quite strong, and the U.S., where much weaker legislative discipline prevails, and to explaining the “wealth-bias” in U.S. politics.

While we are certainly not the first to note, or try to explain, differences in the size of government between the U.S. and Europe, we believe the focus on how legislative discipline may change tax rates through changes in the equilibrium composition of the legislature, is novel.¹ In particular, although the literature on legislative recruitment has concluded “that legislatures worldwide include more of the affluent than the less well-off, ... and more white-collar professionals than blue-collar workers” (Norris, 1997, p. 6), there is still much heterogeneity between legislatures. In particular, in countries such as Italy and Germany, about a quarter of legislators have no post-secondary education, and Britain has, in the past, famously had coal-miner MPs, while in the U.S., there are almost no legislators who are laborers or without college degrees (Wessels, 1997; Carnes, 2011).

Our link to the debate about the “wealth bias” in U.S. politics follows Carnes’s 2011 empirical finding that a legislator’s background matters for their policy preferences as expressed through roll-call votes. We build on this by giving a formal-theoretic basis for why poorer voters would choose to be represented by wealthy legislators with policy preferences that differ from theirs. This is complementary to explanations that focus on differences in campaign resources (Campante, 2010; Bartels, 2007), and the sensitivity of poor voters to the outcomes of richer voters (Bartels, 2007).²

¹Primo and Snyder (2010) investigate the link between party strength and spending in U.S. states, although their theoretical mechanism focuses on the need of legislators in weak-party states to distribute pork in order to secure personal votes from their constituents in place of the party vote.

²In a related analysis, Bai and Lagunoff (2011) explore what data would be needed to uncover a wealth bias in politics.

2 Theory Set Up

2.1 Players and Utilities

Consider a country divided in three electoral districts $j \in J = \{1, 2, 3\}$. Each district has a unit measure of citizens. There are two political parties $\{L, H\}$ that nominate a slate of citizen-candidates, one in each district. Citizens vote, and, if they are elected to the legislature, participate in the legislature's setting of tax and distributive policy.

Parties are partly office motivated, receiving office rents $R > 0$ when a majority of their candidates are elected to the legislature, and they also receive utility that depends on the tax rate implemented by the legislature. In particular, we assume that party L , the party of labor, has a preference for high tax rates, and party H , the party of hoity-toity citizens, has a preference for low tax rates. Labeling the tax rate chosen by the legislature as τ^* , and the most preferred tax rates by each party as (τ_L, τ_H) , the utility of the two parties can be given as:

$$\begin{aligned} U_L &= R\mathbb{I}_{\{\geq 2\}} + f(|\tau^* - \tau_L|) \\ U_H &= R(1 - \mathbb{I}_{\{\geq 2\}}) + f(|\tau^* - \tau_H|) \end{aligned}$$

where $\mathbb{I}_{\{\geq 2\}}$ is an indicator function taking the value of 1 if and only if two or more of party L 's candidates are elected to the legislature, and $f(\cdot)$ is a continuous, decreasing, function.

The utility of citizens is determined by their income, the level of the tax rate, the amount of transfers to their district, and, to a very small extent, their vote. In particular, a citizen i can be one of two observable types $\theta^i \in \{\theta^l, \theta^h\}$, low or high, and will have income given by $y(\theta^i)$, where the function $y(\theta^i)$ takes values:

$$y(\theta^i) = \begin{cases} y\theta & \text{if } \theta^i = \theta^l \\ y\eta\theta & \text{if } \theta^i = \theta^h \end{cases}$$

where $\eta > 1$ is difference in productivity of a high type versus a low type. We will thus

occasionally refer to high types as *rich* or *successful*, and low types as *poor* or *unsuccessful*.

In the first district, which we sometimes refer to as the low district, the median voter is a low type. In the third district, which we sometimes refer to as the high district, the median voter is a high type. The middle district's median voter is either a low type, with common-knowledge probability $p > \frac{1}{2}$, or a high-type with common-knowledge probability $1-p$. Overall, a fraction λ of citizens are low types, so the total income of all three legislative districts together is given by:

$$3\bar{y} = 3(\lambda + (1 - \lambda)\eta)y\theta \quad (1)$$

Each district receives public transfers, determined by the legislature. The legislature balances its budget, and a proportion π^j of tax revenue is distributed to district j , with $\sum_j \pi^j = 1$. Finally, we assume that each citizen gets a small utility $\varepsilon_C > 0$ for voting for the preferred party of their type. Taking all of this together, we can write the utility of low and high type citizens in district j as:

$$\begin{aligned} u_l^j &= (1 - \tau^*)y\theta + g(3\bar{y}\tau^*\pi^j) + \varepsilon_C \mathbb{I}_{\{iL\}} \\ u_h^j &= (1 - \tau^*)y\eta\theta + g(3\bar{y}\tau^*\pi^j) + \varepsilon_C(1 - \mathbb{I}_{\{iL\}}) \end{aligned} \quad (2)$$

where $\mathbb{I}_{\{iL\}}$ is an indicator function taking the value of 1 if and only if citizen i votes for party L , and the function $g(\cdot)$ is strictly increasing, strictly concave, and such that $g'(0) = \infty$. For tractability, we will present our results assuming that $g(x) = 2\sqrt{x}$. Our results will be stated in the limiting case as $\varepsilon_C \rightarrow 0$ vanishes. The ideal tax rates of high and low types, assuming equal distribution between districts, i.e. in the case of $\pi_j = 1/3$ for all j , are given by:

$$\begin{aligned} \tau_l^* &= \frac{\lambda + (1 - \lambda)\eta}{y\theta} \\ \tau_h^* &= \frac{\lambda + (1 - \lambda)\eta}{y\eta^2\theta} \end{aligned} \quad (3)$$

We assume that the ideal tax rates of the parties match those of the low- and high-type citizens: $\tau_L = \tau_l^*$ and $\tau_H = \tau_h^*$.³

³This is not necessary for any result, but is an appealing source of the parties' policy preferences.

If a citizen is elected to the legislature, he will vote, along with the other two legislators from the other two districts, on a tax rate. Legislators get the same utility as any other citizen of their type in their district, thus, their preferred tax rate will be determined by their type and district. Each legislator will then introduce a number of bills with proposals for how the tax revenues should be distributed among the three districts. Given our citizen-candidate approach, each legislator will propose in each of his bills that all tax revenue is allocated to his own district.⁴ We assume that the number of bills introduced by a legislator depends on their type and is given by $\beta(\theta^j)$. We normalize this function such that:

$$\beta(\theta^j) = \begin{cases} \theta & \text{if } \theta^j = \theta^l \\ \beta\theta & \text{if } \theta^j = \theta^h \end{cases} \quad (4)$$

with $\beta \in [1, \bar{\beta}]$.⁵ Thus, high types are more productive workers *and* legislators. The amount of tax revenues that each legislator will be able to apportion to his district is proportional to the number of bills he introduces. In particular, we assume that each legislator will bring to his district a proportion

$$\pi^j = \frac{\beta(\theta^j)}{\sum_j \beta(\theta^j)}$$

of the tax revenues, where $\beta(\theta^j)$ is defined in (4).

The existence of different legislative abilities is the major way in which our model departs from previous work. However, we view this parameter as not only a characteristic of legislators, but of the legislative environment. A natural starting point would be to assume that $\beta = \eta$, and that differences in productivity between the private sector and the legislator are due to institutional details or party structure may affect the value of β . For example, equally productive legislators in two different legislatures may have substantially different β s, if in one legislature party discipline leaves little room for independent action by legislators.

⁴Following the literature on principal-agent problems we refer to legislators (agents) using the pronoun he, and citizens (principals), especially the median voter of a district, using the pronoun she.

⁵Assuming that $\bar{\beta} < 25$ simplifies the analysis at one point, but does not materially affect our results. This is explicated in the footnote accompanying Table 1.

2.2 Timeline

To recap the above, and put it all on a timeline, we note that the model proceeds in four stages, which we pair together into two subgames.⁶ These are:

1. **The election subgame**, which consists of a
 - (a) Party stage, and a
 - (b) Voting stage.

2. **The legislative subgame**, which consists of a
 - (a) Tax-policy stage, and a
 - (b) Distributive stage.

Although our analysis will largely proceed at the *election* and *legislative* subgame level, we fill in the rest of the details here according to the four individual stages listed above.

The party stage. In the party stage, each party simultaneously introduces a slate of three citizen-candidates, one drawn from each legislative district. The strategy for a party can be summarized as the type of citizen they have chosen to run in each district. For example, $\sigma_L = \{\theta^l, \theta^l, \theta^h\}$ indicates that the labor (or low-type) party has chosen to run a low type in the first district, a low type in the second district, and a high type in the third district. In order to differentiate citizens from legislator we abuse notation and use $\sigma_L = \{L, L, H\}$ instead of $\sigma_L = \{\theta^l, \theta^l, \theta^h\}$.

The voting stage. After observing the two candidates chosen by the parties for their district, the citizens vote as if they were pivotal *in their district* for either the low-party or high-party candidate.⁷ Thus, from now on we focus on the median voter in each district.

⁶Note that each stage is a well-defined subgame. We opt for the nomenclature stage for expositional clarity.

⁷As there are only two candidates in each district, sincere and sophisticated voting will be the same. However, as each citizen is never pivotal, there is not necessarily a utility cost in voting for a less preferred candidate. Thus, we assume directly that voters vote as if they were pivotal.

The median voter in each district condition her vote on the utility they expect from each candidate. This will depend on whether each candidate is a low-type or a high-type, as well as the types of the candidates elected in the other two districts.⁸ We assume that the uncertainty about the identity of middle district's median voter is resolved after votes are cast.

The tax-policy stage. Elected legislators vote over a level of taxes using an open rule, taking into account what will happen in the distributive stage. As the majority of the legislature will always be of the same type, they will have the same most-preferred tax rate, which will be the outcome of the tax policy stage. Note here that as legislators are not directly constrained by their party when setting tax rates, when we refer to the majority of the legislature we mean the majority *type*, not the majority *party*. However, legislators will be indirectly constrained by the level of β , which may be determined by the institutional or party structure of the legislature.

The distributive stage. Redistribution is done according to a process whereby each legislator introduces a number of bills with proposals for how tax revenues should be distributed among the three districts. As each legislator will propose in each of his bills that all tax revenue is allocated to his own district, the amount of tax revenues that he will be able to apportion to his district is proportional to the number of bills he introduces.

2.3 Equilibrium

We largely consider pure-strategy, subgame-perfect, equilibria. However, some additional concepts will be useful.

Definition 1 *A subgame-perfect equilibrium is said to be **mostly pure strategy** if:*

1. *Pure strategies are played on the equilibrium path,*

⁸And also, to a vanishingly small extent, the party of each candidate, see (2).

2. Whenever a pure strategy equilibrium exists in an off-path voting stage, one is selected as the off-path play, and
3. Whenever no pure strategy equilibria exist in an off-path voting stage, a mixed strategy equilibrium is selected as the off-path play.

Although, for most parameter values we consider, there exists at least one pure-strategy subgame-perfect equilibrium in each voting stage, there is one parameter value, namely $\beta = 1$ in which there are no pure strategy equilibria in two off-path voting stages. Thus, there are no pure-strategy subgame-perfect equilibria. The concept of mostly-pure-strategy subgame-perfect equilibria will be useful there.

Definition 2 *A subgame-perfect equilibrium is said to be **strong** if whenever coalition-proof pure-strategy equilibria exist in any voting stage, on or off-path, they are played.*

We do not defend this concept now, rather, its use will become clear in the context of the analysis of Section 4.2.

3 Analysis

We begin by solving the outcome of the fairly straightforward legislative subgame, before turning our attention to the more challenging election subgame.

3.1 The Legislative Subgame

The legislative subgame has two stages, a tax-policy stage, followed by a distributive stage. In the distributive stage legislators compete to bring tax revenues to their district through the introduction of distributive legislation. As high types are more *legislatively effective*, i.e. they are more successful in introducing bills, they will have an expected advantage over low-type legislators in bringing transfers to their own district. Legislators of the majority type will take this into account when setting the tax rate in the tax-policy stage.

3.1.1 The Distributive Stage

Each legislator will propose in each of their bills that all tax revenue go to their district.

Thus, each legislator will bring to his district a proportion

$$\pi^j = \frac{\beta(\theta^j)}{\sum_j \beta(\theta^j)}$$

of the tax revenues, where $\beta(\theta^j)$ is defined in (4). It is useful to define six values of π^j for future use:

$$\begin{aligned} \pi_{3L}^L &= \frac{1}{3} \\ \pi_{2L}^L &= \frac{1}{\beta+2} < \frac{1}{3} & \pi_{2L}^H &= \frac{\beta}{\beta+2} > \frac{1}{3} \\ \pi_{2H}^L &= \frac{1}{2\beta+1} < \frac{1}{3} & \pi_{2H}^H &= \frac{\beta}{2\beta+1} > \frac{1}{3} \\ \pi_{3H}^H &= \frac{1}{3} \end{aligned} \tag{5}$$

where the subscript defines the number, and type, of legislators in the majority, and the superscript defines whether district j is represented by a high-type or low-type legislator. For example, π_{2L}^L is the share of tax revenues that a low-type legislator is able to transfer to his district when he belongs to a majority composed of himself and one other low-type legislator. Note that this quantity is necessarily less than the equal (1/3) share that a low-type legislator would obtain if all three legislators were low types. This occurs because the lone high-type legislator is more effective than the majority low-type legislators at appropriating tax revenues for his district.

3.1.2 The Tax-Policy Stage

The majority will take into account their share of tax revenues when setting tax policy. In particular, this leads to four potential values of the tax rate:

$$\begin{aligned}
\tau_{3L}^* &= \tau_l^* \\
\tau_{2L}^* &= 3\pi_{2L}^L \tau_l^* \\
\tau_{2H}^* &= 3\pi_{2H}^H \tau_h^* \\
\tau_{3H}^* &= \tau_h^*
\end{aligned} \tag{6}$$

where τ_l^* and τ_h^* are defined in (3). The equivalence of $\tau_{3H}^* = \tau_h^*$ and $\tau_{3L}^* = \tau_l^*$ comes from the fact that legislators are citizen candidates, and if all three legislators are of the same type they split tax revenues evenly, as in the definition of τ_l^* and τ_h^* .

Note that as $\pi_{2H}^H > \frac{1}{3}$, $\tau_{2H}^* > \tau_h^*$, and as $\pi_{2L}^L < \frac{1}{3}$, $\tau_{2L}^* < \tau_l^*$. In particular, when β is relatively large, it may be the case that $\tau_{2L}^* < \tau_h^*$ (although $\tau_h^* < \tau_l^*$). This occurs because when β is large, almost all tax revenue will go to the high-type legislator's district. Anticipating the outcome of the distributive stage, a majority composed of low-type legislators will set a relatively low tax rate in the tax-policy stage to prevent their districts from being expropriated. Similarly, it may be the case that $\tau_{2H}^* > \tau_l^*$.

3.2 The Election Subgame

Most of the action of our model takes place the election subgame. It is here that the parties propose their slates of candidates, and that the median voters in each district size up their options, and vote for the candidate that will deliver the highest utility for them. Although the citizens move after the parties, it is the citizens' incentives that drive our main result. In equilibrium, the parties understand how the voters will vote, and, as they are partly office motivated, provide the candidates that voters prefer.

3.2.1 The Voting Stage

Although the median voter in a district can only chose a legislator for her district, her utility will depend on the types of the legislators elected from other districts as well, see (5) and

Table 1: Median Voter Incentives

If the other two districts elect	A low type median voter will want to elect a	A high-type median voter will want to elect a
Two low types:	high type ⁹	high type ⁹
A low type and a high type:	low type if: $\frac{\pi_{2H}^H}{\pi_{2L}^L} \leq \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta},$ otherwise, a high type	high type
Two high types:	high type	high type

(6). It is thus quite useful to look at what type of legislator a median voter would want to elect, given the types of legislators elected from the other two districts. As, in equilibrium, a voter’s beliefs about the types of the legislators elected by other districts must be correct, these preferences will shape the median voter’s vote choice.

The preferences of median voters are summarized in Table 1. As noted above, in the first district, which we sometimes refer to as the low district, the median voter is a low type. In the third district, which we sometimes refer to as the high district, the median voter is a high type. The middle district’s median voter is either a low type, with common-knowledge probability p , or a high-type with common-knowledge probability $1 - p$. This uncertainty is

⁹A low-type will only want to elect a high type when $\beta < 25$. If β is higher than this value, then electing a single high-type will depress the tax rate so much that a low-type median voter prefers an even share of the revenues resulting from a higher tax rate to a large share of the revenues from a very low tax rate. Similarly, for a high-type to want to elect a high-type legislator here, it must be the case that either $\eta \geq 2$ or $\beta < ((\eta + 4)/(\eta - 2))^2$. As the minimum value of the right hand side of the latter inequality is 25 when η equals its minimum value of 1, both of these conditions will be satisfied if $\beta < 25$. If $25 < \beta < ((\eta + 4)/(\eta - 2))^2$, or $\beta > 25$, $\eta \geq 2$, none of our results are affected. All these conditions are established in the proof of Proposition 3, which can be found in the appendix.

resolved after votes are cast.

A strategy for the citizens would specify which party's candidate each citizen would vote for after each possible set of actions in the party stage. Fully specifying such strategies would be quite cumbersome, and for expositional purposes we will only care about how the median voter in each district will vote given a pair of strategies in the party stage. We introduce the following notation for the vote profile of the citizens, dependent on the strategies of the party of labor σ_L , and of the hoity-toities σ_H : $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$ which states that the median voter in the low (first) district votes for the L party, and the median voter in the third district votes for the H party candidate. The middle term indicates that when the median voter of the middle district is a low-type, she votes for L , and when she is a high-type, she votes for H .

Of course, just because the median voter of, say, the third (high) district wants to elect a high-type legislator doesn't mean that one will be available for her to elect. This will depend on the slate of candidates introduced in the party stage. If both parties offer legislative candidates of the same type, electing one or the other of these candidates will produce the same utility for each type in that district. Therefore, when this occurs, voters vote according to the tie-breaking party utility, as specified in (2).

3.2.2 The Party Stage

In the party stage parties chose a slate of candidates, one for each district, taking account of how the median voter in each district will vote, and how this will affect their probability of legislative victory (winning at least two legislative seats) and how this will affect the final policy outcome. A strategy for each party is just the type of the legislative candidate in each district. As there are eight potential strategies for each party, this defines sixty-four different possible voting stages.

We now turn to the main results of the paper. We start by examining the case of $\beta = 1$, which implies that the share of tax revenue to each district will be the same no matter the type of the legislator that a district elects. As such, this is a relatively standard game of

redistribution between successful and unsuccessful agents. We then turn to the implications of allowing $\beta > 1$. That is, we examine the implications of competition for tax revenue in legislatures on the composition of those legislatures, for redistributive policy, and for the number of bills introduced in the legislature.

4 Main Results

This section shows that when there is little competition between legislators to bring tax-revenue back to their district, fairly standard redistributive results apply. However, as competition becomes more fierce, no district wants to risk a low-type legislator, and thus, only high-type legislators will be elected in equilibrium. Because high-type legislators are richer, they have a personal preference for lower taxation, which has profound implications for tax policy.

4.1 Standard Redistribution

When $\beta = 1$, then (5) shows that $\pi_{3L} = \pi_{2L}^L = \pi_{2L}^H = \pi_{2H}^L = \pi_{2H}^H = \pi_{3H} = 1/3$. That is, no matter what type of legislator a district elects, they are guaranteed the exact same share of the tax revenue. Thus, the median voter of each district will focus on the effect that her legislator will have on the tax rate. As such, the median voters would choose to elect someone who favors exactly the same tax rate that they do.

Based on the above intuition, we would expect that in any equilibrium the tax rate would be that preferred by the “median of the medians”, in this case, the median voter of the middle district. As this median voter will be a low-type with probability p , and a high-type with probability $1 - p$, we would expect the tax rate to be τ_l^* with probability p , and τ_h^* with probability $1 - p$. As the following proposition shows, this is the case, however, because the model also incorporates the choices of parties, the equilibrium has several additional predictions as well.

Proposition 1 *If $\beta = 1$, then in all mostly-pure-strategy subgame-perfect equilibria:*

1. All legislators from the H party are high-types, and all legislators from the L party are low-types
2. Party L is in the majority with probability p and party H is in the majority with probability $1 - p$, and thus, the tax rate is τ_l^* with probability p , and τ_h^* with probability $1 - p$
3. The number of bills introduced in the legislature is 3θ .

As detailed in the proof, there are four mostly-pure-strategy subgame-perfect equilibria. Among these equilibria, one seems particularly intuitive: both parties introduce the same candidate type in the legislative districts where the median voter's type is known, and play on their advantage with low- or high-types in the middle district.¹⁰ In this equilibrium, the party of labor introduces a candidate slate $\sigma_L = \{L, L, H\}$, the hoity-toity party a slate $\sigma_H = \{L, H, H\}$. The vote profile is $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$. The equilibrium dynamics described in the proposition follow immediately. This equilibrium is also particularly appealing because, as detailed in the next subsection, it continues to be an equilibrium when β is slightly larger than one.

4.2 Redistribution Between Districts

When $\beta > 1$ there is the potential for redistribution not just between rich and poor, but also between districts. This creates different incentives, especially for low-type median voters. In particular, if their vote choice will not change tax policy too much (which is always the case when the other two districts both elect high types or low types and β is close to one), they would prefer to have a high-type representing them because of a high-type's superior ability in procuring transfers for his district.

When $\beta > 1$, the trade-off faced by the low-type median voters between electing a legislator that is able to represent closely their tax policy preferences and electing a legislator

¹⁰We should note that this is not due to median convergence in the low- and high- (first and third) districts, but rather due to the incentives created by off-the-equilibrium path play.

that is able to maximize transfers to their district has profound consequences for the equilibrium of the game. Indeed, even when β increases even slightly above one, while the focal equilibrium outcome $\sigma_L = \{L, L, H\}$, $\sigma_H = \{L, H, H\}$, $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$ is preserved, a new equilibrium is created.¹¹ The new equilibrium outcome is $\sigma_L = \{H, H, H\}$, $\sigma_H = \{H, H, H\}$, $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$. As should be clear, in the new pure-strategy subgame-perfect equilibrium all candidates are high-types and the tax rate will always be τ_h^* . While this illustrates how taking account of distributive concerns between districts can profoundly change the outcome of fairly standard models of redistributive taxation, there are ways in which this new equilibrium is quite odd.

In particular, this equilibrium requires off-the-equilibrium path behavior that is somewhat counter-intuitive. To illustrate this, take the example of the voting stage when $\sigma_L = \{L, L, H\}$, and $\sigma_H = \{H, H, H\}$. Given these party strategies, there are two pure-strategy voting profiles that can be part of an equilibrium: $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$ and $V(\sigma_L, \sigma_H) = \{1, 1/1, 1\}$. If $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$ is played, then $\sigma_L = \{L, L, H\}$, $\sigma_H = \{H, H, H\}$ can be supported as a pure-strategy subgame-perfect equilibrium outcome of the game, while $\sigma_L = \{H, H, H\}$, $\sigma_H = \{H, H, H\}$ cannot.¹² Hence, in order to support the subgame-perfect equilibrium outcome $\sigma_L = \{H, H, H\}$, $\sigma_H = \{H, H, H\}$, the vote profile $V(\sigma_L, \sigma_H) = \{1, 1/1, 1\}$ must be played if party L deviates to $\sigma_L = \{L, L, H\}$ off-the-equilibrium path. However, if instead the low-type median voters in the low and middle districts coordinated to play $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$, this would make them both better off in this particular sub-game (or stage), and in the game over-all, as $\sigma_L = \{H, H, H\}$, $\sigma_H = \{H, H, H\}$ would no longer be part of an equilibrium. As such, the new equilibrium is not a strong subgame-perfect equilibrium, as described in Section 2.3

Furthermore, this rather peculiar behavior by low-type median voters is required in not just one, but four, off-the-equilibrium path subgames in order for $\sigma_L = \{H, H, H\}$, $\sigma_H =$

¹¹At a technical level there are other profound changes as well: the focal equilibria is now a pure-strategy subgame-perfect (rather than mostly-pure-strategy) equilibrium, and some of the voting stages now have two pure-strategy equilibria, as detailed below and in the appendix.

¹²As long as when $\sigma_L = \{L, L, H\}$, $\sigma_H = \{H, H, L\}$ then $V(\sigma_L, \sigma_H) = \{0, 0/1, 0\}$. See the appendix for more details.

$\{H, H, H\}$ to be supported as an equilibrium outcome. As such, we describe the equilibrium here in the interest of both showing how radically altered equilibria may be by considering redistribution between districts, and in the interest of full-disclosure, but from now on restrict attention to strong equilibria.

In order to characterize such equilibria, we introduce one additional piece of notation. Let $u_l(L|L, H)$ denote the utility of a low-type median voter of electing a low-type legislator when the other two districts elect one high-type and one low-type legislator. With this additional notation we define

$$\textbf{Condition A: } p[u_l(L|L, H) - u_l(H|L, H)] + (1-p)[u_l(L|H, H) - u_l(H|H, H)] \geq 0. \quad (7)$$

We then have:

Theorem 1 *Let $\beta > 1$.*

1. *If Condition A holds, then in all strong, pure-strategy, subgame-perfect equilibria the following holds:*
 - (a) *All legislators from the H party are high-types, and all legislators from the L party are low-types*
 - (b) *Party L is in the majority with probability p and party H is in the majority with probability $1 - p$, and thus, the tax rate is τ_{2L}^* with probability p , and τ_{2H}^* with probability $1 - p$*
 - (c) *The expected number of bills introduced in the legislature is $(p(1 - \beta) + 1 + 2\beta)\theta$.*
2. *If, instead, Condition A does not hold, then all, pure-strategy, subgame-perfect equilibria are strong, and in each:*
 - (a) *Only high-type legislators are elected, and thus, the tax rate is τ_h^**
 - (b) *Party L is in the majority with probability p and party H is in the majority with probability $1 - p$,*

(c) *The number of bills introduced in the legislature is $3\beta\theta > (p(1 - \beta) + 1 + 2\beta)\theta$.*

Clearly, it is quite important to understand Condition A, as this sharply delineates equilibrium behavior between a regime where there is a distinct high- and low- party that, in the end cater to the median-of-the-medians, and a regime where all legislators are high types, and taxes are correspondingly lower. Much of the intuition comes from the incentives presented in Table 1. This table shows that a high-type median voter will always prefer a high-type legislator, no matter what types are elected from the other two districts. This occurs because a high-type legislator will always behave optimally from a high-type voters' point of view. Thus, in all subgame-perfect equilibria, regardless of whether Condition A holds, or not, the hoity-toity party will run a high-type in the high district, and this legislator will be elected.

Regardless of the value of β , a low-type median voter will also want to elect a high-type legislator whenever the other two legislative districts elect either no high-type legislators or no low-type legislators. This occurs because the decrease in the tax rate that comes along with such a choice is more than offset (in utility terms) by the increase in the proportion of tax revenue the district will get from electing a high-type rather than a low-type.

The relevant choice of a low-type median voter occurs when his legislator may be pivotal in whether the majority of the legislature is composed of high-type or low-type legislators. That is, when the other districts may elect one high-type and one low-type legislator. However, the median voter of the low district will never be assured, in equilibrium, that the other districts are electing a high-type and a low-type. Indeed, there will always be at most a probability p (i.e., the probability the middle district has a low-type median voter), that the middle district and high district elect one low-type and one high-type legislature.

If the median voter of the middle district turns out to be a high-type, which occurs with probability $1 - p$, and the low district's median voter elected a low type, then she will suffer a utility loss of $u_l(L|H, H) - u_l(H|H, H)$. On the other hand, this may lead to a utility gain (when β is low) of $u_l(L|L, H) - u_l(H|H, L)$, if the median voter of the middle district turns out to be a low-type, which occurs with probability p . When the utility gain is higher than the utility loss, then the low district will want to vote for a low type. Anticipating this,

low-type voters in the middle district will also want to elect a low-type legislator. However, when the gain does not offset the loss, then a low district will vote for a high-type legislator, and since low-type voters in the middle district anticipate that there will be two high-type legislators in the legislature, they also want to elect a high-type legislator.

The economic intuition behind this result is easiest to understand when the probability that the median voter of the middle district is a low-type is relatively high. Indeed, when p approaches 1, then Condition A can be re-written as:

$$\frac{\pi_{2H}^H}{\pi_{2L}^L} \leq \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta} \quad (8)$$

which can also be found in Table 1. The left-hand side is the benefit of electing a high-type legislator: he will be able to transfer to the district a greater share of the tax-revenue raised by the government. The right-hand-side is the benefit of electing a low-type legislator when pivotal: low-types will then be the majority, and set their preferred tax rate.

Note that the value of β largely determines what type of equilibrium outcomes can be supported. In particular, the regime where only high-types are elected to the legislature will occur for high values of β . To see this, note that:

$$\frac{d}{d\beta} \left(\frac{\pi_{2H}^H}{\pi_{2L}^L} \right) = \frac{d}{d\beta} \left(\frac{\beta(\beta + 2)}{2\beta + 1} \right) = \frac{2(\beta(\beta + 1) + 1)}{(2\beta + 1)^2} > 0,$$

so the left-hand-side of (8) is increasing in β . As the right-hand-side can be reduced to $\eta^2/(2\eta + 1)$, it is constant in β . Thus, there exists a level of β^* such that, for $\beta > \beta^*$ the only subgame-perfect equilibrium outcomes will be such that only high-types are elected, and, as a result, there is very little difference between the party of labor, and the hoity-toity party.

Finally, it is easy to show that when there is no legislative discipline, that is when there is nothing that reduces legislative productivity of high types below its “natural” level of $\beta = \eta$, the equilibrium will always feature only high type legislators, and the corresponding tax rate τ_h^* .

Corollary 1 *If $\beta = \eta$, then only high-type legislators will be elected.*

This follows by setting $\beta = \eta$ in the expression that appears in the left-hand-side of (8), and showing that it is always larger than the right-hand-side, which equals $\eta^2/(2\eta - 1)$.

5 Legislative Wages

This section examines how the equilibria described in the previous section change if legislators are paid a wage in addition to their private sector earnings. As legislators are citizen candidates, this wage will drive a wedge between the ideal tax rate of a legislator of a given type and a citizen of the same type. If this wage is high enough, low-type and high-type legislators will become very similar in terms of their tax policy preferences. This will result in low-type voters favoring high-types even when the difference in legislative ability is quite small.

Suppose that each legislator is paid a wage \tilde{w} which we normalize, i.e. $\tilde{w} = wy\theta$. The ideal tax rates of a low- and high-type legislators are thus:

$$\begin{aligned}\tau_{lw}^* &= \frac{3\pi^j(\lambda + (1 - \lambda)\eta)}{(1 + w)^2y\theta} \\ \tau_{hw}^* &= \frac{3\pi^j(\lambda + (1 - \lambda)\eta)}{(\eta + w)^2y\theta}\end{aligned}$$

As expected, the ideal tax rates of both types decrease with the legislative wage, w , although the ideal tax rate of a low-type legislator decreases faster than that of the high-type legislator. This leads to the following result.

Proposition 2 *If $\beta > 1$, and the legislative wage w is high enough, then in all pure-strategy subgame-perfect equilibria only high-type legislators will be elected, and the tax rate will be τ_{hw}^* .*

To understand the intuition note that as p approaches 1, then a low-type median voter whose vote is pivotal for whether the majority of the legislature consists of low- or high-types

will want to elect a high type if:

$$\frac{\pi_{2H}^H}{\pi_{2L}^L} \leq \frac{2\sqrt{\tau_{lw}^* \bar{y}} - \tau_{lw}^* y \theta}{2\sqrt{\tau_{hw}^* \bar{y}} - \tau_{hw}^* y \theta}, \quad (9)$$

which is similar to (8). Note that the left-hand side of (9) is always greater than 1, and as w grows, the right-hand side converges to 1. Thus, as w grows large, eventually (9) will be violated.

However, it is not always the case that increasing legislator wages will make low-type citizens worse off. Indeed, when $w = 0$, the numerator of the right-hand-side of (9) equals a low-type citizen's utility function at its maximal value (when ε_C vanishes). As such, a small increase in the wage rate will not change the numerator when $w = 0$, but it will decrease the denominator.¹³ This makes it possible that a relatively small wage may cause (9) to hold where it would not have before, and lead to a low-type majority in equilibrium. While this low-type majority in the legislature will implement a tax-rate that is different than the ideal tax rate of low-type citizens τ_l^* (due to the legislative wage), it may still make low-type citizens much better off.

Overall, this suggests that there is a downside to paying legislators more: it will make them sufficiently different from the people they represent, resulting in lower taxation than the median voter prefers. Moreover, if the legislative wage is high-enough, it will shrink the difference between all legislators sufficiently that low-type median voters, (correctly) viewing low- and high- type legislators as close enough substitutes on tax policy, that they will opt for the high-type legislator's superior ability to direct tax revenue to their district, depressing tax rates even further.

6 Large Legislatures

¹³In particular, it is straightforward to show that the derivative of the right-hand-side of (9) is positive at $w = 0$. The right hand side is maximized when $w = ((1 - \eta) + \sqrt{\eta^2 - 2\eta + 3})/2$, which is positive and decreasing in η .

Extending the analysis of our model to larger legislatures is by no means a trivial task given the size, and discreteness of the party's strategy spaces.¹⁴ However, examining a simpler but closely-related model furthers the intuition developed in the previous sections, and shows that redistribution between districts is a particularly serious concern with large legislatures.

In particular, consider a model with $2n + 1$ legislative districts. Most legislative district have a low-type median voter, and $z \in [0, n]$ districts have a high type median voter. We abstract from the selection role of parties and assume that each district can vote for one of two candidates: one high-type and one low-type. Everything else remains unchanged.¹⁵ We define two new quantities related to those found in (5):

$$\pi_{(n+1)L}^L = \frac{1}{n\beta + (n+1)} \quad \pi_{(n+1)H}^H = \frac{\beta}{n\beta + (n+1)}$$

The incentives for different types of median voters, shown in Table 2, are similar to those in Table 1. In particular, low-type median voters wish to elect high type legislators, except, possibly when they are pivotal. Given this model we have that

Proposition 3 *If $\beta > 1$ and*

$$\frac{\pi_{(n+1)H}^H}{\pi_{(n+1)L}^L} \leq \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta} \quad (10)$$

1. *Then, in every pure-strategy coalition-proof equilibrium the legislature will be composed of $n + 1$ low types and n high types, and the tax rate will be $n\pi_{(n+1)L}^L \bar{\tau}_l^* < \tau_l^*$.*
2. *If (10) does not hold, then the unique equilibrium will be for every district to elect a high-type legislator, and the tax rate will be τ_h^* . If $z/(2n + 1) > (\eta^2 - \eta + 1)/(\beta - 1)$, then this unique equilibrium is coalition proof.*

High-type median voters always elect a high-type legislator. Thus, when (10) holds, $n - z$ of the low-type median voters will elect high-type legislators. This makes the remaining $n + 1$

¹⁴With $2n + 1$ legislative districts there are up to $2^{4(2n+1)}$ potential pure-strategy, subgame-perfect equilibria, depending on how many districts have an unknown median voter.

¹⁵As parties are effectively nonexistent in this model, in (2) we set $\varepsilon_C = 0$.

Table 2: Median Voter Incentives

If the other districts elect	A low type median voter will want to elect a	A high-type median voter will want to elect a
A low-type majority:	high type	high type
No Clear Majority:	low type if: $\frac{\pi_{(n+1)H}^H}{\pi_{(n+1)L}^L} \leq \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta},$ otherwise, a high type	high type
A high-type majority:	high type	high type

low-type median voters pivotal, and thus, they will prefer to elect a low-type legislator. On the other-hand, if (10) does not hold, then low-type median legislators always wish to elect high-type legislators, no matter who is elected by every-other district. As electing a low-type legislator is strictly dominated, the unique equilibrium will be for every district to elect a high-type legislator.

Note the close similarity between (10) and (8). While the right-hand-side is precisely the same, the left-hand-side is still increasing in β , albeit at a faster rate:¹⁶

$$\frac{d}{d\beta} \left(\frac{\pi_{(n+1)H}^H}{\pi_{(n+1)L}^L} \right) = \frac{d}{d\beta} \left(\frac{\beta(n\beta + (n+1))}{(n+1)\beta + n} \right) = \frac{n(1 + \beta^2 + n(1 + \beta)^2)}{((n+1)\beta + n)^2} > 0.$$

This suggests that the larger is n , and thus the size of the legislature, the more sensitive

¹⁶Note, in particular that

$$\frac{d^2}{d\beta dn} \left(\frac{\pi_{(n+1)H}^H}{\pi_{(n+1)L}^L} \right) = \frac{\beta(1 + \beta^2) + n(\beta^3 + 3\beta^2 + \beta - 1)}{((n+1)\beta + n)^3} > 0.$$

the equilibrium is to increase in β . Stated differently, holding β constant, as n grows there should be some point where equilibria switch from the mixed legislature, to a legislature composed only of high types. The next corollary makes this point precise.

Corollary 2 *If $\beta > \eta^2/(2\eta - 1)$ then there exists an n^* , such that if $n > n^*$ only high-types are elected in equilibrium. Otherwise, for all $n > 1$, in all coalition-proof equilibria $n + 1$ low-type and n high-type legislators are elected.*

This result is a consequence of the fact that $\lim_{n \rightarrow \infty} \pi_{(n+1)H}^H / \pi_{(n+1)L}^L = \beta$ and $(2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta) / (2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta) = \eta^2 / (2\eta - 1)$. It is worth noting that the condition $\beta > \eta^2 / (2\eta - 1)$ is always satisfied when $\beta = \eta$, and as η grows large will be satisfied if $\beta > \eta/2$.

A Proofs

Before proceeding to proofs it is useful to define some notation that will be useful here, but is not particularly useful in the main document. In particular, define

$$q \equiv 1 - p,$$

the probability the middle-district's median voter is a high-type, and:

$$\begin{aligned} \varepsilon_{3L} &\equiv f(|\tau_l^* - \tau_l^*|) & \varepsilon_{0H} &\equiv f(|\tau_h^* - \tau_l^*|) \\ \varepsilon_{2L} &\equiv f(|\tau_l^* - \tau_{2L}^*|) & \varepsilon_{1H} &\equiv f(|\tau_h^* - \tau_{2L}^*|) \\ \varepsilon_{1L} &\equiv f(|\tau_l^* - \tau_{2H}^*|) & \varepsilon_{2H} &\equiv f(|\tau_h^* - \tau_{2H}^*|) \\ \varepsilon_{3L} &\equiv f(|\tau_l^* - \tau_h^*|) & \varepsilon_{0H} &\equiv f(|\tau_h^* - \tau_h^*|) \end{aligned}$$

Note that when β is close to one, these can be easily ranked:

$$\begin{aligned} \varepsilon_{3L} &> \varepsilon_{2L} > \varepsilon_{1L} > \varepsilon_{0L} \\ \varepsilon_{0H} &< \varepsilon_{1H} < \varepsilon_{2H} < \varepsilon_{3H}, \end{aligned}$$

although when β is larger, other orderings are possible.

Additionally, define $\xi \equiv \lambda + (1 - \lambda)\eta$, so that we have:

$$\begin{aligned} \bar{y} &= \xi y \theta \\ \tau_l^* &= \frac{\xi}{y \theta} \\ \tau_h^* &= \frac{\xi}{y \eta^2 \theta} \end{aligned} \tag{11}$$

as defined in (1) and (3).

Proof of Proposition 1:

Table A.1 displays the payoffs of the parties when they play various actions, based on

the only pure-strategy subgame-perfect equilibrium of each voting subgame, where it exists. For two of the parties' action pairs, $\sigma_L = \{H, L, H\}$, $\sigma_H = \{L, L, L\}$ and $\sigma_L = \{H, H, H\}$, $\sigma_H = \{L, H, L\}$ (displayed in the table with dashed green lines), there exist no pure-strategy subgame-perfect equilibria, so the payoff from the unique mixed strategy equilibrium (as $\varepsilon \rightarrow 0$) are displayed instead, as suggested by the concept of mostly-pure-strategy subgame-perfect equilibria.

From inspection of this payoff matrix it can be seen that there are four mostly-pure-strategy subgame-perfect equilibria (bordered with thick red lines): when $\sigma_L = \{L, L, L\}$ or $\sigma_L = \{L, L, H\}$, and $\sigma_H = \{L, H, H\}$ or $\sigma_H = \{H, H, H\}$, and $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$. All these equilibria have the properties described in the proposition. ■

Proof of Theorem 1: Table A.2 displays the payoffs each party would receive if they played various actions when $\beta > 1$ but (7) holds, and additionally

$$q(u_i(H|H, L) - u_i(L|H, L)) + p(u_i(H|L, L) - u_i(L|L, L)) < 0, \quad (12)$$

while Table A.3 shows the case where $\beta > 1$, (7) holds, but (12) does not. Note that in both cases there are three pure strategy equilibria, two of which are discussed in the text. As we are interested in strong equilibria, we must consider whether an equilibrium is supported by coalition-proof pure-strategy equilibria of the voting game, when coalition-proof pure-strategy equilibria exist.

Table A.2: Subgame-perfect continuation payoffs for parties when $\beta > 1$, (7) and (12) hold.

Low Party, High Party	L, L, L	L, L, H	L, H, L	L, H, H	H, L, L	H, L, H	H, H, L	H, H, H
L, L, L	$pR + \varepsilon_{3L},$ $qR + \varepsilon_{0H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $qR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + p\varepsilon_{2L} + q\varepsilon_{1L},$ $qR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$
L, L, H	$R + \varepsilon_{2L},$ ε_{1H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$R + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $q\varepsilon_{2H} + p\varepsilon_{1H}$	$pR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $qR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$R + \varepsilon_{2L},$ ε_{1H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$R + p\varepsilon_{2L} + q\varepsilon_{1L},$ $q\varepsilon_{2H} + p\varepsilon_{1H}$	$pR + p\varepsilon_{2L} + q\varepsilon_{1L},$ $qR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$
L, H, L	$R + \varepsilon_{2L},$ ε_{1H}	$qR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $pR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{2L},$ ε_{1H}	$R + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
L, H, H	$R + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $q\varepsilon_{2H} + p\varepsilon_{1H}$	$qR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $pR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{2L},$ ε_{1H}	$R + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$R + \varepsilon_{1L},$ ε_{2H}	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, L, L	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$\varepsilon_{1L},$ $R + \varepsilon_{1H}$	$p\varepsilon_{2L} + q\varepsilon_{1L},$ $R + q\varepsilon_{2H} + p\varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, L, H	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$p\varepsilon_{2L} + q\varepsilon_{1L},$ $R + q\varepsilon_{2H} + p\varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, H, L	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, H, H	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$

Note that the following discussion applies to both Table A.2 and Table A.3, and the mathematics supporting the assertions in this discussion can be found in the proof of Proposition 3. First, consider the pure-strategy equilibrium when $\sigma_L = \{L, L, H\}$, $\sigma_H = \{H, H, H\}$. This equilibrium is supported by the voting profile $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$, which is coalition proof, rather than $V(\sigma_L, \sigma_H) = \{1, 1/1, 1\}$ which is not. This equilibria also requires that the voting profile be $V(\sigma_L, \sigma_H) = \{0, 0/1, 0\}$ when $\sigma_L = \{L, L, H\}$, $\sigma_H = \{H, H, L\}$, which is coalition proof, rather than $V(\sigma_L, \sigma_H) = \{1, 1/1, 0\}$, which is not. Note that in the off-path case where $\sigma_L = \{L, L, L\}$, $\sigma_H = \{H, H, H\}$ either pure-strategy equilibrium of the voting game will support the equilibrium at $\sigma_L = \{L, L, H\}$, $\sigma_H = \{H, H, H\}$, so we do not need to check which, if either, is coalition proof. The preceding discussion also establishes that $\sigma_L = \{L, L, H\}$, $\sigma_H = \{L, H, H\}$, $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$ is a strong pure-strategy subgame-perfect equilibrium, and that $\sigma_L = \{H, H, H\}$, $\sigma_H = \{H, H, H\}$, $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$ is not a strong pure-strategy subgame-perfect equilibrium.¹⁷ Note that all strong pure-strategy subgame-perfect equilibria identified above have the conditions identified in the proposition.

When (7) does not hold there are two cases: when (8) holds, and when it does not. When (8) holds, then the payoffs of the party game are illustrated in Table A.4. Inspection shows there is a single pure-strategy subgame-perfect equilibrium, and, as it is not supported by any subgames where there are more than a single pure-strategy equilibrium, it is by definition strong.

¹⁷Note that we do not need to check whether either of the pure-strategy subgame equilibrium of the voting stage determined by $\sigma_L = \{H, L, L\}$, $\sigma_H = \{L, H, H\}$ or $\sigma_L = \{H, L, H\}$, $\sigma_H = \{L, H, H\}$ are coalition proof, as either of the two possible pure-strategy equilibria of these subgames will support the equilibrium $\sigma_L = \{L, L, H\}$, $\sigma_H = \{L, H, H\}$, $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$. Moreover, although the pure-strategy equilibria of the subgames defined by $\sigma_L = \{H, H, H\}$, $\sigma_H = \{L, L, L\}$ and $\sigma_L = \{H, H, H\}$, $\sigma_H = \{L, L, H\}$ needed to support $\sigma_L = \{H, H, H\}$, $\sigma_H = \{H, H, H\}$, $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$ as a pure-strategy subgame-perfect equilibrium are not coalition proof, this does not need to be checked explicitly, as it is sufficient to show that only one of the off-path equilibria needed to support this pure-strategy subgame-perfect equilibrium is not coalition proof to show that it is not strong.

Table A.4: Subgame-perfect continuation payoffs for parties when (7) does not hold, but (8) does.

Low Party, High Party	L, L, L	L, L, H	L, H, L	L, H, H	H, L, L	H, L, H	H, H, L	H, H, H
L, L, L	$pR + \varepsilon_{3L},$ $qR + \varepsilon_{0H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $qR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$p\varepsilon_{2L} + q\varepsilon_{1L},$ $R + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
L, L, H	$R + \varepsilon_{2L},$ ε_{1H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$R + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $q\varepsilon_{2H} + p\varepsilon_{1H}$	$pR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $qR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$R + \varepsilon_{2L},$ ε_{1H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
L, H, L	$R + \varepsilon_{2L},$ ε_{1H}	$qR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $pR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{2L},$ ε_{1H} $p\varepsilon_{2L} + q\varepsilon_{1L},$ $R + q\varepsilon_{2H} + p\varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
L, H, H	$R + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $q\varepsilon_{2H} + p\varepsilon_{1H}$	$qR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $pR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, L, L	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $pR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $qR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, L, H	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, H, L	$\varepsilon_{2L},$ $R + \varepsilon_{1H},$ $qR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $pR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$qR + p\varepsilon_{2L}$ $+q\varepsilon_{1L},$ $pR + q\varepsilon_{2H}$ $+p\varepsilon_{1H}$	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$
H, H, H	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$

When (8) does not hold, the payoffs from the party game are illustrated in Table A.5. As each voting stage has a single pure-strategy equilibrium, all pure-strategy subgame-perfect equilibria will be strong. When β is small enough so that

$$\begin{aligned} \varepsilon_{3L} > \varepsilon_{2L} &> \varepsilon_{1L} > \varepsilon_{0L} \\ \varepsilon_{0H} < \varepsilon_{1H} &< \varepsilon_{2H} < \varepsilon_{3H}, \\ &\text{or,} \\ \varepsilon_{3L} > \varepsilon_{1L} &> \varepsilon_{2L} > \varepsilon_{0L} \\ \varepsilon_{0H} < \varepsilon_{2H} &< \varepsilon_{1H} < \varepsilon_{3H}, \end{aligned}$$

then inspection shows that there are two pure-strategy subgame-perfect equilibria: $\sigma_H = \{H, H, H\}$, $\sigma_L = \{H, H, L\}$ or $\{H, H, H\}$, and $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$. If, instead, β is large enough so that:

$$\begin{aligned} \varepsilon_{1L} > \varepsilon_{3L} &> \varepsilon_{0L} > \varepsilon_{2L} \\ \varepsilon_{2H} < \varepsilon_{0H} &< \varepsilon_{3H} < \varepsilon_{1H}, \end{aligned}$$

then there is a single pure-strategy subgame-perfect equilibria: $\sigma_H = \{H, H, H\}$, $\sigma_L = \{H, H, H\}$, and $V(\sigma_L, \sigma_H) = \{0, 0/1, 1\}$. Note that all pure-strategy subgame-perfect equilibria where (7) does not hold are strong, and have the properties described in the proposition. ■

Proof of Proposition 2: Note that the right-hand side of (9) can be expressed as:

$$\frac{2\sqrt{\tau_{lw}^* \bar{y}} - \tau_{lw}^* y \theta}{2\sqrt{\tau_{hw}^* \bar{y}} - \tau_{hw}^* y \theta} = \frac{\frac{2\xi}{1+w} - \frac{\xi}{(1+w)^2}}{\frac{2\xi}{\eta+w} - \frac{\xi}{(\eta+w)^2}} = \frac{(1+2w)(\eta+w)^2}{(2(\eta+w)-1)(1+w)^2}$$

and the limit of this quantity, as $w \rightarrow \infty$ is 1. Moreover, it is a continuous function of w .

Table A.5: Subgame-perfect continuation payoffs for parties when (8) does not hold.

Low Party, High Party	L, L, L	L, L, H	L, H, L	L, H, H	H, L, L	H, L, H	H, H, L	H, H, H
L, L, L	$pR + \varepsilon_{3L},$ $qR + \varepsilon_{0H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$\varepsilon_{1L},$ $R + \varepsilon_{2H}$	$\varepsilon_{2L},$ $R + \varepsilon_{1H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$\varepsilon_{1L},$ $R + \varepsilon_{2H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
L, L, H	$R + \varepsilon_{2L},$ ε_{1H}	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$R + \varepsilon_{1L},$ ε_{2H}	$\varepsilon_{1L},$ $R + \varepsilon_{2H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
L, H, L	$R + \varepsilon_{2L},$ ε_{1H}	$R + \varepsilon_{1L},$ ε_{2H}	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$\varepsilon_{1L},$ $R + \varepsilon_{2H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$\varepsilon_{1L},$ $R + \varepsilon_{2H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
L, H, H	$R + \varepsilon_{1L},$ ε_{2H}	$R + \varepsilon_{1L},$ ε_{2H}	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, L, L	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$\varepsilon_{1L},$ $R + \varepsilon_{2H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$	$pR + \varepsilon_{2L},$ $qR + \varepsilon_{1H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$\varepsilon_{1L},$ $R + \varepsilon_{2H}$	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, L, H	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$R + \varepsilon_{0L},$ ε_{3H}	$\varepsilon_{0L},$ $R + \varepsilon_{3H}$
H, H, L	$R + \varepsilon_{1L},$ ε_{2H}	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$R + \varepsilon_{1L},$ ε_{2H}	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{1L},$ $qR + \varepsilon_{2H}$	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$
H, H, H	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$R + \varepsilon_{0L},$ ε_{3H}	$pR + \varepsilon_{0L},$ $qR + \varepsilon_{3H}$

As the left-hand side can be expressed as:

$$\frac{\pi_{2H}^H}{\pi_{2L}^L} = \frac{\beta(2 + \beta)}{2\beta + 1} > 1$$

which is greater than 1 for $\beta > 1$. Thus, as w grows, the left-hand-side will eventually be less than the right-hand-side, which guarantees that (7) will not hold, and then all pure-strategy subgame-perfect equilibria will be as described in the second part of Theorem 1. ■

Proof of Proposition 3: In this environment citizen utility is given by:

$$\begin{aligned} u_l &= (1 - \tau^*)y\theta + 2\sqrt{(2n + 1)\bar{y}\tau^*\pi^j} \\ u_h &= (1 - \tau^*)y\eta\theta + 2\sqrt{(2n + 1)\bar{y}\tau^*\pi^j} \end{aligned} \tag{13}$$

as analogues to (2) when there are $2n + 1$ districts. Define $\pi_{rk}^{k'}$ as the share to a district represented by a legislator of type $k' \in \{L, H\}$ when the majority is composed of $r \geq n + 1$ $k \in \{L, H\}$ legislators. Taking first order conditions this implies that the tax rate set by the legislature will be:

$$\tau_{rL}^* = (2n + 1)\pi_{rL}^L\tau_l^*$$

when the majority of the legislature is low types, and

$$\tau_{rH}^* = (2n + 1)\pi_{rH}^H\tau_h^*$$

when the majority of the legislature are high types, where τ_l^* and τ_h^* are the tax rates set by the legislature when it is composed entirely of low types or high types respectively (and thus, the share to each district is $\frac{1}{2n+1}$), as defined in (11).

Now consider the incentives of a high type median voter when the majority of the legislature is low types, and there are $j < n$ high types that have been elected to the legislature. Using the notation defined for Theorem 1, the utility that a high-type median voter will get

when electing a high-type legislator is:

$$\begin{aligned}
u_h(H|(2n+1-(j+1))L, jH) &= (1 - (2n+1)\pi_{(2n+1-(j+1))L}^L \tau_l^*)\eta y\theta \\
&\quad + 2\sqrt{(2n+1)\bar{y}(2n+1)\pi_{(2n+1-(j+1))L}^L \tau_l^* \pi_{(2n+1-(j+1))L}^H} \\
&= \eta y\theta - (2n+1)\xi\eta\pi_{(2n+1-(j+1))L}^L \\
&\quad + 2(2n+1)\xi\sqrt{\pi_{(2n+1-(j+1))L}^L \pi_{(2n+1-(j+1))L}^H}
\end{aligned}$$

and, correspondingly, the utility that a high-type median voter will get when electing a low-type legislator is

$$u_h(L|(2n+1-(j+1))L, jH) = \eta y\theta - (2n+1)\xi\eta\pi_{(2n+1-j)L}^L + 2(2n+1)\xi\pi_{(2n+1-j)L}^L.$$

The difference between electing a high-type and a low-type legislator results in a utility difference of

$$(2n+1)\xi \left[2 \left(\sqrt{\pi_{(2n+1-(j+1))L}^L \pi_{(2n+1-(j+1))L}^H} - \pi_{(2n+1-j)L}^L \right) + \eta \left(\pi_{(2n+1-j)L}^L - \pi_{(2n+1-(j+1))L}^L \right) \right]$$

which, substituting leads to

$$\frac{(2n+1)\xi}{(2n+1-(j+1) + (j+1)\beta)(2n+1-j+j\beta)} \left[2(\sqrt{\beta}-1)(2n+1+j(\beta-1)) + (\eta-2)(\beta-1) \right]$$

which will be positive whenever

$$2(\sqrt{\beta}-1)(2n+1+j(\beta-1)) + (\eta-2)(\beta-1) > 0. \quad (14)$$

As the first term above is positive, the whole thing will be positive if $\eta > 2$. If, instead, $\eta < 1$, then the right-hand-side of (14) will be minimized when $n = 1$ and $j = 0$. When this

is the case, then we are considering the three district case again, and (14) will hold whenever

$$\beta < \left(\frac{\eta + 4}{\eta - 2} \right)^2.$$

Note that setting $\eta = 1$ both minimizes the left-hand-side, and gives the corresponding expression for when a low-type will want to elect a high-type. Doing so implies that both a low-type and a high-type will want to elect a high-type when low-types are in the majority, and doing so will not change the majority, whenever $\beta < 25$, which it is by assumption. Note that this also serves as the conditions in footnote accompanying Table 1.

Next, we consider which type a high-type would like to elect if the majority are high-types, and other districts have elected $j' < n$ low-type legislators. The utility a high-type would get from electing a high-type legislator is given by:

$$u_h(H|j'L, (2n + 1 - (j + 1))H) = \eta y \theta - \frac{(2n + 1)\xi}{\eta} \pi_{(2n+1-j')H}^H + \frac{2(2n + 1)\xi}{\eta} \pi_{(2n+1-j')L}^H$$

and the utility of electing a low-type legislator is:

$$\begin{aligned} u_h(L|j'L, (2n + 1 - (j + 1))H) &= \eta y \theta - \frac{(2n + 1)\xi}{\eta} \pi_{(2n+1-(j'+1))H}^H \\ &\quad + \frac{2(2n + 1)\xi}{\eta} \sqrt{\pi_{(2n+1-(j'+1))H}^L \pi_{(2n+1-(j'+1))H}^H}. \end{aligned}$$

The utility difference between electing a high-type and a low-type legislator thus simplifies to:

$$\frac{(2n + 1)\xi}{\eta((2n + 1 - j')\beta + j')((2n + 1 - (j' + 1))\beta + j' + 1)} \left[2(\beta - \sqrt{\beta})((2n + 1)\beta + j'(1 - \beta)) + \beta(1 - \beta) \right]$$

which will be positive when

$$2(\beta - \sqrt{\beta})((2n + 1)\beta + j'(1 - \beta)) + \beta(1 - \beta) > 0. \quad (15)$$

The first term is positive, and will be minimized when $n = 1$ and $j' = 0$ (as in the three district case). Then, (15) reduces to:

$$5\beta + 1 - 6\sqrt{\beta} > 0.$$

As the left-hand-side above equals zero when $\beta = 1$, and the derivative at $\beta = 1$ is positive, it must be the case that (15) holds for all $\beta > 1$, $n \geq 1$ and $0 \geq j' < n$.

The utility difference for a low-type median voter can be shown to be:

$$\frac{(2n+1)\xi}{\eta^2((2n+1-j')\beta+j')((2n+1-(j'+1))\beta+j'+1)} \left[2\eta(\beta-\sqrt{\beta})((2n+1)\beta+j'(1-\beta)) + \beta(1-\beta) \right].$$

As the term in brackets has already been shown to be positive when $\eta = 1$, and η in the above expression multiplies a positive quantity, it must be the case that the term in brackets is positive for all $\eta > 1$. Thus both high and low-type median voters prefer to elect a high-type when the majority is composed of high-types, and doing so will not change which type is the majority.

Next, consider the preferences of a high-type median voter when other districts have elected n high-types and n low types. Then, her utility from electing a high-type is

$$u_h(H|nL, nH) = \eta y \theta - \frac{(2n+1)\xi}{\eta} \pi_{(n+1)H}^H + 2 \frac{(2n+1)\xi}{\eta} \pi_{(n+1)H}^H$$

and her utility from electing a low type is

$$u_h(L|nL, nH) = \eta y \theta - (2n+1)\xi \eta \pi_{(n+1)L}^L + 2(2n+1)\xi \pi_{(n+1)L}^L.$$

Thus, the utility difference is

$$\begin{aligned} u_h(H|nL, nH) - u_h(L|nL, nH) &= \frac{(2n+1)\xi}{\eta} \left[2(\pi_{(n+1)H}^H - \eta \pi_{(n+1)L}^L) + (\eta^2 \pi_{(n+1)L}^L - \pi_{(n+1)H}^H) \right] \\ &= \frac{(2n+1)\xi}{\eta} \left[\pi_{(n+1)H}^H + \eta(\eta-2)\pi_{(n+1)L}^L \right] \end{aligned}$$

where the term in brackets is minimized when $\eta = 1$. When this is the case, the term in brackets is:

$$\begin{aligned}\pi_{(n+1)H}^H - \pi_{(n+1)L}^L &= \frac{\beta}{(n+1)\beta + n} - \frac{1}{n\beta + (n+1)} \\ &= \frac{n(\beta^2 - 1)}{((n+1)\beta + n)(n\beta + (n+1))} > 0.\end{aligned}$$

So a high-type median voter will always want to elect a high-type, even when her choice will be pivotal in shifting the majority of the legislature from low-types to high-types.

Finally, we consider the choice of a low-type median voter when her choice will be pivotal in shifting the majority of the legislature from low-types to high-types. The utility she would get from different types of legislators is given by:

$$\begin{aligned}u_l(L|nL, nH) &= (1 - (2n+1)\pi_{(n+1)L}^L \tau_l^*)y\theta + 2\sqrt{(2n+1)\bar{y}(2n+1)\pi_{(n+1)L}^L \tau_l^* \pi_{(n+1)L}^L} \\ u_l(H|nL, nH) &= (1 - (2n+1)\pi_{(n+1)H}^H \tau_h^*)y\theta + 2\sqrt{(2n+1)\bar{y}(2n+1)\pi_{(n+1)H}^H \tau_h^* \pi_{(n+1)H}^H}\end{aligned}$$

and thus, $u_l(L|nL, nH) > u_l(H|nL, nH)$ when

$$\frac{\pi_{(n+1)H}^H}{\pi_{(n+1)L}^L} < \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y\theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y\theta}$$

which is the same as (10).

When (10) holds, then pure-strategy equilibria will have two forms. Either $n+1$ districts will elect low type legislators and n districts will elect high-type legislators, or all $2n+1$ districts will elect high type legislators.

Consider the first type. This is an equilibrium as anyone who is electing a high-type legislator is not pivotal, and thus would prefer to elect a high-type. Any median voter who is electing a low-type legislator is pivotal, so, as (10) holds, they prefer to elect a low-type.

Consider now the second type. This is an equilibrium as the median voter in every district is not pivotal, they prefer to elect a high type.

Now, we show there exist no other pure-strategy equilibria. Take as a candidate equilib-

rium one where $j < n$ districts elect high types, and the rest of the districts elect high-types. Then the median voter of every district that is electing a low-type is not pivotal, and would prefer to elect a high-type. Thus, this cannot be an equilibrium. Similar logic applies when the candidate equilibrium calls for $j' < n$ districts to elect low types, and the rest to elect high-types.

Finally, we show that the equilibrium of the second type is not coalition proof, but equilibria of the first type are. In the equilibrium of the second type the tax rate is τ_h^* and each district gets an equal share of tax revenue. It will be a profitable deviation for $n + 1$ low-type districts elected low-types if

$$\frac{\pi_{(2n+1)H}^H}{\pi_{(n+1)L}^L} < \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta}.$$

As $\pi_{(2n+1)H}^H = \frac{1}{3} < \pi_{(n+1)H}^H$, this implies that

$$\frac{\pi_{(2n+1)H}^H}{\pi_{(n+1)L}^L} < \frac{\pi_{(n+1)H}^H}{\pi_{(n+1)L}^L} < \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta}.$$

as (10) holds. Thus, there is a profitable deviation for $n + 1$ low types, and the equilibrium of the second type is not coalition proof.

To see that all equilibria of the first type are coalition proof, consider a potential deviation in which the low-type voters in j of the districts that are supposed to elect low-type legislators (in equilibrium) instead defect and vote for, and elect, high-type legislators. This will be a profitable deviation if:

$$\frac{\pi_{(n+j)H}^H}{\pi_{(n+1)L}^L} > \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta}.$$

However, as $\pi_{(n+j)H}^H < \pi_{(n+1)H}^H$, it follows from (10) that

$$\frac{\pi_{(n+j)H}^H}{\pi_{(n+1)L}^L} < \frac{\pi_{(n+1)H}^H}{\pi_{(n+1)L}^L} < \frac{2\sqrt{\tau_l^* \bar{y}} - \tau_l^* y \theta}{2\sqrt{\tau_h^* \bar{y}} - \tau_h^* y \theta}$$

so there is no coalition of low-type voters who were supposed to vote for low-type candidates

who would be made strictly better off by voting for (and electing) high-type candidates. As high-type voters do not affect the outcomes in any district, we do not need to consider deviations by them.

Next, consider a deviation in which the low-type voters in j' districts that are supposed to elect high-type legislators (in equilibrium) instead defect and vote for, and elect, low-type legislators. Had these voters complied with their equilibrium strategies, they would have received:

$$u_l(H|nL, nH) = y\theta - (2n + 1)\xi\pi_{(n+1)L}^L + 2(2n + 1)\xi\sqrt{\pi_{(n+1)L}^H\pi_{(n+1)L}^L}$$

while deviating gives:

$$u_l(L|(n + j')L, (n - j')H) = y\theta - (2n + 1)\xi\pi_{(n+j'+1)L}^L + 2(2n + 1)\xi\pi_{(n+j'+1)L}^L.$$

After substitutions, the utility gain from sticking with the equilibrium strategy is:

$$\frac{(2n + 1)\xi}{((n + 1) + n\beta)((n + 1 + j') + (n - j')\beta)} \left[2(\sqrt{\beta} - 1)((n + 1 + j') + (n - j')\beta) + j'(\beta - 1) \right]$$

This term in brackets is minimized when $n = 1$, $j' = 1$, which reduces the term in brackets to $6\sqrt{\beta} - 5 - \beta$, which is non-negative as long as $\beta \in [1, 25]$, which it is, by assumption. Thus, there is no coalition of low-type voters who were supposed to vote for high-type candidates who would be made strictly better off by voting for (and electing) low-type candidates. Note that none of the above relationships depend on the number of high-type median voters, z , which will always elect high-types in equilibria. In particular, everything above holds for $z = 0$.

As all coalition-proof equilibria are also equilibria, and we have identified all pure-strategy equilibria, we have thus identified all pure-strategy coalition-proof equilibria when (10) holds.

When (10) does not hold, then electing a low-type legislator is strictly dominated, so every district will want to elect a high-type legislator. As such, this is the unique equilibrium. As

the payoff to defection for low types is largest when all defect to vote for a low-type legislator, this equilibrium will be coalition proof when this deviation is not profitable for a low type. Specifically, if

$$\begin{aligned}
u_l(H|(2n)H) - u_h(L|(2n-z)L, zH) &= \frac{(2n+1)\xi}{\eta^2} \left[2(\eta\pi_{(2n+1)H}^H - \eta^2\pi_{(2n+1-z)}^L) \right. \\
&\quad \left. + (\eta^2\pi_{(2n+1-z)}^L - \pi_{(2n+1)H}^H) \right] \\
&= \frac{(2n+1)\xi}{\eta^2} \left[(\eta-1)\pi_{(2n+1)H}^H - \eta^2\pi_{(2n+1-z)}^L \right] \\
&= \frac{(2n+1)\xi}{\eta^2} \left[\frac{\eta-1}{2n+1} - \frac{\eta^2}{(2n+1-z) + z\beta} \right] > 0
\end{aligned}$$

the the equilibria will be coalition proof. Simplifying leads to the following condition for coalition-proofness

$$\frac{z}{2n+1} > \frac{\eta^2 - \eta + 1}{\beta - 1},$$

which is the same as found in the proposition. ■

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