

History Dependence in Networks of Close Relationships

Theory, and application to job referrals*

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September 3, 2017

Abstract

We develop a model of costly network formation in which agents learn about the quality of their matches. By retaining good connections, agents become increasingly reluctant to form matches of unknown quality, leading their networks to be front-loaded with agents they met near the beginning of their careers. This reluctance combined with turnover naturally gives rise to “cohort attachment”: new agents form links with each other because the agents already there are reluctant to form links with them. When members of a network formed within an organization are subsequently split across many organizations, the desire to renew their successful working relationships leads to job referrals. Using matched employer-employee data from Brazil, we find that the presence of a hiring-cohort former co-worker increases the probability of job acquisition at a specific hiring plant nearly three times more than the presence of a non-hiring-cohort former co-worker. We attempt to mitigate lack of random assignment of former co-workers to job seekers by controlling for observable similarities and by using placebo co-workers, placebo hiring plants, and peers-of-peers instruments for presence of former co-workers.

Keywords: networks, history dependence, job referrals

JEL Classification: D85, J63, J64

*Our thanks to Nageeb Ali, Roy Allen, Zachary Breig, Gordon Dahl, Paul Niehaus, John Rehbeck, Jeffrey Shrader, Joel Sobel, Alexis Toda, Joel Watson, and participants in seminars at Board of Governors, Caltech, Duke, Tufts, and UCSD for many helpful comments. We are responsible for any errors.

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1 Introduction

People form close social and work relationships inside organizations such as firms and schools. These networks influence how happy and productive they are in the organizations. A distinguishing feature of close relationships is that they require significant time and energy. In this paper we argue that these relationships are also persistent. We show how this persistence of close relationships, in combination with their time-intensity, shapes the way agents form their networks of close relationships and the resulting patterns of history dependence in these networks. Moreover, we provide evidence that these relationships continue to have value even outside their original organizational contexts.¹

In our model agents learn whether they get along well or work productively together by trying to do so. If they discover they are well matched, they continue to socialize or work collaboratively in the future, given the opportunities. Denoting a focal agent by ego and designating the others in the organization as alters, we consider the set of alters with whom ego has learned he is well matched to constitute his network of close relationships. Ego can expand this network by trying out relationships with alters of unknown match quality and learning with which new alters match quality is good. Adding members to his network becomes increasingly costly, however, because close interaction with each one eventually interferes with close interaction with the others, given limited time and energy. Considering an ego entering a new environment, he will be most open to trying out relationships at the beginning, and less open later when his network is growing large. Agents' networks thus tend to be front-loaded with people they met near the beginning of their organizational careers. Turnover within the organization erodes this front-loading as the oldest relationships are replaced by recent ones, leading to a U-shaped pattern of history dependence for the networks of agents with careers of moderate length.

When turnover brings a new cohort of agents into the organization, they find that the agents already there are not very open to trying out new relationships, so the new agents try out relationships with each other. A pattern of network links (close relationships) forms within the organization in which within-cohort links are overrepresented. Our model thus gives rise to predictions about the cross-section pattern of network links within an organization as well as predictions regarding how individual networks evolve over time.

¹Costa, Kahn, Roudiez, and Wilson (2016) provide evidence that networks of close relationships formed through serving in the same company in the civil war led veterans to co-locate for mutual assistance after the end of the war.

We will give the name “cohort attachment” to the tendency for within-cohort links to be over-represented within an organization. The concept is recognized in sociology, though the phrase “cohort attachment” is not used. [Wagner, Pfeffer, and O’Reilly III \(1984, p. 76\)](#) write, “Thus, because of the effects of free communication capacity and interest in forming relationships, persons who enter [the organization] at roughly the same time are more likely to communicate with each other than with those who entered either much earlier or later.” This idea is used by [Zenger and Lawrence \(1989\)](#) to examine the impact of tenure similarity (equivalent to time-of-entry similarity) on subsequent communication. They find that tenure similarity strongly predicts the frequency with which engineers and engineering managers in the research division of a medium-sized U.S. electronics firm communicate outside of their project groups. We were able to find one example of the use of the concept of cohort attachment in economics.² [Bandiera, Barankay, and Rasul \(2008, Table 4\)](#) find that “same arrival date” is a strong predictor of friendship among college students working on seasonal contracts picking fruit on a UK farm, controlling for a wide range of ascriptive characteristics and potential correlates such as same living site. They go on to use this indicator as a “plausibly exogenous” measure of network links when analyzing the impact of network links on worker productivity.

An advantage of the prediction of cohort attachment over the other predictions of our model is that it can be tested without detailed, retrospective surveys of the agents in an organization, making it feasible to use data from many organizations. It is simple to extend our model to allow members of ego’s network formed within an organization who are subsequently split across many organizations to be his “contacts.” The desire of contacts to renew their successful working relationships leads to job referrals.³ This application to job referrals allows us to fit our cohort attachment prediction into an existing empirical literature on job referrals from former co-workers, which includes [Cingano and Rosolia \(2012\)](#), [Glitz \(2017\)](#), [Eliason, Hensvik, Kramarz, and Skans \(2017\)](#), [Hensvik and Skans \(2016\)](#), and [Saygin, Weber, and Weynandt \(2014\)](#).⁴

²Because it is useless to search for the phrase “cohort attachment,” it is entirely possible that we missed many other examples.

³As we discuss at the beginning of Section 6, workers do not have incentives to provide “negative referrals” if not forced to renew unsuccessful relationships when former co-workers with whom they are poorly matched are hired.

⁴The teamwork motivation for referral has recently been confirmed experimentally by [Pallais and Sands \(forthcoming\)](#). In randomized controlled trials with referred job applicants, they report (p. 4), “each referral completed one task with her referrer and one task with another randomly-chosen referrer. Referred workers performed substantially better when paired with their own referrers.” They also report (pp. 39-40), “referrers were more than twice as likely to want to partner again with their own referral as with someone else’s referral. Similarly, referred workers were substantially more likely to want to work again with their own referrer than with someone else’s referrer.” On the other hand,

We use matched employer-employee data from Brazil to investigate the impact of former co-workers on the probabilities of job acquisition at specific hiring plants for workers from closing firms, an investigation most similar to [Saygin, Weber, and Weynandt \(2014\)](#). We find that under our baseline specification the presence of a hiring-cohort former co-worker increases the probability of job acquisition at a specific hiring plant nearly three times more than the presence of a non-hiring-cohort former co-worker. We attempt to mitigate lack of random assignment of former co-workers to job seekers by controlling for observable similarities and by using placebo co-workers, placebo hiring plants, and peers-of-peers instruments for presence of former co-workers ([Bramoullé, Djebbari, and Fortin 2009](#), [De Giorgi, Pellizzari, and Redaelli 2010](#)).

Our work is closely related to, and has implications for, the peer effects literature. Both are concerned with networks formed as a result of being in the same place at the same time. The current state of the art in the peer effects literature is to examine peer groups created by random assignment (see [Sacerdote 2014](#) for a survey). Typically random assignment occurs at the beginning of the agents' tenure in an organization. A popular example is random assignment of college freshmen to dorm rooms. The alters to which ego is randomly assigned are then found to influence a wide range of his behaviors, from binge drinking to buying a new car. The results of our model suggest that this influence would be much weaker if the random assignments occurred at the ends instead of the beginnings of organizational careers, because egos will be less open to establishing new relationships with the alters to whom they have been assigned. At the same time, persistence of close relationships suggests that it would be worth pursuing follow-up studies of the influence of randomly assigned peers.

In the next section we develop our model of network formation and history dependence for one firm, and derive results for the longitudinal and cross-sectional structure of agents' networks. [Section 3](#) extends the model to allow for job referrals across firms. Our data on job acquisitions and former co-workers are described in [Section 4](#). We establish a baseline specification for the impact of former co-workers on job acquisition in [Section 5](#). [Section 6](#) distinguishes between cohort and non-cohort former co-workers and carries out robustness checks. [Section 7](#) concludes.

Pallais and Sands (p. 1) “do not find evidence that referrals exert more effort because they believe their performance will affect their relationship with their referrer or their referrer’s position at the firm.”

2 Network Formation Within One Firm

2.1 Model assumptions

We will consider the formation of personal networks by agents within an organization. We will call this organization a firm with a view to our later empirical application. However, we believe that our model applies to network formation in other institutional settings as well.

A key inspiration for our model is [Jovanovic \(1979\)](#). In his model, one worker meets with one firm, and the pair learn about the quality of their match. Roughly speaking, if they learn that the quality of their match is good, they stay together, and if they learn that the quality of their match is bad, they separate. In our model, matches are between workers (agents) within a firm. Well matched agents become members of each others' networks (stay together), and poorly matched agents avoid each other in the future (separate). Different from [Jovanovic \(1979\)](#), an agent can in principle form matches with any number of other agents, up to the limit of all the agents in the firm.

We will follow the evolution of agents' networks in the firm over time $t = \{0, 1, 2, \dots\}$. We will assume these agents are symmetric and form a continuum of size N . The continuum assumption allows us to avoid integer problems. In this section, we will ignore agents outside the boundary of the firm.

In every period, risk-neutral agents engage in pairwise work relationships or matches.⁵

Assumption 1. *Every match is one of two types determined by the surplus it yields to the matched parties in the period in which it occurs: high quality yielding y_H or low quality yielding y_L ($y_H > y_L > 0$).⁶ The unconditional probability that a match is high quality is $p \in (0, 1)$.*

The match surplus can be thought of as net of any benefit derived by the firm.

Assumption 2. *Every match is of equal value to both parties, i.e., the matched parties divide the surplus equally.*

When the context is appropriate, it is possible to interpret this assumption as the outcome of Nash bargaining with a disagreement point of $(0, 0)$. For example, we could suppose that if the matched

⁵We believe that work relationships are ubiquitous even where employees appear to work in isolation, as in a typical cubicle environment, for example. Employees find others with whom they work well solving non-routine problems or fill in for each other. They interact during breaks and lunch, where good relationships boost morale and reduce absenteeism.

⁶This follows the [Moscarini \(2005\)](#) simplification of [Jovanovic \(1979\)](#).

parties cannot agree on who deserves how much credit, they cannot turn in their project to their boss to get paid. In other contexts the benefits of the match are non-monetary, so we are effectively assuming that the “technology of friendship” divides the surplus equally.

Assumptions 1 and 2 imply that, in the period in which the match occurs, each agent receives $\frac{y_H}{2}$ when the match is high quality and $\frac{y_L}{2}$ when the match is low quality. We assume that all matches contribute equally, regardless of type, to an agent’s time and energy cost. Recalling that every agent is symmetric, let z_t be the total number of matches formed by an agent in period t :

Assumption 3. *The cost to an agent of forming z_t matches is $c(z_t)$, where $c(0) = 0$, $c'(z) > 0$, $c''(z) > 0$ and $\lim_{z \rightarrow \infty} c'(z) = \infty$.*

We assume $c''(z) > 0$ because, as the number of work relationships grows, the agent gets tired, has scheduling conflicts, etc.

Agents learn their match qualities with other agents by experience:

Assumption 4. *At the end of every period, the qualities of all unknown matches formed in that period are revealed.*

The firm undergoes a constant, exogenous rate of worker turnover:

Assumption 5. *At the beginning of every period $t = \{1, 2, \dots\}$, every agent separates from the firm with probability δ , and the departing agents are replaced by a cohort of size δN , where $\delta \in [0, 1]$.*

This assumption serves two purposes. First, it creates network decay at a constant rate δ : an agent who remains with the firm finds that a share δ of the agents with whom he knows he is well matched disappears each period. Second, it creates a cohort structure for the firm in which every cohort except the founding ($t = 0$) cohort is of equal initial size, and cohort size declines at a constant rate with tenure. When we analyze the cohort shares of agents’ networks, the patterns generated by the interplay of forces in our model will stand out more clearly against this simple baseline. We could decouple hiring from network decay, allowing the firm to grow or shrink in any period, without affecting the longitudinal results in the next subsection, but we will note in subsection 2.3 that some of our cross-sectional results could be affected.

It is useful to consider what happens with extreme values of δ . If $\delta = 1$, the firm is re-created from scratch every period. Since the firm and its agents have no history, there can be no analysis

of history dependence in the agents' networks. If $\delta = 0$, the firm consists of a fixed set of agents whose networks do not decay. This polar case is of some interest and will be covered in the next subsection.

2.2 Longitudinal results

Let us call the agent on whose decisions we are focusing ego and all other agents alters. In this subsection, without loss of generality we select ego from the founding cohort. This saves on notation because period t is identical to ego's tenure. In the next subsection we consider egos from later cohorts and introduce notation that allows us to distinguish ego tenure from time period.

In each period t , ego inherits from the previous period knowledge that allows him to partition alters into three sets: alters with whom he knows he is well matched, alters with whom his match quality is unknown, and alters with whom he knows he is poorly matched. We call the set of alters with whom he knows he is well matched at the end of the period his **network** and denote its size by n . We denote the size of the set of alters unknown to ego by u . The decisions that each agent needs to make in any period are how many matches z_t to form and with whom. Clearly he prefers to match with alters within his network before trying matches with unknown alters, and prefers trying matches with unknown alters before matching with alters with whom he knows he is poorly matched. Noting that ego inherits a network of size $(1 - \delta)n_{t-1}$ from the previous period, we can consider three cases: i) $z_t \leq (1 - \delta)n_{t-1}$; ii) $(1 - \delta)n_{t-1} < z_t \leq (1 - \delta)n_{t-1} + u_t$; and iii) $z_t > (1 - \delta)n_{t-1} + u_t$. We rule out case iii) by imposing an additional condition on the cost function, derived in Appendix A and expressed in terms of the model parameters, that prevents the number of matches ego desires to form from exceeding $(1 - \delta)n_{t-1} + u_t$ in equilibrium. We will show below that case i) never obtains. Therefore, at the margin, ego always matches with an unknown alter (case ii). Inframarginally, ego matches with any alter within his network with probability one.⁷

One of the most robust features of social networks is "homophily": the tendency for egos to be linked to alters who are similar to them along observable dimensions (McPherson, Smith-Lovin, and Cook 2001). It is possible to incorporate homophily into our model. Suppose we divide our continuum of agents into two types. Matches within type are identical to matches between types, except that the former (latter) are of high quality with probability $p''(p')$, where $p'' > p'$. Agents

⁷Note that ego's desire to match with alters in his network is always reciprocated.

therefore match within type if they can. If case ii) prevails within type then agents never match across types, so all matches are of high quality with probability $p = p''$. The same reasoning holds for more than two types, but as the number of types increases it becomes increasingly unrealistic for there to be enough unknowns within type for case ii) to hold. In other words, we can incorporate homophily into our model by reinterpreting p as the probability that a match is of high quality within type and strengthening the condition in Appendix A sufficiently to ensure that case ii) prevails within type.

We denote by x_t the number of matches ego chooses to form in period t with alters of unknown match quality. He meets x_t alters uniformly at random, and then incurs matching costs $c(z_t) = c(x_t + (1 - \delta)n_{t-1})$. At the end of the period match qualities are revealed and surplus is divided. Ego's total per-period payoff is thus given by the sum of his payoffs from matching within his network and matching outside his network less his matching costs,

$$(1 - \delta)n_{t-1} \frac{y_H}{2} + x_t \frac{py_H + (1 - p)y_L}{2} - c(x_t + (1 - \delta)n_{t-1}).$$

His network size evolves according to

$$n_t = (1 - \delta)n_{t-1} + px_t. \quad (1)$$

We assume that ego maximizes the discounted sum of his per-period payoffs. We also assume that if he separates from the firm he is immediately hired by another firm, at which he accumulates a new network. (We consider the possibility of unemployment in section 3.) Ego's value function is then given by

$$V(n_{t-1}) = \max_{x_t} \left\{ (1 - \delta)n_{t-1} \frac{y_H}{2} + x_t \frac{py_H + (1 - p)y_L}{2} - c(x_t + (1 - \delta)n_{t-1}) + \beta[(1 - \delta)V(n_t) + \delta V(0)] \right\}, \quad (2)$$

where β is the constant discount factor.

The first-order condition yields

$$\frac{py_H + (1 - p)y_L}{2} + \beta(1 - \delta)V'(n_t)p = c'(x_t^* + (1 - \delta)n_{t-1}).$$

Note that

$$V'(n_{t-1}) = (1 - \delta) \frac{y_H}{2} - (1 - \delta) c'(x_t^* + (1 - \delta)n_{t-1}) + \beta(1 - \delta) V'(n_t)(1 - \delta) \\ + \left[\frac{py_H + (1 - p)y_L}{2} + \beta(1 - \delta) V'(n_t)p - c'(x_t^* + (1 - \delta)n_{t-1}) \right] \frac{\partial x_t^*}{\partial n_{t-1}}.$$

The coefficient on $\frac{\partial x_t^*}{\partial n_{t-1}}$ equals zero by the first order condition. We also use the first-order condition to substitute for $c'(x_t^* + (1 - \delta)n_{t-1})$, obtaining

$$V'(n_{t-1}) = (1 - \delta) \frac{y_H}{2} - (1 - \delta) \left[\frac{py_H + (1 - p)y_L}{2} + \beta(1 - \delta) p V'(n_t) \right] + \beta(1 - \delta) V'(n_t)(1 - \delta) \\ = (1 - \delta)(1 - p) \frac{y_H - y_L}{2} + \beta(1 - \delta)^2(1 - p) V'(n_t).$$

This is a linear difference equation for $V'(n_t)$, which admits a constant solution

$$V'(n_{t-1}) = V'(n_t) = \frac{(1 - \delta)(1 - p)}{1 - \beta(1 - \delta)^2(1 - p)} \frac{y_H - y_L}{2}.$$

The constant solution is the only solution that satisfies the transversality condition.⁸ We can substitute it back into the first-order condition to obtain

$$\frac{py_H + (1 - p)y_L}{2} + \beta(1 - \delta)p \frac{(1 - \delta)(1 - p)}{1 - \beta(1 - \delta)^2(1 - p)} \frac{y_H - y_L}{2} = c'(x_t + (1 - \delta)n_{t-1}) \equiv c'(z^*). \quad (3)$$

Equation (3) states that ego sets the marginal cost of a match with an unknown alter equal to the current period payoff plus the expected future payoff to having a larger network.

We see from equation (3) that ego forms a constant total number of matches z^* in every period.⁹

Equation (4) then yields the number of random matches that ego forms in any period:

$$x_t = z^* - (1 - \delta)n_{t-1}. \quad (4)$$

⁸We can show that $V'(n_t)$ grows at rate $[\beta(1 - \delta)^2(1 - p)]^{-1} > 1$ unless it is constant. But by the transversality condition, $[\beta(1 - \delta)]^t V'(n_t)$ must be bounded, and since $\beta(1 - \delta) \times [\beta(1 - \delta)^2(1 - p)]^{-1} = [(1 - \delta)(1 - p)]^{-1} > 1$, this is impossible. Hence the only possibility is $V'(n) = \text{constant}$.

⁹The additional condition on the cost function that rules out case iii) above also ensures the existence of a z^* that solves equation (3).

We can substitute equation (4) into equation (1), yielding

$$n_t = pz^* + (1 - p)(1 - \delta)n_{t-1}.$$

We can then derive the complete time paths for network size and for the number of random matches ego forms in each period:

$$n_t = \sum_{\tau=0}^t (1-p)^\tau (1-\delta)^\tau pz^* \quad x_t = z^* - (1-\delta) \sum_{\tau=0}^{t-1} (1-p)^\tau (1-\delta)^\tau pz^*. \quad (5)$$

Note that the expression for n_t gives the value of network size at the end of the period. In particular, for $t = 0$ the expression yields $n_0 = pz^*$, but the value of network size at the beginning of period 0 is zero, which also implies $x_0 = z^*$.

As $t \rightarrow \infty$, network size and the number of matches of unknown quality ego forms approach their steady state values:

$$n_t \rightarrow \bar{n} = \frac{p}{[\delta + p(1 - \delta)]} z^* \quad x_t \rightarrow \bar{x} = \frac{\delta}{[\delta + p(1 - \delta)]} z^*. \quad (6)$$

We see from equations (5) and (6) that n_t increases monotonically from zero to its steady state value, which never exceeds z^* . It follows that case i) above ($z_t \leq (1 - \delta)n_{t-1}$) never obtains. Note that steady state network size increases with the probability of a good match and decreases with the rate of network decay. If the network does not decay ($\delta = 0$), then in the limit all matches are within network and random matches drop to zero. The firm becomes completely static, with a fixed set of agents and fixed relationships between them.

Inspection of equation (5) establishes our first proposition:

Proposition 1. *Ego becomes monotonically less open over time to meeting alters of unknown match quality.*

This occurs because ego's network size increases monotonically with time whereas his optimally chosen capacity for work relationships remains unchanged.

Clearly ego's network is valuable to him, in that the same number of work relationships without a network yields less benefit. We can compute the value of a network of size n_{t-1} explicitly by

comparing $V(n_{t-1})$ to $V(0)$.¹⁰ Since

$$V'(n_{t-1}) = \frac{(1-\delta)(1-p)}{1-\beta(1-\delta)^2(1-p)} \frac{y_H - y_L}{2}, \text{ then}$$

$$V(n_{t-1}) = \frac{(1-\delta)(1-p)}{1-\beta(1-\delta)^2(1-p)} \frac{y_H - y_L}{2} n_{t-1} + V(0).$$

It follows that

$$V(n_{t-1}) - V(0) = \frac{(1-p)(1-\delta)}{1-\beta(1-\delta)^2(1-p)} \frac{(y_H - y_L)}{2} n_{t-1}.$$

Inspection of this expression establishes:

Proposition 2. *The value of ego's network is increasing in (last period's) network size n_{t-1} , decreasing in the rate of network decay δ , decreasing in the probability of a good match p , decreasing in the rate at which future payoffs are discounted (increasing in β), and increasing in the difference between good and bad match values $y_H - y_L$.*

Since n_{t-1} is monotonically increasing with time, we have the following corollary:

Corollary 1. *The value of ego's network is monotonically increasing with his tenure at the firm.*

We conclude this subsection with our results on history dependence. In our model an ego with tenure t looking at his network retrospectively will see that he met the alters at various times t' . We denote the number of matches that were first formed in t' that are still in ego's network at the end of t by $n_t(t') = px_{t'}$ if $t = t'$ and $(1-\delta)^{t-t'} px_{t'}$ if $t > t'$.

Definition. History dependence $HD_t(t') \equiv \frac{n_t(t')}{n_t}$, *the probability that a member of ego's network in t resulted from a random meeting from a given previous period t' .*

Substituting for $x_{t'}$ in the expression for $n_t(t')$ using equation (4) yields $HD_t(t') = \frac{(1-\delta)^{t-t'} p[z^* - (1-\delta)n_{t'-1}]}{n_t}$.

Note that if $\delta = 1$, $HD_t(t') = 0$ for $t > t'$: if network decay is complete, there is no history dependence.

We see that the past time period t' has two counteracting influences on our measure of history dependence. On the one hand, since $n_{t'-1}$ increases with t' , $HD_t(t')$ tends to decrease with t' , reflecting the “front-loading” of agents' networks caused by persistence of relationships and time

¹⁰It is straightforward to show that $V(0) = \frac{z^* c'(z^*) - c(z^*)}{(1-\beta)}$.

constraints as discussed above. On the other hand, since $(1 - \delta)^{t-t'}$ increases with t' , $HD_t(t')$ tends to increase with t' , showing how a constant rate of decay of network relationships tends to establish a more conventional pattern of history dependence where more recent meetings are more influential. We also see that $HD_t(t')$ is decreasing in t , so the longer is an agent's tenure in an organization the less is the influence on his network of meetings from any particular time in the past.

The influences of t' and t on our measure of history dependence can be summarized in the following proposition:

Proposition 3. *Assume $p > \frac{\delta}{(1-\delta)}$. For $\delta > 0$, there exists a $\underline{t}' \geq 1$ such that $HD_t(t')$ is monotonically decreasing in t' for $t' < \underline{t}'$ and monotonically increasing in t' for $t' > \underline{t}'$. Moreover, there exists a $\underline{t} > \underline{t}'$ such that $HD_t(0) > HD_t(t)$ for $t < \underline{t}$ and $HD_t(0) < HD_t(t)$ for $t > \underline{t}$.*

The proof of Proposition 3, and all remaining proofs, are in Appendix B.

Proposition 3 shows that when an agent's tenure in an organization is sufficiently short ($t \leq \underline{t}$), representation of alters in his network is least influenced by his most recent meetings and most influenced by his very first meetings. With longer tenure ($t > \underline{t}$), the share of alters resulting from his most recent meetings increases relative to less recent meetings, creating a U-shaped pattern of history dependence where ego's network is dominated by alters he met at the beginning and most recent periods of his organizational career. Eventually ($t > \underline{t}$), the influence of the distant past diminishes sufficiently that the most recent meetings account for the largest share of alters of any period.

Let us consider two informative special cases. It is helpful if from this point forward we denote firm age by T .

Example 1: $\delta = 0$. We have $n_t(t') = pz^*[1 - \sum_{\tau=0}^{t'-1} (1-p)^\tau p] = pz^*[1 - 1 + (1-p)^{t'}] = pz^*(1-p)^{t'}$. That is, as we move from the past toward the present, $HD_t(t')$ decreases at rate $(1-p)$.

Example 2: $\delta > 0$, T large so that $n_T \approx \bar{n}$. In this case $\bar{x} = \frac{\delta \bar{n}}{p}$, $HD_T(T) \approx \frac{p\bar{x}}{\bar{n}} = \delta$, and $HD_T(t') \approx \delta(1-\delta)^{T-t'}$. That is, as we move t' from the present toward the past, $HD_T(t')$ decreases at rate $(1-\delta)$.

The second example shows that, for any positive rate of network decay, a conventional pattern of network history dependence will be established if a sufficient amount of time passes. For relatively short time horizons or small rates of network decay, the greatest representation in ego's network will be from alters he met early in his career at the firm.

Figure 1: History Dependence

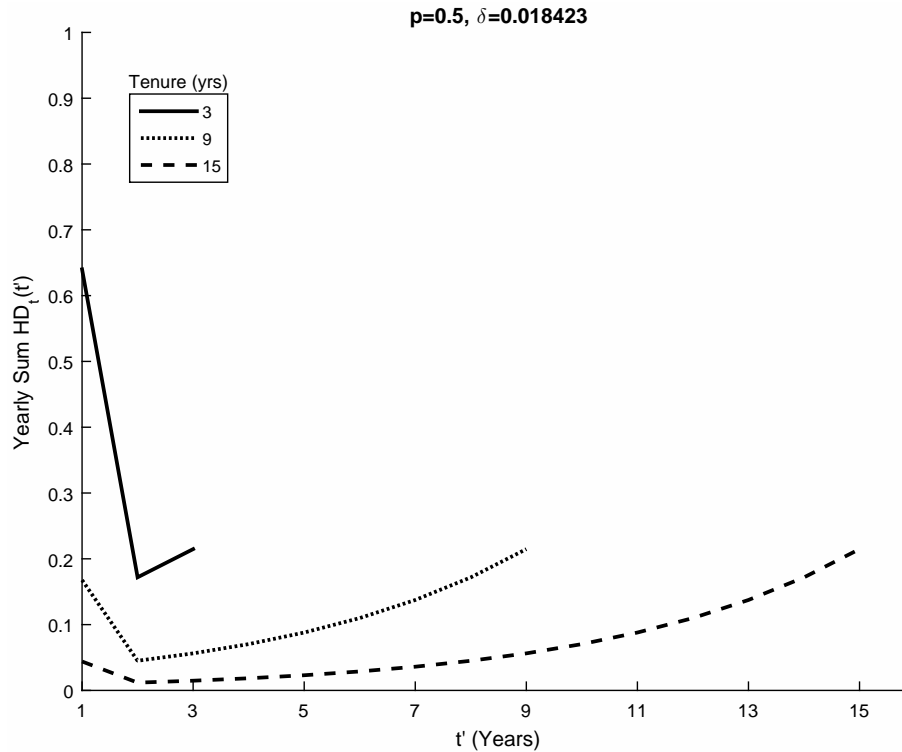


Figure 1 calculates the time pattern of history dependence for various lengths of ego tenure at the firm, where time is measured in years to build intuition. The calculation assumes an even bet that matches are of high quality ($p = 0.5$) and a 20 percent rate of network decay per year ($\delta \approx 0.018$, where the underlying periods are months as in our data). At three years of tenure, more than 60 percent of ego's network consists of alters he met in his first year at the firm. We also see that the U-shaped pattern of history dependence already appears, with greater representation of alters met during ego's third year at the firm than during his second. At nine years of tenure, representation of alters met during ego's most recent year at the firm finally surpasses representation of alters met during his first year. After fifteen years of tenure ego's pattern of history dependence is dominated by network decay.¹¹

¹¹Figure 1 shows that after a relatively short length of time ego's network is essentially in the steady state given by equation (6), so that the values of $HD_t(t')$ are virtually equal across the different tenures for $t' = t$.

2.3 Cross-sectional results

Proposition 3 is of limited empirical applicability. Absent retrospective interviews, we cannot see in what periods ego met randomly with which alters.¹² Alters are, however, horizontally differentiated by cohort of entry to the firm, which is much more easily observable. A natural analog to $HD_t(t')$ is the share of each cohort in ego's network at time t . This equals the probability, conditional on remaining in the firm at time t , that a given alter in each cohort belongs to ego's network, multiplied by cohort size and divided by ego's network size. Insofar as one can observe alters' exit from as well as entry to the firm, the conditional probability is of independent empirical interest.

Let us denote cohort by c . Tenure of an agent in cohort c is given by $t - c$. For the founding cohort, tenure is given by $t - 0 = t$. However, we can no longer afford the notational convenience of using the founding cohort to represent all cohorts. We will now denote network size and number of matches with agents whose match quality is unknown by n_t^c and x_t^c , respectively. To compute n_t^c and x_t^c , we can use equation (5) substituting $t - c$ for t . For example, $x_c^c = z^*$, and $x_c^{c-1} = z^* - (1 - \delta)pz^*$.

Let $P_t^c(c')$ be the probability that a given alter in cohort c' is in the network of a given ego in cohort c at the end of period t , conditional on his remaining with the firm. Let L_t^c be the number of matches with unknown alters formed in period t by agents whose match quality is unknown to a given ego in cohort c . Noting that $P_t^c(c')$ equals p times the probability that a given alter in cohort c' who remains with the firm is of known match quality to a given ego in cohort c at the end of period t , we have

$$P_t^c(c') = \begin{cases} 0 & \text{if } t < \max\{c, c'\} \\ P_{t-1}^c(c') + [1 - \frac{P_{t-1}^c(c')}{p}]px_t^c \frac{x_t^{c'}}{L_t^c} & \text{if } t \geq \max\{c, c'\} \end{cases} \quad (7)$$

$$L_t^c = \begin{cases} N(1 - \delta)^t x_t^0 + \sum_{c'=1}^t \delta N(1 - \delta)^{t-c'} x_t^{c'} & \text{if } t = c \\ N(1 - \delta)^t [1 - \frac{P_{t-1}^c(0)}{p}]x_t^0 + \sum_{c'=1}^t \delta N(1 - \delta)^{t-c'} [1 - \frac{P_{t-1}^c(c')}{p}]x_t^{c'} & \text{if } t > c \end{cases} \quad (8)$$

Equations (7) and (8) provide recursive solutions for $P_t^c(c')$ and L_t^c . We can use these solutions to compute the share $S_t^c(c')$ of any cohort c' in the network of an ego in cohort c at the end of period

¹²The time may come, however, when retrospective interviews are not necessary, because sociometric badges will be used to track employee locations and behavior (Waber, Aral, Olguin Olguin, Wu, Brynjolfsson, and Pentland 2011).

t :

$$S_t^c(c') = P_t^c(c')N^{c'}(1 - \delta)^{t-c'}/n_t^c \quad (9)$$

where $N^{c'} = N$ if $c' = 0$ and $N^{c'} = \delta N$ if $c' > 0$.

Despite the complex, recursive formulas for $P_t^c(c')$ and $S_t^c(c')$, we are able to derive some useful analytical results. First, we show that the conditional probability that an alter from ego's own cohort is a member of his network is greater than the conditional probability that an alter from any incumbent cohort is a member of ego's network:

Proposition 4. $P_t^c(c) > P_t^c(c')$ for all $c' \in [0, c - 1]$, for all $t \geq c$.

The intuition for Proposition 4 is provided by Proposition 1: alters become monotonically less open to meetings with unknowns as their tenure increases. Proposition 4 applies to incumbent cohorts. Does the intuition for Proposition 4 apply to later cohorts, since ego becomes monotonically less open to meetings with unknowns as his tenure increases? Yes, but this intuition is insufficient. It is possible that, because some alters are known to him, fewer total meetings with unknowns will be available to ego when he first meets with later cohorts than were available when he first met with his own cohort, raising $P_{c'}^c(c')$ relative to $P_c^c(c)$ for $c' > c$ and creating the possibility that $P_t^c(c') > P_t^c(c)$ for some c', t . The condition stated in Lemma 1 eliminates this possibility:

Lemma 1. If z^*/N is sufficiently small, $x_t^c/L_t^c > x_{t+b}^c/L_{t+b}^c$ for all $b \in [1, T - c]$, for all $t \geq c$.

The Lemma states that the ratio of desired to available meetings with unknowns by ego in cohort c is greater in period t than b periods later. It follows from diminishing openness to meetings with unknowns that desired meetings decrease from period t to $t + b$, and the condition that z^*/N is sufficiently small prevents any reduction in available meetings from overturning the result. Note that if we were to weaken Assumption 5 by decoupling firm hiring from network decay (layoffs), firm shrinkage could undermine Lemma 1. This in turn would make it possible for the next two propositions (but not Propositions 1-4) to fail.

With Lemma 1 in place, we can prove a parallel to Proposition 4 for cohorts that arrive after ego's cohort:

Proposition 5. If z^*/N is sufficiently small, $P_t^c(c) > P_t^c(c')$ for all $c' \in [c + 1, T]$, for all $t \geq c'$.

Together, Propositions 4 and 5 demonstrate *cohort attachment*: conditional on remaining with the firm, a member of ego’s own cohort is more likely to belong to his network than is a member of any other cohort.

Remark. Consider a change in perspective from the egocentric networks of the agents in the firm to the network of relationships in the firm as a whole, and measure “clustering” by the average probability that agents j and k have a relationship given that both have a relationship with i (Jackson 2008, p. 35). Since probabilities of relationships in our model are independent, as in a random graph, this clustering measure simply equals the average probability that two agents have a relationship. Considering a cohort or collection of cohorts as subnetworks of the firm network, Propositions 4 and 5 imply that clustering is greater for any one cohort than for any collection of cohorts.

Our results for cohort shares of ego’s network differ from our results for conditional probabilities of belonging to ego’s network because they incorporate network decay. Over time, network decay erodes the dominant position of ego’s own cohort in his network and opens up space for more recent cohorts.

Proposition 6. *Consider a firm of age $T \geq 2$. If z^*/N is sufficiently small and $\delta[1 + \delta p/2(1 - p)] < 1/2$, then $S_t^c(c')$ reaches its maximum over cohorts $c' \in [1, T]$ for cohort c for at least the two most recent cohorts, i.e., $c \in [T - 1, T]$.*

The condition on δ in Proposition 6 is needed because otherwise the higher conditional probability that an alter in ego’s network is in ego’s own cohort than in the next cohort is dominated by turnover of his own cohort.¹³ However, any positive rate of turnover must eventually cause the own cohort network share to fall below the share of the most recent cohort.

Proposition 7. *If z^*/N is sufficiently small and $T - c$ is sufficiently large, $S_T^c(T) > S_T^c(c')$ for all cohorts $c' \in [1, c]$.*

The founding cohort ($c = 0$) is excluded from Propositions 6 and 7 because, under the condition on δ given in Proposition 6, it is of larger initial size than all the other cohorts.

The two propositions combined predict that the networks of more recent cohorts are dominated by agents that joined the firm at or near the same time as themselves, the networks of the oldest

¹³It can be shown that the upper bound on δ in Proposition 6 lies between 0.472 and 0.5, depending on the value of p .

cohorts are dominated by the most recent cohorts, and the networks of agents with intermediate tenure are dominated by a combination of cohorts close to their own and the most recent cohorts.

Figure 2: Cohort Network Shares

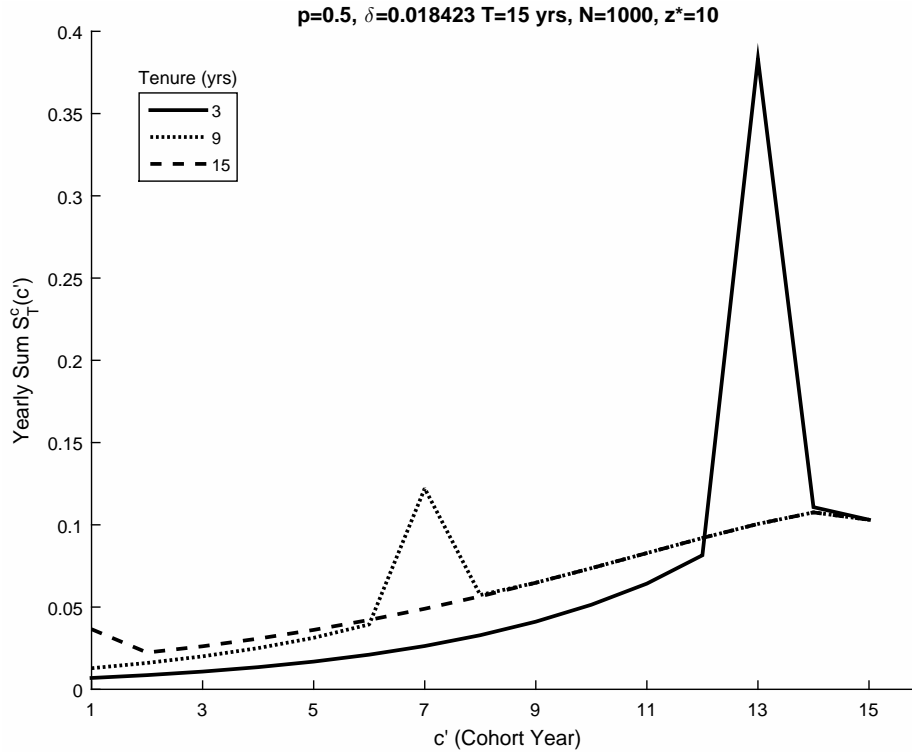


Figure 2 illustrates Propositions 6 and 7. It imitates Figure 1: the parameters p and δ are the same, the firm is 15 years old, the underlying periods are months that are aggregated into years to build intuition, and the plots are for egos with 3, 9, and 15 years tenure. More specifically, the egos we plot are assigned to the seventh month of their cohort years, so that the egos with 3, 9, and 15 years tenure entered the firm in the seventh month of years 13, 7, and 1, respectively. We see that the share of own cohort year in the network of the ego with 3 years tenure is four times that of any other cohort year, whereas the share of own cohort year in the network of the ego with 9 years tenure is only slightly larger than the shares of the most recent cohorts, and the most recent cohorts are clearly the largest in the network of the ego with 15 years tenure. Figure 2 also shows that by the end of their cohort years the egos' networks are essentially in the steady state given by equation (6), so that the network shares of all cohorts arriving later are virtually equal across the egos despite their different tenures. The reason for the downturn in cohort network share at T is that for the first few periods after an alter enters the firm his probability of having met ego

increases at a faster rate than δ .

3 Contacts and Job Referral

Job referrals are the canonical application of network models in economics (e.g., [Calvo-Armengol and Jackson 2004](#)). The small but growing empirical literature cited in our Introduction specifically analyzes job referrals to ego from alters he met in previous employment.¹⁴ This indicates the importance of exactly the kind of history-dependent network formation we emphasize in this paper. However, a job referral necessarily connects an ego outside a firm to alters inside the firm, whereas our model has focused entirely on network formation and operation within a firm (or, more broadly, within any one organization).

Let us extend our model to include many firms, finite in number. We assume that an agent can be employed by at most one firm in any period. An agent who does not work for any firm in a given period is unemployed in that period. Firms can form or dissolve. We assume an exogenous, constant probability of firm dissolution.

When alters in ego's network separate from him because they or ego leave the firm or because the firm dissolves, they become the *contacts* of ego. A contact is different from an alter in ego's network in his current firm because, in the current period, ego cannot form a match with him. Ego therefore neither derives value nor incurs costs from the contacts in his network in the current period. We assume that ego's contacts return to unknown match quality at a constant, exogenous rate, as ego and alters "drift apart" over time following separation from their common employer.

We only consider referrals of unemployed agents to firms.¹⁵ The pool of unemployed is filled by the exogenous separations of the previous section and by exogenous firm dissolution.¹⁶ The pool of unemployed is drained by firms replacing the workers who separated from them and by the founding of new firms. The number of hires, respectively δN and N , is exogenous.

Consistent with the referral literature, we assume an information structure such that a firm only knows of contacts that are brought to its attention by its current employees. In particular, the firm is

¹⁴There is a much broader job referral literature, which covers all types of connections between egos and alters rather than focusing on those formed through previous work at a common employer. For surveys see [Ioannides and Loury \(2004\)](#) and [Topa \(2011\)](#).

¹⁵Referrals of employed agents bring up interesting additional issues that we hope to address in future work.

¹⁶The small empirical literature cited above has only considered referrals of egos who are unemployed because their previous firms dissolved. By focusing on egos whose firms have closed, these papers avoid a potential selection bias from studying egos who have been laid off from thriving firms.

unaware of contacts that may exist between the unemployed workers themselves. If the firm knew of such contacts, it might want to hire a “ready-made” network of unemployed workers. Under our assumed information structure, it is clear that the interests of the firm’s employees and the firm are aligned. The employees want the firm to be aware of their contacts among the unemployed, and the firm wants to hire the unemployed workers with the greatest mass of contacts among its employees.

We denote the mass of contacts of unemployed worker i at firm j in period t by m_{ijt} . Firm j hiring in period t will rank the unemployed workers by m_{ijt} and hire until all vacancies are filled or until the firm exhausts all unemployed workers with $m_{ijt} > 0$, in which case we assume it chooses randomly among the remaining unemployed workers.¹⁷ The two cases have different implications for the relationship between m_{ijt} and the probability that unemployed worker i is hired by firm j in period t . In the case where all unemployed workers with $m_{ijt} > 0$ are hired, this probability takes a discrete jump when m_{ijt} increases from zero to positive, then remains constant. In the case where not all unemployed workers with $m_{ijt} > 0$ are hired, this probability strictly increases with m_{ijt} .¹⁸

Recall that ego’s contacts are alters who were formerly in his networks. When ego meets alters, then, he recognizes that in the future they may become contacts who help him out of unemployment through referral. We greatly simplify ego’s decision problem by assuming there is only one round of referral, so that workers hired through referral cannot refer other workers in turn. In this case, an alter’s contacts are of no consequence to ego: if an alter is hired because of his contacts, he cannot refer an unemployed ego by assumption. Alters in his firm therefore remain symmetric from ego’s point of view within the three sets delineated in the previous section: alters with whom he knows he is well matched, alters with whom his match quality is unknown, and alters with whom he knows he is poorly matched.

We would like the results of the previous section, especially cohort attachment, to continue to apply to the networks that egos form with workers who were of unknown match quality to them when they or the workers joined their firms. The key to retaining these results is retaining Proposition 1: ego becomes monotonically less open over time to meeting alters of unknown match

¹⁷In a richer model unemployed workers could be distinguished not only by m_{ijt} but also by some ϵ_{ijt} that is independent of m_{ijt} . ϵ_{ijt} could, for example, reflect idiosyncratic firm-specific training costs.

¹⁸Saygin, Weber, and Weynandt (2014) use as their proxy for referral an indicator for whether the number of former co-workers at the hiring firm is positive, which corresponds to our case where all unemployed workers with $m_{ijt} > 0$ are hired.

quality.

We can see immediately that ego's behavior in the previous section is not qualitatively changed by the prospect of dissolution of his employer. From ego's point of view separation from his current firm and dissolution of his current firm are equivalent, so he can add the probabilities together, leaving the results obtained in the previous section qualitatively unchanged.

Likewise, being hired through referral does not in itself qualitatively change ego's subsequent behavior, provided his mass of contacts at the hiring firm is small (in particular, smaller than his steady state network size). It only means that his initial network size is positive rather than zero.

The two changes made to our model in this section that threaten Proposition 1 are 1) ego's network at his current employer grows when his employer hires his contacts as well as when he matches with unknowns, and 2) ego has an additional incentive to match with unknowns, because they could turn out to be useful contacts in the future. 1) becomes a problem if the firm tends to hire ego's contacts early rather than late in his tenure, because this will cause his demand for matches with unknowns to increase rather than decrease over time. 2) becomes a problem if ego's marginal value of adding contacts grows with his stock of contacts, causing his desired number of matches z^* to increase over time.

In Appendix C, we re-solve ego's problem in the previous section incorporating the changes to our model in this section, under the assumptions that 1) the firm hires an exogenous mass of ego's contacts each period, and 2) ego's marginal value of adding contacts does not increase with his stock of contacts. We show that ego's desired total number of matches remains constant if his marginal value of adding contacts is constant, and decreases over time if his marginal value of adding contacts is decreasing. An additional condition limiting the decrease in firm hiring of ego's contacts ensures that his demand for matches with unknowns declines monotonically with time. Hence these assumptions and condition are sufficient to retain our qualitative results from the previous section for ego's network with workers who were of unknown match quality to him when he or the workers joined the firm.

If the cohort attachment result continues to apply to ego's network in any firm, it will extend to his contacts as well. That is, just as co-workers in ego's cohort are more likely to belong to his network, former co-workers who were in ego's cohort are more likely to be his contacts. This prediction does not apply to former co-workers who referred or were referred by ego.

4 Data: Brazilian Work Histories

Our data derive from the linked employer-employee records RAIS (*Relação Anual de Informações Sociais* of the Brazilian labor ministry *MTE*). By Brazilian law, every private or public-sector employer must report this information every year.¹⁹ This paper uses the data from 1994 to 2001. The data set extends back to 1986, but important variables are missing prior to 1994 so those years are not used in the analysis.

The use of an employer-employee data set provides the distinct advantage of being able to track workers through their job histories rather than relying on survey data to construct the set of former co-workers. This can be done because a job observation in RAIS is identified by the employee ID, the employer's tax ID (CNPJ), and dates of job accession and separation. The rules on tax ID assignments make it possible to identify new firms (the first eight digits of the tax ID) and new plants within firms (the last six digits of the tax ID), which is crucial since plants are the level at which relationships are most likely to be formed. RAIS also records comprehensive individual employee information on demographic characteristics, earnings, industry, location, and occupation.

RAIS does not include the large Brazilian informal sector. Our sample of workers is therefore selected for participation in the formal sector.²⁰ We shall see below that the key to identification of the impact of former co-workers on job referral is variation in their presence and number within a job seeker-hiring plant pair. We will miss the contribution to this variation of former co-workers from the informal sector.

We restrict our sample of workers to males, working more than 20 hours per week, in job spells lasting more than three months. Our aims are to focus on workers who are likely to have a strong attachment to the formal sector labor force and who will want to return to it after their firms close,²¹

¹⁹RAIS primarily provides information to a federal wage supplement program (*Abono Salarial*), by which every employee with formal employment during the calendar year receives the equivalent of a monthly minimum wage. RAIS records are then shared across government agencies. An employer's failure to report complete workforce information can, in principle, result in fines proportional to the workforce size, but fines are rarely issued. In practice, employees and employers have strong incentives to maintain complete RAIS records because payment of the annual public wage supplement is exclusively based on RAIS. The ministry of labor estimates that well above 90 percent of all formally employed individuals in Brazil are covered in RAIS throughout the 1990s.

²⁰For an extensive discussion of the choice between the formal and informal sector in Brazil see [Menezes-Filho, Muendler, and Ramey \(2008\)](#), [Bosch and Maloney \(2010\)](#), [Bosch and Esteban-Pretel \(2012\)](#).

²¹[Bosch and Maloney \(2010\)](#) use another Brazilian dataset that measures informal employment, Pesquisa Mensual do Emprego (PME), and suggest that transition probabilities between the formal sector, informal sector and unemployment are considerably different between men and women.

and to give them enough time to form working relationships.

We use data from only five of Brazil’s 26 states: Acre, Ceará, Espírito Santo, Mato Grosso do Sul, and Santa Catarina. This is justified primarily by concerns regarding the computational time and resources necessary to track worker job histories.²² The five states were chosen because they represent different geographic (see Figure E.1) and demographic circumstances in Brazil. Estimates are pooled across states with each state considered in isolation, so obtaining a job outside of the state is not considered. In this respect out-of-state jobs are like informal sector jobs. Figure E.2 shows that there is substantial migration in Brazil in 2000, but at relatively low levels in the chosen states. The states used had total populations of 0.7, 8.4, 3.5, 2.4, and 6.2 million, respectively, in 2010, with corresponding densities of 4, 57, 76, 7, and 65 per km² (Instituto Brasileiro de Geografia e Estatística, 2010 Census of Brazil). Similar projects used employer-employee data sets from Austria and Sweden (Saygin, Weber, and Weynandt 2014, Eliason, Hensvik, Kramarz, and Skans 2017, Hensvik and Skans 2016), which have populations (densities) of 8.4 (102) and 9.4 (23) million (per km²) in 2010, respectively (World Bank WDI 2014). To the best of our knowledge, ours is the first paper to conduct this type of analysis outside of Europe.

4.1 Displaced Workers

Within the universe of workers, those of interest are individuals who enter a new job following unemployment resulting from firm closure. Firm closure occurs in year t if the firm last appears in the data in year t . To avoid including small firms that are slowly failing, we also require that at least five employees work at the closing firm in its last year. We do not include individual plant closures because they can represent consolidation by the employer, creating correlated hiring that resembles referrals. Closures create plausibly exogenous unemployment and reduce concerns regarding selection into job transition. Closures have the added benefit of providing a natural set of comparison workers for each ego.

Following Schwerdt (2011), we include all workers who were at the closing firm in the last year it appeared in the data. Not all of these workers pass through a spell of unemployment before obtaining their new jobs. The sample includes closing firms from 1998-1999 to allow for a minimum of four years (1994-1998) of work history and two years (2001-1999) to obtain another

²²In terms of complexity, matching workers to former co-workers is an $O(n^2)$ task. In subsection 6.4 we track the former co-workers of former co-workers, an $O(n^3)$ task, in order to construct a peers-of-peers instrument.

job. For consistency across workers, we only consider four years of work history prior to the month they left the closing firm and the first job acquired within two years of leaving the closing firm. This is similar to the selection procedure in [Saygin, Weber, and Weynandt \(2014\)](#), but they use five years prior and one year after because of a longer panel and greater re-employment rate (possibly because of the lack of a large Austrian informal sector).

Each worker leaving a closing firm in the closure year is referred to as an *ego*. If an ego is at multiple closing firms then we only use his observation at the last closing firm observed in the data. Additionally, if the ego leaves the closing firm because of death or retirement he is excluded from the sample.

4.2 Historic Co-workers

Having identified the egos of interest, the next step is to identify their historic co-workers. We trace each ego back to all plants in his employment history prior to his employment at the closing firm.²³ Historic co-workers are those who overlapped with ego at the same plant for more than three months. For brevity, we refer to historic co-workers as *alters*. According to our theory, the set of alters for a given ego are that ego's *possible* contacts: the alters constitute the universe of workers who *could have been* in ego's networks at his past employers. As is the case in most of the literature (e.g., [Cingano and Rosolia \(2012\)](#), [Saygin, Weber, and Weynandt \(2014\)](#), [Kramarz and Skans \(2014\)](#) and [Eliason, Hensvik, Kramarz, and Skans \(2017\)](#)), we do not have information as to which (if any) alters were the *actual* sources of referrals for ego. We drop from the sample any egos with zero alters.²⁴

Within the set of alters, we define the subset of *cohort alters* as those alters who started $+/- 2$ months from ego at the historic plant at which they first worked together.²⁵ As noted in Section 3, later jobs at which ego and alter overlap again could reflect referrals between them, to which our model of relationship-building between unknowns does not apply. The two-month window is suggested by our (admittedly arbitrary) rule that at least three months are required to form a work

²³This is true for all but the first column in Table 4, where to be consistent with the previous literature closure co-workers are included if they are not also egos.

²⁴Our preferred specifications below will include ego fixed effects, which would absorb these egos if we kept them in the sample.

²⁵If ego had multiple jobs at the historic plant, we use the one with the earliest start date. If an alter had multiple jobs that overlapped with ego at the historic plant, we use the highest-paying one, or the one with longest tenure if there is a tie, or the one with the earliest start date if there is yet another tie.

relationship. This window ensures that when the co-worker arrives, ego will not have finished forming his first round of work relationships, or that when ego arrives, the co-worker will not have finished forming his first round of work relationships. We experimented with a six-month window and obtained qualitatively similar but quantitatively weaker results, as would be expected.

We call the plants where an alter (cohort alter) is employed at the time ego leaves the closing firm the *alter plants (cohort-alter plants)* of a given ego. These are the plants to which ego might receive a referral. We require that the alter be at the plant at least three months before ego leaves the closing firm in order to ensure that contemporaneous movement effects do not exist.

We are interested in estimating the impact that the presence of alters or cohort alters has on the probability that ego is hired by the plant that employs them. We will want to control for other reasons an ego from a specific closure might be hired at a specific plant. The most effective way to do this is to include closure-hiring plant fixed effects in our estimation procedure, in which case the impacts of alters and cohort alters can only be identified from variation across workers from the same closure in the presence of alters and cohort alters at the hiring plant. If there is no such variation, the closure-hiring plant pair does not contribute to identification of the effects of interest. This reasoning led [Saygin, Weber, and Weynandt \(2014\)](#) to define, for each closure, *potential plants* as plants where at least one ego has an alter. Egos from the same closure have different sets of alter plants, but the same set of potential plants. For a specific ego, all alter plants have at least one alter, but a potential plant can have zero alters if it is the alter plant of another ego from the same closure. We adopt this ego-potential formulation and specify our dependent variable as equal to one if ego obtains a job at one of his potential plants and zero otherwise.

4.3 Summary Statistics

Our sample selection procedure yields 38,603 egos at 1,672 closures with 51,315 unique potential plants.²⁶ A closing firm has a mean (median) of 23 (9) egos and 199 (83.5) potential plants (see Table 1). The mean (median) ego has 253 (16) alters, has worked at 1.0 (1) historic plants, and has 24 (3) alter-plants and 5 (1) cohort-alter plants. 25.7 percent of egos found a job at a potential plant, 7.6 percent at an alter plant, and 4.1 percent at a cohort-alter plant. If an ego did not find a job at a potential plant, he either found a job at a non-potential plant or in the informal sector, or

²⁶For an extensive comparison of the selection differences between this paper and [Saygin, Weber, and Weynandt \(2014\)](#) see Table D.1.

Table 1: Ego and Closing Firm Statistics

	Mean	SD	Median
<u>Egos (N=38,603)</u>			
Alters	253	968	16
Cohort-alters	36	119	2
Potential Plants	765	833	443
Alter Plants	24	52	3
Cohort-alter Plants	5	13	1
Start at Potential Plant	.257		
... at Alter Plant	.076		
... at Cohort-alter Plant	.041		
Age (at Closure)	33	10	32
Average Monthly Wage (at Closure)	382.16	528.57	232.20
Tenure (at Closure)	32	41	18
Historic Plants	1	.05	1
Avg. Historic Plant Size	454	1625	96
Avg. Tenure at Historic Plants (Months)	34	40	20
Unemployment Spell (Months)	7.6	6.3	6
Return to a Historic Plant	.021		
<u>Closing Firms (N=1,672)</u>			
Potential Plants	199	321	83.5
Egos	23	52	9

¹ In the year ego left the closing firm.

² In the month ego left the closing firm.

remained unemployed.

As seen in column (1) of Table 2, at the time of leaving the closure the typical ego is in the lowest education group and has an occupation in the “Manufacturing and Transport” category which includes “workers in industrial production, machine and vehicle operators, and similar workers” (Muendler, Poole, Ramey, and Wajnberg 2004). Column (2) shows that workers in the potential hiring plants, though similar to the egos, are more educated and less concentrated in the Manufacturing and Transport occupation group.

Table 2: Ego and Potential Characteristics

	Egos (1)	Potentials (2)
<u>Age Breakdown</u>		
18 – 24	.202	.237
25 – 29	.207	.203
30 – 39	.331	.304
40 – 49	.171	.160
50 – 64	.076	.071
≥ 65	.005	.005
<u>Education Breakdown</u>		
Middle School or less	.755	.687
Some High School	.192	.255
Some College	.015	.020
College Degree	.037	.037
<u>Occupation Breakdown</u>		
Scientists and Technicians	.038	.051
Executive and Government	.023	.024
Administrative and Clerical	.125	.144
Commerce	.066	.130
Personal Services	.115	.151
Agriculture	.040	.053
Manufacturing and Transport	.592	.446
Obs	38,603	72,332

The potential characteristics are the average fraction of employees in a category.

The number of potential observations is larger than the 51,315 unique potentials because some potentials are in the sample twice as a destination for a closure in each of the possible closure years (1998-1999)

The ego-potential pair is the primary unit of observation. Each of the 38,603 egos is paired with all potential plants from his closure. Since the mean number of these potential plants is 765 (see Table 1), we obtain a large sample size of 29.5 million. The sample size for a given closure scales quickly in the number of egos because one additional ego adds observations for the new

ego with that ego's alter plants, all other egos' alter plants, and for other egos with the new ego's alter plants. Table 3 summarizes the main variables of interest in the regressions of the next two sections of this paper, for the whole sample of ego-potential pairs (Column 1) and subsets with an alter (Column 2), non-cohort alter (Column 3), and cohort alter (Column 4). The dependent variable, job acquisition, equals one if a given ego obtains a job at a given potential plant and zero otherwise. We see from column (1) that the mean probability of this event for the whole sample is 0.03 percent. Only 3.1 percent of ego-potential pairs have an alter and only 0.7 percent have a cohort alter. Different egos from a closure have divergent job histories and thus different sets of co-workers, and since potential plants are the union of alter plants the different sets of co-workers result in a low level of possible contacts. Closures have an average of 23 egos. If each had a unique alter plant then there would be an alter at $\frac{1}{23}$ (4.4 percent) of the 23^2 (529) observations, which is comparable to the 3.1 percent observed in the data.

In our regressions we will control for several measures of ego's compatibility with the potential plant, as discussed more in Section 5. For the whole sample the mean percentages of a potential plant's employees that are in the same age group, education group, and occupation group as the ego are respectively 23.0, 52.5, and 33.0.²⁷ 41.5 percent of potential plants are in the same municipality as the ego at the time of firm closure,²⁸ and 0.07 percent are also historic plants of the ego. Column (2) of Table 3 shows that all of these compatibility measures increase when conditioning on the presence of at least one alter, and increase still further in column (4) when conditioning on the presence of at least one cohort alter. At the same time, the chance that ego obtains a job at a specific potential plant increases to 0.3 percent for potentials with at least one alter and 0.8 percent for potentials with at least one cohort alter. This shows that possible contacts, compatibility and job acquisition are positively correlated and a regression framework is needed to sort out the contact effect on job acquisition from the compatibility effect.

We also see from Table 3 that the number of alters increases from column (2) to column (4). Hence controlling for the number of non-cohort alters, not only the presence of at least one, will be important when estimating the impact of cohort alters on job acquisition. The variables listed below Potential is Historic Plant will be defined when used in subsection 6.3.

²⁷The groups are defined in Table 2. Occupational classifications in RAIS follow the Classificação Brasileira de Ocupações (CBO). This paper uses the 1994 CBO, which has more than 350 categories.

²⁸The municipality is the smallest administrative unit in Brazil. In 2000 Brazil had 5,507 municipalities, and the five states used in this analysis had 22 (Acre), 184 (Ceará), 77 (Espírito Santo), 77 (Mato Grosso do Sul), and 293 (Santa Catarina) (IBGE 2000).

Table 3: Ego-Potential Statistics

		Alts \geq 1	Non-Coh. Alts \geq 1	Coh. Alts \geq 1
	(1)	(2)	(3)	(4)
Job Acquisition	.0003	.003	.003	.008
Alters \geq 1	.031	1	1	1
Alters		5.207	5.917	17.455
Non-Coh. Alters \geq 1	.026	.849	1	.342
Non-Coh. Alters			5.415	
Coh. Alters \geq 1	.007	.229	.092	1
Coh. Alters				2.656
% Same Age Group	.230	.244	.244	.248
% Same Education Group	.525	.569	.557	.632
% Same Occupation Group	.330	.398	.392	.453
Same Municipality (Indic)	.415	.519	.523	.521
Potential is Historic Plant (Indic)	.0007	.019	.022	.060
Num Alt. First Worked in Same Age Grp \geq 1	.009	.301	.308	.403
Num Alt. First Worked in Same Age Grp	4.116	4.134	4.582	10.577
Num Alt. First Worked in Same Edu. Grp \geq 1	.020	.636	.625	.747
Num Alt. First Worked in Same Edu. Grp	4.209	4.226	4.829	11.759
Num Alt. First Worked in Same Occ. Grp \geq 1	.018	.561	.554	.656
Num Alt. First Worked in Same Occ. Grp	4.594	4.639	5.290	13.290
Num Alt. with Multiple Overlaps \geq 1	.003	.106	.114	.100
Num Alt. with Multiple Overlaps	.023	.742	.856	2.673
Avg Total Overlap (Months)		23.767		
Avg Separation (Months)		32.923		
Avg Tenure (Months)		29.338		
Avg Non-Coh. Tot. Overlap			25.195	
Avg Non-Coh. Sep.			33.145	
Avg Non-Coh. Tenure			30.515	
Avg Coh. Tot. Overlap				19.115
Avg Coh. Sep.				31.590
Avg Coh. Tenure				27.052
Obs	29,519,778	909,063	771,835	208,487

The mean of the sample of ego-potential characteristics for: the whole sample of ego-potential pairs (Col. 1), the subsets with an alter (Col. 2) and non-cohort-alter (Col. 3), cohort-alter (Col. 4).

5 Impacts of Historic Co-workers on Job Referral

We want to compare the impact on job acquisition of cohort alters to the impact of non-cohort alters. In this section, however, we omit the distinction between cohort and non-cohort alters in order to stay as close as possible to the previous literature and thereby establish a benchmark. We also introduce changes in specification relative to the previous literature that we view as improvements, but that do not qualitatively affect the differences between the impacts of cohort and non-cohort alters estimated in the next section.

In the following regression framework, the dependent variable equals one for the ego-potential pair for which ego acquires a job and zero otherwise. If an ego does not acquire a job at any of his potential plants, the dependent variable equals zero for all of his ego-potential pairs. We use a linear probability model for ease of interpretation.²⁹ Our procedure is similar to that of [Saygin, Weber, and Weynandt \(2014\)](#), so we first conduct a similar regression to establish a baseline:³⁰

$$J_{ikf} = \alpha + \beta_{alt} \mathbb{1}\{G_{if} \geq 1\} + \theta_{kf} + \epsilon_{ikf}, \quad (10)$$

where

J_{ikf} is an indicator for whether ego i acquires a job at potential plant f following closure k

G_{if} is the number of i 's alters at plant f at the time that ego i leaves closure k

$\mathbb{1}\{\cdot\}$ is an indicator function

θ_{kf} is a vector of fixed effects for each closing firm-potential plant pair

ϵ_{ikf} is an error term.

The coefficient of interest is β_{alt} , the increase in probability of job acquisition at a specific potential plant given that there exists at least one alter. In column (1) of Table 4 we include alters (historic co-workers) from the closing firm if they are not also egos in order to increase comparability to the previous literature, specifically [Cingano and Rosolia \(2012\)](#) and [Saygin, Weber, and Weynandt \(2014\)](#). The point estimate of 0.0010 is more than three times the mean probability of 0.0003.

Column (2) of Table 4 restricts alters to those who were met prior to ego's tenure at the closing firm, a restriction we will maintain for the remainder of this paper. This restriction better captures our intention that the source of identifying variation be differences in job histories between egos from the same closure. Moreover, the purpose of using closing firms is to avoid selection across workers, and alters who left closing firms for other jobs prior to the closing year may be positively selected relative to the workers who remained until the closing year. Using our more restrictive definition of alters results in a smaller sample because we lose some potential plants that only employ closure alters. We see that the point estimate of the coefficient of interest is slightly larger.

²⁹For a discussion of the use of the linear probability model for binary outcomes see [Wooldridge \(2001\)](#) Section 15.2.

³⁰[Saygin, Weber, and Weynandt \(2014\)](#) use a fixed effect transformation from [Kramarz and Skans \(2014\)](#) for estimation that reduces the data to closing firm-potential firm observations.

Table 4: Ego-Potential Job Acquisition

	(1)	Non-Clos. (2)	Ego FE (3)	Compatibility (4)	Size (5)
Non-Ego Alters ≥ 1	.0010 (.0001)***				
Alters ≥ 1		.0012 (.0001)***	.0013 (.0001)***	.0009 (.0001)***	.0004 (.00009)***
(Log) Alters					.0022 (.0003)***
% Same Age Group				.0001 (.00002)***	.0001 (.00002)***
% Same Education Group				.00008 (.00002)***	.00008 (.00002)***
% Same Occupation Group				.0003 (.00004)***	.0003 (.00004)***
Same Municipality (Indic)				.0012 (.0005)***	.0012 (.0005)***
Potential is Historic Plant (Indic)				.0192 (.0019)***	.0142 (.0019)***
Obs.	29,519,890	29,519,778	29,519,778	29,519,778	29,519,778
R^2	.2696	.2403	.2432	.2438	.244
Closure \times Potential (FE)	336,597	332,590	332,565	332,565	332,565
Egos (FE)	-	-	38,603	38,603	38,603
Closures (cluster)	1,724	1,697	1,672	1,672	1,672

Note: Standard errors clustered at the closing firm level. All columns contain closing firm \times potential plant fixed effects. Columns (3)-(5) also contain ego fixed effects. The dependent variable is an indicator for an ego's job acquisition at the potential plant.

The next departure from the previous literature is to introduce ego fixed effects:

$$J_{ikf} = \alpha + \beta_{alt} \mathbb{1}\{G_{if} \geq 1\} + \phi_i + \theta_{kf} + \epsilon_{ikf} \quad (11)$$

where ϕ_i is a vector of fixed effects for each ego.³¹ Inclusion of ego fixed effects is important since egos differ in their probabilities of being hired at any given potential plant regardless of contacts, for example because they differ in their numbers of potential plants or because they differ in their probabilities of finding a formal sector job. The point estimate of the coefficient of interest in column (3) of Table 4 is little changed from column (2).

The two sets of fixed effects account for similarities between the closure and destination plant and for the ego's idiosyncratic characteristics, but not for the compatibility between ego i and the

³¹The Stata command `reghdfe` is used throughout this paper because of its ability to accurately estimate a model with two high dimensional fixed effects (Correia 2015).

potential hiring plant f , which varies by ego within a closure-potential pair. Without relying on a contact, it is plausible that f targets individuals like i , or that i is more likely to look to plant f , if f has more employees like i . We therefore control for observable compatibility between ego and the potential plant:³²

$$J_{ikf} = \alpha + \beta_{alt} \mathbb{1}\{G_{if} \geq 1\} + \delta H_{if} + \phi_i + \theta_{kf} + \epsilon_{ikf} \quad (12)$$

where H_{if} are measures of compatibility. The measures are the percentages of potential plant workers in, respectively, the same age group, education group, and occupation group as ego;³³ an indicator for whether ego worked at the closure in the same municipality as the potential;³⁴ and an indicator for whether ego has ever worked at the potential plant in the sample period.

We see in column (4) of Table 4 that, as expected, the inclusion of the compatibility controls decreases the estimate of the contact effect because it accounts for mobility that is not truly associated with contacts. This decrease is largely driven by the inclusion of the indicator for whether the potential plant is also a historic plant, that is, for employee “recall.” Previous work controlled for specific characteristics of the ego and potential plant independently, but did not address these baseline compatibilities (Saygin, Weber, and Weynandt 2014, Kramarz and Skans 2014, Hensvik and Skans 2016, Eliason, Hensvik, Kramarz, and Skans 2017).

Our last change in this section to the job referral specification prevailing in the literature is suggested by the analysis in section 3. There we noted that, depending on whether the mass of unemployed workers with positive masses of contacts at a hiring firm is smaller or larger than its mass of openings, the firm will hire all the unemployed workers with positive masses of contacts or only those with the largest masses of contacts. The implication for empirical specification of the job referral equation is that the probability that an unemployed ego is hired at a potential plant can be influenced by the number of alters he has there in addition to whether that number is positive. We therefore add to our job referral equation the interaction of the indicator for whether the number of alters is positive with the log of the number of alters:³⁵

³²Targeting on unobservable characteristics is addressed in Sections 6.1-6.4.

³³The groups are defined in Table 2.

³⁴Each closing firm can have multiple municipalities and so this is not collinear with the fixed effects.

³⁵Because ego’s alters at the potential plant are only *possible* contacts, the impact of this additional variable may reflect an increase in the probability that ego has one actual contact as well as the impact of more contacts.

$$J_{ikf} = \alpha + \beta_{alt} \mathbb{1}\{G_{if} \geq 1\} + \beta_{log} \log(G_{if}) \times \mathbb{1}\{G_{if} \geq 1\} + \delta H_{if} + \phi_i + \theta_{kf} + \epsilon_{ikf} \quad (13)$$

The addition of the interaction term changes the interpretation of β_{alt} from the impact of having any alters to the impact of having one alter, while the interaction term captures the impact of increases in the number of alters above one.³⁶ Comparing column (5) to column (4), we see that the impact of having one alter is less than half the impact of having any alters. Conditional on having at least one alter, every doubling of the number of alters increases the probability that the unemployed ego will be hired at the potential plant by $0.69 \times 0.0022 = 0.0015$.

6 Distinguishing Between the Impacts of Cohort and Non-Cohort Alters

Our theory predicts that a possible contact is more likely to be an actual contact if he was from the same hiring cohort at the ego's historic plant. Ego's cohort alters should therefore be more likely to generate referrals for him than his non-cohort alters. Using the specification of Column (5) in Table 4 as a baseline, we decompose alters by their cohort status:

$$\begin{aligned} J_{ikf} = & \alpha + \beta_{coh} \mathbb{1}\{C_{if} \geq 1\} + \beta_{logcoh} \log(C_{if}) \times \mathbb{1}\{C_{if} \geq 1\} \\ & + \beta_{noncoh} \mathbb{1}\{G_{if} - C_{if} \geq 1\} + \beta_{lognoncoh} \log(G_{if} - C_{if}) \times \mathbb{1}\{G_{if} - C_{if} \geq 1\} \\ & + \delta H_{if} + \phi_i + \theta_{kf} + \epsilon_{ikf} \end{aligned} \quad (14)$$

where C_{if} is the number of i 's cohort-alterers at plant f at the time that ego i leaves closure k .

Column (1) of Table 5 shows that the impact on the probability that ego is hired at a potential plant of having one cohort alter is nearly three times the impact of having one non-cohort alter. However, the point estimate of the effect of the (log) number of cohort alters conditional on there being at least one is only slightly greater than the corresponding point estimate for non-cohort alters. (Of ego-potential observations with a cohort (non-cohort) alter, 18.1 (22.7) percent have

³⁶Of ego-potential observations with an alter, 22.8 percent have more than one.

more than one.) The former result is strongly supportive of our theory, the latter result is at best weakly supportive. We shall see below that the former result is robust across different samples and estimation strategies whereas the latter result is not.

Table 5: Cohort vs. Non-Cohort

	Base	Pot. Size \leq 24	Pot. Size $>$ 24	Split
	(1)	(2)	(3)	(4)
Coh. Alters \geq 1	.0011 (.0002)***	.0007 (.0002)***	.0012 (.0002)***	.0012 (.0002)***
(Log) Coh. Alters	.0017 (.0008)**	.0106 (.0034)***	.0014 (.0009)	.0016 (.0009)*
Non-Coh. Alters \geq 1	.0004 (.0001)***	.0003 (.00008)***	.0004 (.0001)***	
(Log) Non-Coh. Alters	.0015 (.0003)***	.0020 (.0007)***	.0015 (.0004)***	
Incumb-Coh. Alters \geq 1				.0007 (.0001)***
(Log) Incumb-Coh. Alters				.0006 (.0004)*
Later-Coh. Alters \geq 1				.0005 (.0001)***
(Log) Later-Coh. Alters				.0014 (.0005)***
Obs.	29,519,778	14,780,878	14,738,383	29,519,778
R^2	.244	.2529	.2458	.244
Closure \times Potential (FE)	332,569	143,043	189,454	332,569
Egos (FE)	38,603	38,350	38,069	38,603
Closures (cluster)	1,672	1,639	1,592	1,672

Note: Standard errors clustered at the closing firm level. All columns contain closing firm \times potential plant and ego fixed effects. Controls for the compatibility between the ego and potential (% Same Age Group – Potential is Hist. Pl. (Indic) from Table 4) are included, but not shown. The dependent variable is an indicator for an ego's job acquisition at the potential plant.

According to our theory, the result that cohort alters have a larger impact than non-cohort alters on the probability that ego obtains a job at a potential plant stems entirely from the greater probability that ego matched with cohort alters at their historic plant and not from any difference in probability that the quality of a match was good. A cohort alter is both more likely than a non-cohort alter to know that ego is well matched with him and that ego is poorly matched with him, but only the former affects referrals. That is, alters whose match qualities with ego are poor, like alters with whom he never matched, refer him with zero probability. The key is that a worker (and his employer) is indifferent between adding to his plant workers of unknown match quality and

workers with whom he knows he is poorly matched, given that enough unknowns are available with whom he can match at the margin. If the plant hires workers known to be poorly matched with alter, they match with other incumbent workers.

This highlights the importance of the assumption of our model that parameters are such that there are always enough unknowns available for matching. It seems plausible that, if this assumption were to be violated, it would more likely happen for small than large hiring plants.³⁷ With this in mind, in Columns (2) and (3) of Table 5 we split the sample of ego-potential observations into those for which the potential plant size is less than or equal to the median of the sample (24 employees) and those for which the potential plant size is greater than the median. For the larger plants, the impact of one cohort alter is substantially greater than for the smaller plants, and the impact of one non-cohort alter is slightly greater. Nevertheless, for the smaller plants the impacts on the probability that ego is hired remain positive and very precisely estimated for both types of alters. Moreover, for both types of alters the impacts of the (log) number of alters conditional on there being at least one are greater for the smaller than for the larger plants. We conclude that the evidence for treating smaller plants differently from larger plants is not strong enough to override the virtues that follow from treating all plants equally: simplicity and avoidance of ad hoc departures from our model.

We need to address another ambiguity that arises in applying our model to our data. We noted in Section 2 that the intuition provided by our model for why a cohort alter is more likely to be in ego's network than a non-cohort alter is the same whether the non-cohort alters arrived at the firm before or after ego. However, *proof* that a cohort alter is more likely to be in ego's network than a non-cohort alter requires Assumption 5 for a later cohort alter. Assumption 5 specifies a constant rate of turnover for a firm, hence equal size of every hiring cohort except the founding one. Needless to say, this assumption does not hold in our data. We therefore need to allow for the possibility that cohort attachment holds relative to incumbent but not later alters, i.e., not impose equality of coefficients for incumbent and later alters. Moreover, even with Assumption 5 our model makes no prediction regarding the probability that a given incumbent alter will be in ego's

³⁷If the assumption were violated, then if a mass of workers with whom alter is poorly matched were hired, he will in general match with some fraction of that mass (or reduce his mass of matches accordingly). In contrast, if a mass of workers with whom alter is well matched is hired, he still matches with all of them. Hiring workers poorly matched to current workers thus hurts them (and the employer) less than hiring workers well matched to current workers helps them (and the employer). This suggests that an increased probability of having matched with an alter will generate a higher probability of referral from that alter even if the assumption of enough unknowns fails for the hiring plant.

network relative to a given later alter.

In Column (4) of Table 5, we divide non-cohort alters into incumbent alters and later alters. We see that the impact of one cohort alter on the probability that ego is hired is much more different from the impact of one incumbent or one later alter than latter are from each other. The coefficient for one incumbent alter is larger than the coefficient for one later alter, but a test of their equality shows that the difference is not statistically significant (two-sided t -test p -value = 0.1450). In contrast, tests of equality between the coefficient for one cohort alter and the coefficient for one incumbent alter or one later alter show that the differences are highly significant (p -values 0.0179 and 0.0003, respectively). The coefficient on the (log) number of alters conditional on there being at least one is much smaller for incumbent alters than later alters, but a test of their equality shows that the difference is not statistically significant—in fact, one cannot reject equality between any pair of coefficients on the log alter variables (later = incumbent p = 0.2596, later = cohort p = 0.8785, incumbent = cohort p = 0.3811). In light of these results we will continue to aggregate all non-cohort alters, thereby maintaining simplicity for robustness tests.³⁸

In the remainder of this section we employ diverse strategies to try to rule out alternative explanations for why the presence of cohort alters raises the probability that a worker who loses his job due to firm closure is hired at a given potential plant. We examine (i) placebo histories (subsection 6.1), (ii) placebo destinations (subsection 6.2) (iii) alter characteristics (subsection 6.3), and (iv) instruments for alter presence (subsection 6.4).

6.1 Placebo Histories

The fact that a potential plant has hired one or more workers from a historic firm of ego may indicate that the potential plant is searching for skills the historic firm also sought or that were developed there. In this case the association of the presence of alters from the historic firm with the hiring of ego does not indicate that the alters referred ego. Similarly, the historic firm may have sought or developed specific skills in workers hired at specific times, and if the potential plant is seeking these specific skills it could account for the stronger association of the presence of cohort alters with the hiring of ego. To address these concerns, in this subsection we use placebo histories for the egos. A placebo history assigns an ego the same employment spells at his historic firms,

³⁸There are about the same number of ego-potential plant observations with at least one incumbent alter of ego (421,429) as with at least one later cohort alter of ego (424,720).

but at other plants.³⁹ The placebo alters are workers who were at the historic firms at the same time as the ego, but in different plants, and the placebo cohort alters in particular were in the same hiring cohort but at different plants. “True” alters are excluded from the set of placebo alters. The set of potential plants is constructed in the same way as the set of potential plants in the baseline specification, but using placebo alters in place of true alters.

Table 6: Placebo History

	True Hist. (1)	Plac. Hist. (2)
Coh. Alters ≥ 1	.0012 (.0002)***	-.00006 (.0001)
(Log) Coh. Alters	.0026 (.0009)***	.00005 (.0004)
Non-Coh. Alters ≥ 1	.0003 (.0001)**	.00002 (.00006)
(Log) Non-Coh. Alters	.0015 (.0004)***	.00006 (.0002)
Obs.	8,911,746	7,938,624
R^2	.2624	.2507

Note: Standard errors clustered at the closing firm level. All columns contain closing firm \times potential plant and ego fixed effects. Controls for the compatibility between the ego and potential (% Same Age Group – Potential is Hist. Pl. (Indic) from Table 4) are included, but not shown. The dependent variable is an indicator for an ego’s job acquisition at the potential plant. The dependent variable is an indicator for an ego’s job acquisition at the potential plant.

Column (1) of Table 6 reproduces the results from Table 5 for the subset of egos who have placebo histories, which requires that their historic firms had multiple plants. This accounts for the roughly two-thirds reduction in sample size. The coefficients on the indicator for at least one cohort alter and on the log number of non-cohort alters stay essentially the same as for the full sample of egos, whereas the coefficient on the log number of cohort alters increases substantially and the coefficient on the indicator for at least one non-cohort alter decreases substantially. Column (2) of Table 6 shows the results from using placebo alters in place of true alters. The closures and egos are the same as in Column (1), but the set of potential plants is different.⁴⁰ The impacts of

³⁹For use of a similar test to study the employment outcomes of referred and non-referred employees see [Hensvik and Skans \(2016\)](#) and [Eliason, Hensvik, Kramarz, and Skans \(2017\)](#).

⁴⁰We drop closures and egos when the closure \times potential fixed effects or ego fixed effects are identified from only one observation, leading to the lower numbers of closures and egos in Column (2) relative to Column (1). Dropping singletons is standard in the literature because it allows for an efficiency gain (see [Correia \(2015\)](#) for a survey).

all placebo alter variables, particularly cohort alter variables, are statistically insignificant, as we would expect if job referrals from contacts were the cause of the association between employment of alters at potential plants and their hiring of egos.

6.2 Placebo Potential Plants

Table 7: Placebo Potential Plants

	True Dest. (1)	Plac. Dest. (2)
Coh. Alters ≥ 1	.0010 (.0004)**	.0005 (.0005)
(Log) Coh. Alters	.0043 (.0018)**	-.0010 (.0020)
Non-Coh. Alters ≥ 1	.00008 (.0002)	-.0005 (.0008)
(Log) Non-Coh. Alters	.0017 (.0008)**	-.0008 (.0011)
Obs.	5,533,474	5,533,474
R^2	.209	.3034

Sample restricted to those potential plants with other plants in the same firm X year, but a different municipality.

Note: Standard errors clustered at the closing firm level. All columns contain closing firm \times potential plant and ego fixed effects. Controls for the compatibility between the ego and potential (*% Same Age Group – Potential is Hist. Pl. (Indic)* from Table 4) are included, but not shown. The dependent variable is an indicator for an ego's job acquisition at the potential plant.

Workers from the same historic firm may find the same hiring firm attractive, and this effect may be especially strong for workers from the same hiring cohort of the historic firm. Once again, the association of the presence of alters from the historic firm with the hiring of ego would not indicate that the alters referred ego. Following a strategy similar to that of the previous subsection, we can restrict potential plants to those belonging to multiplant firms and use the plants at which the alters do not work as placebos. That is, we can examine whether the alter variables that predict hiring at the potential plants also predict hiring at other plants within the same firms. An extra benefit of this placebo test is that it provides a check on our claim in Section 3 that contacts refer former co-workers in order to renew their good working relationships. This would not succeed if the referred workers took jobs at plants in the hiring firms different from the ones at which the referring workers are employed. For this reason we strengthen the placebo test by excluding

other plants in the same firms as the potential plants that are also in the same municipalities, since employment at these might allow the egos and alters to work together on a regular basis. We thus restrict attention to potential plants within multi-plant firms that have plants in multiple municipalities.

Column (1) of Table 7 reproduces the results from Table 5 for the subset of potential plants belonging to multiplant, multi-municipality firms. The coefficients on the indicator for at least one cohort alter and on the log number of non-cohort alters stay essentially the same as for the full sample of potential plants, whereas the coefficient on the log number of cohort alters greatly increases and the coefficient on the indicator for at least one non-cohort alter becomes statistically insignificant. In Column (2), the dependent variable is replaced by an indicator for whether ego is hired at another plant of the potential firm in a different municipality. The coefficients on all the alter variables become statistically insignificant, as would be expected if the association between employment of alters at potential plants and their hiring of egos reflects referrals to renew working relationships.

6.3 Alter Characteristics

In Section 2 we showed that we could incorporate homophily into our model, so that egos will form their networks with alters who are similar to them in observable characteristics. In our data, it is reasonable to believe that a possible contact (alter) is more likely to be an actual contact if in the same age group, education group, or occupation group as ego. We measure these characteristics at the historic plant where ego was first employed with alter, in contrast to the compatibility measures which are computed for potential plant workers at the time ego left his closing firm. Table 3 shows that, for ego-potential hiring plant pairs with alters, the share with at least one alter who was in ego's age, education, or occupation group at their historic plant is 30.1, 63.6, and 56.1 percent, respectively.

We also see from Table 3 that, for ego-potential plant pairs with alters, 10.6 percent have an alter who was employed with ego at more than one historic plant. This might have occurred because alter referred ego or vice-versa, or because alter and ego are exceptionally similar along unobservable dimensions. In either case, if these alters are cohort alters they could cause cohort alters to be associated with a higher probability of ego job acquisition for reasons unrelated to our Propositions 4 and 5.

In column (2) of Table 8, we add eight variables to the specification in column (1): indicators for the presence at the potential hiring plant of at least one alter in the same age group, education group, and occupation group, and with overlapping employment at multiple historic plants, and the interactions of these four indicators with the log numbers of these alters. The coefficients on all eight variables are positive, and for each characteristic group at least one coefficient is statistically significant (though only marginally for log number alters with multiple overlaps). The coefficient on the indicator for the presence of at least one cohort alter falls by half but continues to be precisely estimated, and the coefficient on the interaction of this indicator with the log number of cohort alters becomes statistically insignificant. The impacts of non-cohort alters on the probability of ego obtaining a job at the potential plant are entirely absorbed by the new variables.

Belonging to the same hiring cohort is not the only potentially important temporal aspect of the relationship between ego and alter. The amount of time they overlapped at the historic plant should also increase the probability that alter was in ego's network. The longer they were separated after working together at the historic plant, the more likely that alter is no longer a contact for ego.

We measure both overlap and separation in months, averaged over cohort or non-cohort alters at the potential hiring plant. (Recall that there is only one cohort (non-cohort) alter for 81.9 (77.3) percent of ego-potential hiring plant pairs where a cohort (non-cohort) alter is present.) We end measured separation in the month ego leaves the closing firm and begins his job search, which is when contacts become relevant. Since to be included in our sample an alter must be present at a potential plant at the time ego leaves the closing firm, job search would add the same number of months to measured separation for all alters. However, the number of months between when a potential plant hires an alter and when ego leaves the closing firm can vary. If the potential plant is targeting skills that ego and alter acquired at the historic plant, ego's skills may have deteriorated relative to alter during these months. We therefore add these months as a control variable so that months of separation will more accurately reflect the impact of relationship decay.

In column (3) of Table 8, we add the following variables to the specification of column (2): log average months of overlap of cohort and non-cohort alters with ego at their historic plants, where overlap is totaled over all common employment if ego and alter worked together more than once; log average months of separation of ego from cohort and non-cohort alters, where separation is measured from last job together; and log average months between when cohort and non-cohort alters were hired at the potential plant and when ego left the closing firm, which is equivalent to

Table 8: Alters' Characteristics

	Base (1)	Homophily (2)	Char (3)
Coh. Alters ≥ 1	.0011 (.0002)***	.0005 (.0002)***	.0038 (.0018)**
(Log) Coh. Alters	.0017 (.0008)**	-.0007 (.0010)	-.0007 (.0010)
Non-Coh. Alters ≥ 1	.0004 (.0001)***	.0001 (.0001)	.0011 (.0007)
(Log) Non-Coh. Alters	.0015 (.0003)***	-.0014 (.0004)***	-.0013 (.0004)***
Num Alt. First Worked in Same Age Grp ≥ 1		.0004 (.0002)***	.0004 (.0001)***
(Log) Num Alt. First Worked in Same Age Grp		.0011 (.0008)	.0011 (.0008)
Num Alt. First Worked in Same Edu. Grp ≥ 1		.0001 (.0001)	.0001 (.0001)
(Log) Num Alt. First Worked in Same Edu. Grp		.0022 (.0005)***	.0022 (.0005)***
Num Alt. First Worked in Same Occ. Grp ≥ 1		.0003 (.0001)**	.0003 (.0001)**
(Log) Num Alt. First Worked in Same Occ. Grp		.0016 (.0004)***	.0016 (.0004)***
Num Alt. with Multiple Overlaps ≥ 1		.00002 (.0004)	-.0002 (.0004)
(Log) Num Alt. with Multiple Overlaps		.0023 (.0012)*	.0022 (.0012)*
(Log) Avg Coh. Total Overlap (Months)			.0002 (.0003)
(Log) Avg Non-Coh. Total Overlap (Months)			.00004 (.00009)
(Log) Avg Coh. Separation (Months)			-.0007 (.0004)
(Log) Avg Non-Coh. Separation (Months)			-.0002 (.0002)
(Log) Avg Coh Tenure (Months)			-.0005 (.0003)*
(Log) Avg Non-Coh. Tenure (Months)			-.0001 (.0001)
Obs.	29,519,778	29,519,778	29,519,778
R^2	.244	.2441	.2441

Note: Standard errors clustered at the closing firm level. All columns contain closing firm \times potential plant and ego fixed effects. Controls for the compatibility between the ego and potential (*% Same Age Group – Potential is Hist. Pl. (Indic)* from Table 4) are included, but not shown. The dependent variable is an indicator for an ego's job acquisition at the potential plant.

alter tenure at the potential plant at the time ego left the closing firm.⁴¹ Each of these six variables takes the value zero if none of the corresponding type of alter is present at the potential plant and

⁴¹Overlap is measured by job \times months so that overlapping in multiple jobs simultaneously (a rare event) is double counted. This choice does not influence the results.

should therefore be interpreted as an interaction with the indicator for alter presence. Positive (negative) estimated coefficients therefore tend to lower (raise) the estimated coefficients on the alter presence indicators. The means of the six variables conditional on presence of the relevant type of alters are given in columns (3) and (4) of Table 3.⁴²

We see that all of the new variables in column (3) of Table 8 have the expected signs, with the largest coefficient (in absolute value) on log average months of separation of ego from cohort alters, but only the coefficient on log average cohort alter tenure is even marginally statistically significant ($t=1.87$ compared to 1.62 for log average months of separation). Given the collinearity that exists among these variables (e.g., length of separation and length of alter tenure are identical if alter leaves the historic plant before ego and goes directly to the potential plant), we feel these results will not bear much interpretive weight.

6.4 Instrument for Alters' Location

In subsections 6.1 and 6.2 we presented evidence that our results cannot be explained by correlated unobservable characteristics of ego and cohort or non-cohort alters at the firm level. However, there may still be sufficient heterogeneity of plants within firms for correlated unobservables at the plant level to remain a viable alternative explanation for our findings. In this subsection we employ an instrumental variables strategy to address this concern. The literature suggests using “peers-of-peers” instruments (Bramoullé, Djebbari, and Fortin 2009, De Giorgi, Pellizzari, and Redaelli 2010) for the location of alters at potential hiring plants. This means using the location of the alters' alters as instruments for the location of the alters. Use of a peers-of-peers instrument is a network analog to use of a time-lag as an instrument, as in Altonji and Card (1991).

An alter's alter is a worker who is an alter of one of ego's alters, but not an alter himself. Alter's alters are restricted to those that the alters met before starting at the employer where they are located at the time ego leaves the closing firm. If an alter's alters were included from that final job spell then they would artificially predict the location of the alter. Our instruments are

⁴²Comparing these columns, we see that the mean overlap for non-cohort alters is six months greater than the mean overlap for cohort alters. This is a surprise, because if length of overlap is determined by ego leaving the historic plant, overlap for cohort alters will be greater: about equal relative to incumbent alters and greater relative to cohorts that arrive later. Further examination of the data shows that this within-plant effect is dominated by a between-plant effect. Some plants hire later cohort alters but other plants do not. The former plants tend to be thriving relative to the latter plants, so workers stay at the former plants longer and accumulate more months of overlap. There are also many more non-cohort alters relative to cohort alters at the former plants because there is extensive hiring of later cohort alters, causing mean overlap for non-cohort alters to be greater than for cohort alters.

modeled after [De Giorgi, Pellizzari, and Redaelli \(2010\)](#), who study a student’s choice of college major relative to the choice of their peers. Their instrument is the fraction of unique excluded peers-of-peers (equivalent to alters’ alters) choosing a major.

Table 9: Alters’ Statistics

	Frac. w/ ≥ 1	Count ¹
Coh. Alts’ Coh. Alts	.047	7.056
Coh. Alts’ Non-Coh. Alts	.083	15.863
Non-Coh. Alts’ Coh. Alts	.099	9.896
Non-Coh. Alts’ Non-Coh. Alts	.155	23.529

Obs. 29,519,778

¹ Conditional on having at least one.

For the four endogenous variables, W_{if} ([cohort alters, non-cohort alters] \times [≥ 1 , log]), we use eight instruments Z_{if} ([cohort alters’ cohort alters, cohort alters’ non-cohort alters, non-cohort alters’ cohort alters, non-cohort alters’ non-cohort alters] \times [≥ 1 , log]). Table 9 summarizes the fraction of observations with at least one of each type of alters’ alter and the mean number of that type, given there exists at least one. For example, 15.5% of potential plants have a non-cohort alter’s non-cohort alter, with an average of 23.5, if there is at least one. The instrumental variable coefficients are estimated using the two-step feasible generalized method of moments (IV-GMM) because it is more efficient than standard two-stage least squares when the number of instruments is greater than the number of endogenous variables.⁴³

Our instruments will be relevant – alter’s alters will predict the locations of alters – if the referral mechanism we are studying is operative.⁴⁴ The validity of our instruments is more problematic. The exclusion restriction is likely to be satisfied because we are using the set of alters’ alters and the set of alters, so that the instrument encompasses all avenues through which alters’ alters could influence ego. Conditional independence is more suspect. By construction, alters’ alters are employees with whom ego has never worked. Nevertheless, alters’ alters are not randomly assigned to alters. Given this concern about instrumental validity, it is safest to view the following results as complementary to our other robustness checks rather than convincing on their own.

⁴³For more on the rationale for and implementation of the IV-GMM estimator, see [Baum, Schaffer, Stillman, et al. \(2003\)](#).

⁴⁴However, note that in the reduced form ego acquires a job at a potential plant as the result of two rounds of referrals, from alter’s alter to alter and from alter to ego. In our model extension in section 3, to maintain tractability we ruled out more than one round of referral. We leave further extension to more than one round of referral to future research.

Table 10: Instrumental Variable First-stage and Reduced Form

	Coh. Alts ≥ 1	(Log) Coh. Alts	Non-Coh. Alts ≥ 1	(Log) Non -Coh. Alts	Reduced ¹
	(1)	(2)	(3)	(4)	(5)
Coh. Alts'	-.0046	-.0116	-.0019	-.0124	-.00004
Coh. Alts ≥ 1	(.0009)***	(.0005)***	(.0018)	(.0013)***	(.00003)
(Log)0536	.0448	.0131	.0340	.0004
	(.0016)***	(.0028)***	(.0025)***	(.0061)***	(.0001)***
Coh. Alts'	.0016	-.0035	-.0065	-.0031	-3.00e-06
Non-Coh. Alts ≥ 1	(.0006)***	(.0003)***	(.0019)***	(.0011)***	(.00003)
(Log) ..	.0135	.0028	.0114	.0263	.00003
	(.0008)***	(.0010)***	(.0013)***	(.0027)***	(.00005)
Non-Coh. Alts'	-.0062	-.0041	-.0078	-.0246	.00003
Coh. Alts ≥ 1	(.0006)***	(.0004)***	(.0028)***	(.0008)***	(.00003)
(Log) ..	.0183	.0188	.0765	.1153	.0002
	(.0009)***	(.0010)***	(.0015)***	(.0035)***	(.00008)***
Non-Coh. Alts'	-.0004	.0001	-.0086	-.0033	.00009
Non-Coh. Alts ≥ 1	(.0002)*	(.0002)	(.0019)***	(.0007)***	(.00003)***
(Log) ..	-.0009	-.0060	.0246	-.0142	-.00003
	(.0005)*	(.0004)***	(.0019)***	(.0019)***	(.00004)
Obs.	29,519,778	29,519,778	29,519,778	29,519,778	29,519,778
R^2	.218	.3791	.3316	.469	.2438

Note: Standard errors clustered at the closing firm level. All columns contain closing firm \times potential plant and ego fixed effects. Controls for the compatibility between the ego and potential (*% Same Age Group – Potential is Hist. Pl. (Indic)* from Table 4) are included, but not shown. The dependent variable is an indicator for an ego's job acquisition at the potential plant.

¹ The dependent variable is an indicator for an ego's job acquisition at the potential plant.

Our first-stage results are shown in columns (1) – (4) of Table 10. We see that the instruments are indeed predictive of the presence of cohort alters and non-cohort alters at potential plants. Column (5) presents the reduced form estimates. All significant reduced form coefficients have the right sign.

Table 11 compares the IV-GMM estimates to the OLS estimates from Table 5. We see that the Kleibergen-Paap F-statistic for the estimation, which is the best summary of the first stage in the IV-GMM setting, is highly significant at 74.87 and thus provides substantial support for the relevance of the instruments.

The IV-GMM coefficients on both cohort alter variables are roughly three times the OLS coefficients, and the IV-GMM coefficient on the indicator for the presence of at least one non-cohort alter is roughly six times the OLS coefficient. The coefficient on the log of the number of non-cohort alters when there is at least one changes from positive and significant to negative and insignificant. This is a puzzle, but the overall message of Table 11 is clear: the presence of cohort alters raises the

Table 11: Instrumental Variable

	OLS	IV-GMM
	(1)	(2)
Coh. Alters ≥ 1	.0011 (.0002)***	.0038 (.0015)**
(Log) Coh. Alters	.0017 (.0008)*	.0040 (.0026)
Non-Coh. Alters ≥ 1	.0004 (.0001)***	.0024 (.0008)***
(Log) Non-Coh. Alters	.0015 (.0003)***	-.0014 (.0011)
Obs.	29,519,778	29,519,778
R^2	.244	.2436
K-P F (weak) id		74.87

Note: Standard errors clustered at the closing firm level. All columns contain closing firm \times potential plant and ego fixed effects. Controls for the compatibility between the ego and potential (*% Same Age Group – Potential is Hist. Pl. (Indic)*) from Table 4 are included, but not shown. The dependent variable is an indicator for an ego's job acquisition at the potential plant. The dependent variable is an indicator for an ego's job acquisition at the potential plant.

probability of ego's job acquisition at the potential plant more than the presence of non-cohort alters, and the impact of contacts is estimated to be much greater when using instrumental variables. The latter fact suggests a local average treatment effect (LATE) interpretation. Suppose that, conditional on overall hiring levels, some potential plants tend to hire more through referrals relative to going directly to the labor market. This unobserved choice, not present in the analysis of Section 3 but modeled in papers such as Galenianos (2014), would not be captured in the closure \times potential fixed effects. The LATE would be isolating the impact of alters on the probability that ego starts at plants that used referrals to hire said alters. If this is the case, correlation in the tendency to use referrals will cause the IV-GMM estimates of the impacts of contacts to be greater than the OLS estimates.

7 Conclusions

When chance meetings reveal compatibility, the agents involved have incentives to maintain their relationships. Accumulating relationships becomes increasingly costly, however, causing agents to become less open to chance meetings over time. The interaction of this dynamic with turnover

leads to Proposition 3 in our paper, describing for an egocentric network in an organization the time pattern of history dependence as a function of ego's tenure in the organization. Mutual openness of newly arrived agents also leads to the cross-section prediction of "cohort attachment," a tendency for members of ego's hiring cohort to be disproportionately represented in his network. In an extension of our model we allowed members of ego's network who are subsequently split across many organizations to be his "contacts." The desire of contacts to renew their successful working relationships leads to job referrals. Using matched employer-employee data from Brazil, we found that the presence of a hiring-cohort former co-worker increases the probability of job acquisition at a specific hiring plant nearly three times more than the presence of a non-hiring-cohort former co-worker. We attempted to mitigate lack of random assignment of former co-workers to job seekers by controlling for observable similarities and by using placebo co-workers, placebo hiring plants, and peers-of-peers instruments for presence of former co-workers.

The lack of externalities in our model means that agents build their egocentric networks optimally from the social point of view. It would be straightforward to extend our model so that the network of relationships in the firm as a whole has a function that each agent in the continuum fails to internalize when making his decisions, thereby opening up space for policy. An often-studied function is diffusion of information. Suppose the owner(s) of the firm are concerned about "siloing" (Tett 2015) of information within cohorts. They can respond by spreading out hiring over time, causing reduced cohort size to shrink the share of within-cohort matches endogenously. In a multi-plant firm, rotating workers from the same hiring cohort to different plants can accomplish the same aim, though firms must be wary of the extent to which workers might react by reducing their desired numbers of matches.

In addition to homophily, non-mechanical theories of network formation emphasize contingency (e.g., Small and Sukhu 2016) and strategy (e.g., Jackson and Wolinsky 1996). Random matching with agents of unknown match quality places our model clearly in the former group. Our contribution shows that contingency does not imply complete unpredictability. On the contrary, the time structure induced by the persistence and costliness of relationships allows us to make a number of testable predictions.

In our empirical work it has been convenient for us to focus on the prediction of cohort attachment, the impact of which could be estimated using publicly available data. In the future, surveys of individuals in organizations could map out the times at which they met the alters in their

networks and thus directly test Proposition 3. Data at the firm rather than individual level could also be relevant if the agents in our model were firms instead of individuals, establishing relationships with other firms. Proposition 3 then suggests, for example, that the networks of young firms would be dominated by the clients and suppliers with which they were matched at startup, whereas the networks of firms that survive to “maturity” would be dominated by more recent clients and suppliers. The generality of our framework should accommodate many applications.

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Appendices

A Conditions Ensuring Sufficient Availability of Agents of Unknown Match Quality

We derive conditions on the cost of matching that ensure that agents will not desire to form so many matches that they have to form matches they know to be of low quality. That is, our sufficient conditions ensure that in equilibrium there are more agents of unknown match quality to ego than the number of matches he desires to form outside his network.

We start with the special case $\delta = 0$, so the set of agents is fixed. We assume that enough unknowns are available to ego in every period, then derive a condition under which this assumption holds. Under this assumption, ego builds his network as described by equations (4) and (5) (with $\delta = 0$). In period 0 he meets with $x_0 = z^*$ unknowns, yielding a network of size $n_1 = pz^*$ in period 1. In period 1 he meets with $x_1 = z^* - pz^*$ unknowns, and so on. Thus in any period t one can compute the total number of matches ego has formed with unknowns by dividing n_t from equation (5) by p . As shown in equation (6), with $\delta = 0$ n_t reaches its maximum, steady state value at z^* . Hence ego can never have matched with more than z^*/p unknowns. Since all agents are initially unknown to ego, the condition $z^*/p \leq N$ is sufficient to ensure that ego never runs out of unknowns with whom to match.

Now we consider the main case $\delta > 0$, so that agents enter the firm in cohorts. We will show that the candidate sufficient condition $z^*/p \leq N$ is too weak when small δ is combined with large p , and derive an alternative sufficient condition. Recall that $P_t^c(c')$ is the probability that a given alter in cohort c' is in the network of a given ego in cohort c at the end of period t , conditional on the alter remaining with the firm. It follows that $P_t^c(c')/p$ is the probability that an ego in cohort c has met with a given alter in cohort c' by the end of period t , conditional on the alter remaining with the firm. Since $P_t^c(c')/p$ was derived by dividing the demand for matches by ego with a given unknown alter by the supply of matches by all unknown alters, we want a condition that ensures $P_t^c(c')/p \leq 1$.

Consider the polar case $p = 1$. Derivation of the sufficient condition is greatly simplified by Propositions 4 and 5, which imply $P_t^c(c)/p > P_t^c(c')/p$ for $c' \neq c$. (Proposition 5 itself is conditioned on “ z^*/N small enough,” but it can be shown that for $p = 1$ this condition is satisfied

if $z^*/N < 1$, which is equivalent to our weaker sufficient condition.) We therefore only need a condition sufficient for $P_t^c(c)/p \leq 1$.

When $p = 1$ every alter is of high match quality. Each agent therefore makes z^* matches with alters of high match quality in his entry period and δz^* replacement matches in every subsequent period. It follows that the number of matches with unknowns sought by every incumbent alter in the period ego enters the firm (and every subsequent period) is fixed at its minimum value δz^* . Using this information along with equations (7) and (8) allows us to compute $P_c^c(c)/p = z^*/[N(1 - \delta)\delta + \delta N] = z^*/\delta N[1 + (1 - \delta)]$. Setting $z^* = \delta N[1 + (1 - \delta)]$ therefore yields $P_c^c(c)/p = 1$. From equation (7) we see that $P_c^c(c)/p = 1$ implies $P_t^c(c)/p = 1$ for all $t > c$. We also see from equation (7) that $P_t^c(c)/p$ is monotonically increasing in z^* for all $t \geq c$. Hence $z^* \leq \delta N[1 + (1 - \delta)]$ is sufficient to ensure $P_t^c(c)/p \leq 1$ for all $t \geq c$ when $\delta > 0$ and $p = 1$. Finally, inspection of equations (7) and (8) shows that reducing p below 1 decreases $P_c^c(c)/p$ since the numerator remains unchanged and the denominator increases, after which $P_t^c(c)/p$ cannot rise above 1 for $t > c$ provided $z^* \leq \delta N[1 + (1 - \delta)]$.

We can now summarize. For $p \in (0, 1)$ and $\delta = 0$, the condition $z^* \leq pN$ is sufficient to ensure that ego never runs out of unknowns with whom to match. For $p \in (0, 1)$ and $\delta > 0$, the condition $z^* \leq pN$ may be too weak if $\delta N[1 + (1 - \delta)] < pN$ or $\delta < 1 - \sqrt{1 - p}$, in which case we use $z^* \leq \delta N[1 + (1 - \delta)]$ as our condition sufficient to ensure that ego never runs out of unknowns with whom to match.

We must translate these conditions on z^* into the primitives of our model. We assume that the cost of matching rises sufficiently fast that the desired number of matches z^* falls below the levels in the relevant inequalities:

Assumption A. For $\delta \in (0, 1 - \sqrt{1 - p})$, we assume $c'(\delta N[1 + (1 - \delta)]) \geq \frac{py_H + (1-p)y_L}{2} + \beta(1 - \delta)p \frac{(1-\delta)(1-p)}{1-\beta(1-\delta)^2(1-p)} \frac{y_H - y_L}{2}$. For $\delta = 0$ or $\delta \geq 1 - \sqrt{1 - p}$, we assume $c'(pN) \geq \frac{py_H + (1-p)y_L}{2} + \beta(1 - \delta)p \frac{(1-\delta)(1-p)}{1-\beta(1-\delta)^2(1-p)} \frac{y_H - y_L}{2}$.

The right-hand sides of the inequalities in Assumption A equal $c'(z^*)$ by equation (3), and therefore imply the inequalities $\delta N[1 + (1 - \delta)] \geq z^*$ and $pN \geq z^*$, respectively. Assumption A can be thought of as an addendum to Assumption 3 in Section 2.

B Proofs

Proposition 3. Assume $p > \frac{\delta}{(1-\delta)}$. For $\delta > 0$, there exists a $\underline{t}' \geq 1$ such that $HD_t(t')$ is monotonically decreasing in t' for $t' < \underline{t}'$ and monotonically increasing in t' for $t' > \underline{t}'$. Moreover, there exists a $\underline{t} > \underline{t}'$ such that $HD_t(0) > HD_t(t)$ for $t < \underline{t}$ and $HD_t(0) < HD_t(t)$ for $t > \underline{t}$.

Proof. Substituting $n_{t'-1}$ and n_t into the definition of $HD_t(t')$:

$$\begin{aligned}
HD_t(t') &= \frac{(1-\delta)^{t-t'} p [z^* - (1-\delta) \sum_{\tau=0}^{t'-1} (1-p)^\tau (1-\delta)^\tau p z^*]}{\sum_{\tau=0}^t (1-p)^\tau (1-\delta)^\tau p z^*} \\
HD_t(t') &= \frac{(1-\delta)^{t-t'} [1 - p(1-\delta) \sum_{\tau=0}^{t'-1} (1-p)^\tau (1-\delta)^\tau]}{\sum_{\tau=0}^t (1-p)^\tau (1-\delta)^\tau} \\
HD_t(t') &= \frac{(1-\delta)^{t-t'} [1 - p(1-\delta) \frac{1-(1-p)^{t'}(1-\delta)^{t'}}{1-(1-p)(1-\delta)}]}{\frac{1-(1-p)^{t+1}(1-\delta)^{t+1}}{1-(1-p)(1-\delta)}} \\
HD_t(t') &= \frac{(1-\delta)^{t-t'} [\delta + p(1-\delta) - p(1-\delta)[1 - (1-p)^{t'}(1-\delta)^{t'}]]}{1 - (1-p)^{t+1}(1-\delta)^{t+1}} \\
HD_t(t') &= \underbrace{\frac{(1-\delta)^t}{1 - (1-p)^{t+1}(1-\delta)^{t+1}}}_A [(1-\delta)^{-t} \delta + (1-\delta)p(1-p)^{t'}] \tag{B.1}
\end{aligned}$$

Inspection of equation (B.1) shows that $HD_t(t')$ is increasing in t' for t' sufficiently large. Straightforward computation shows that $HD_t(0) > HD_t(1)$ given $p > \frac{\delta}{(1-\delta)}$. Moreover, if we treat t' as continuous and differentiate $HD_t(t')$ twice with respect to t' , we obtain $A[(1-\delta)^{-t} \delta [\ln(1-\delta)]^2 + (1-\delta)p(1-p)^{t'} [\ln(1-p)]^2] > 0$. Thus $HD_t(t')$ is strictly convex in continuous t' and has a global minimum, and in discrete time reaches a minimum for some $\underline{t}' \geq 1$.

It follows from the first part of the proposition that $HD_t(0) > HD_t(t)$ for $t < \underline{t}'$, hence $\underline{t} > \underline{t}'$. Next, we can use equation (B.1) to show that the inequality $HD_t(0) < HD_t(t)$ reduces to $\delta + (1-\delta)p < (1-\delta)^{-t} \delta + (1-\delta)p(1-p)^t$. From the first part of the proposition the right-hand side of this inequality is monotonically increasing in t for $t > \underline{t}'$. Since, in fact, the right-hand side of this inequality increases without bound in t , the existence of a \underline{t} as described in the second part of the proposition follows. \square

Proposition 4. $P_t^c(c) > P_t^c(c')$ for all $c' \in [0, c-1]$, for all $t \geq c$.

Proof. We will show that $P_t^c(c) - P_t^c(c') > 0$ for $c' < c$, $t \geq c$. The proof proceeds by induction. For the base case $t = c$, from equation (7) we have $P_c^c(c) - P_c^c(c') = p x_c^c (x_c^c - x_c^{c'}) / L_c^c > 0$,

because $x_c^c > x_c^{c'}$ for $c' < c$ by equation (5). For the inductive step for period $t > c$, use equation (7) to compute $P_t^c(c) - P_t^c(c') > 0$ as

$$[P_{t-1}^c(c) - P_{t-1}^c(c')] + [1 - \frac{P_{t-1}^c(c)}{p}]px_t^c \frac{x_t^c}{L_t^c} - [1 - \frac{P_{t-1}^c(c')}{p}]px_t^{c'} \frac{x_t^{c'}}{L_t^c}.$$

Let $k_c = \frac{P_{t-1}^c(c)}{p}$, $k_{c'} = \frac{P_{t-1}^c(c')}{p}$, $m_c = x_t^c \frac{x_t^c}{L_t^c}$ and $m_{c'} = x_t^{c'} \frac{x_t^{c'}}{L_t^c}$. Note that $0 < k_{c'} < k_c < 1$ by the inductive hypothesis. Additionally, $0 < m_{c'} < m_c < 1$ because $x_t^c > x_t^{c'}$ for $c' < c$, $t \geq c$ by equation (5). Substituting $k_c, k_{c'}, m_c$ and $m_{c'}$ into the expression above, we have

$$p\{(k_c - k_{c'}) + [(1 - k_c)m_c - (1 - k_{c'})m_{c'}]\} = p\{(1 - m_c)(k_c - k_{c'}) + (1 - k_{c'})(m_c - m_{c'})\} > 0.$$

□

Lemma 1. *If z^*/N is sufficiently small, $x_t^c/L_t^c > x_{t+b}^c/L_{t+b}^c$ for all $b \in [1, T - c]$, for all $t \geq c$.*

Proof. Rearrange the inequality as $\frac{L_{t+b}^c}{L_t^c} > \frac{x_{t+b}^c}{x_t^c}$. From equation (8), we have

$$\frac{L_{t+b}^c}{L_t^c} = \frac{N(1 - \delta)^{t+b}[1 - \frac{P_{t+b-1}^c(0)}{p}]x_{t+b}^0 + \sum_{c'=1}^{t+b} \delta N(1 - \delta)^{t+b-c'}[1 - \frac{P_{t+b-1}^c(c')}{p}]x_{t+b}^{c'}}{N(1 - \delta)^t[1 - \frac{P_{t-1}^c(0)}{p}]x_t^0 + \sum_{c'=1}^t \delta N(1 - \delta)^{t-c'}[1 - \frac{P_{t-1}^c(c')}{p}]x_t^{c'}}.$$

Note that N factors out of both the numerator and denominator of this expression. Likewise, z^* factors out of every x , hence z^* factors out of both the numerator and denominator. Changes in N or z^* therefore affect $\frac{L_{t+b}^c}{L_t^c}$ only through the terms containing P . From equation (7), we see that we can make any P arbitrarily small by shrinking z^*/N .

Letting z^*/N approach zero, we have

$$\begin{aligned} \frac{L_{t+b}^c}{L_t^c} &\approx \frac{N(1 - \delta)^{t+b}x_{t+b}^0 + \sum_{c'=1}^{t+b} \delta N(1 - \delta)^{t+b-c'}x_{t+b}^{c'}}{N(1 - \delta)^t x_t^0 + \sum_{c'=1}^t \delta N(1 - \delta)^{t-c'}x_t^{c'}} \\ &= \frac{N(1 - \delta)^{t+b}x_{t+b}^0 + \sum_{a=1}^b \delta N(1 - \delta)^{t+b-a}x_{t+b}^a + \sum_{c'=1+b}^{t+b} \delta N(1 - \delta)^{t+b-c'}x_{t+b}^{c'}}{N(1 - \delta)^t x_t^0 + \sum_{c'=1}^t \delta N(1 - \delta)^{t-c'}x_t^{c'}}. \end{aligned}$$

We will show that this last expression is greater than $\frac{x_{t+b}^c}{x_t^c}$, from which it follows that $\frac{L_{t+b}^c}{L_t^c} > \frac{x_{t+b}^c}{x_t^c}$ for z^*/N sufficiently small.

Note that the last term in the numerator of the expression equals the last term in the denominator of the expression because $x_{t+\tau}^{c'} = x_t^{c'-\tau}$. From this fact and $\frac{x_{t+b}^c}{x_t^c} < 1$, it follows that a sufficient

condition for the expression to be greater than $\frac{x_{t+b}^c}{x_t^c}$ is

$$N(1-\delta)^{t+b}x_{t+b}^0 + \sum_{a=1}^b \delta N(1-\delta)^{t+b-a}x_{t+b}^a - N(1-\delta)^t x_t^0 \frac{x_{t+b}^c}{x_t^c} \geq 0,$$

$$\text{or } (1-\delta)^b x_{t+b}^0 + \sum_{a=1}^b \delta(1-\delta)^{b-a}x_{t+b}^a - x_t^0 \frac{x_{t+b}^c}{x_t^c} \geq 0.$$

Using the fact that $(1-\delta)^b = 1 - \sum_{a=1}^b \delta(1-\delta)^{b-a}$, and dividing through by x_t^0 , the sufficient condition becomes

$$\frac{x_{t+b}^0}{x_t^0} + \sum_{a=1}^b \delta(1-\delta)^{b-a} \frac{x_{t+b}^a - x_{t+b}^0}{x_t^0} - \frac{x_{t+b}^c}{x_t^c} \geq 0.$$

Note that by example 1 of Proposition 3, the left-hand side goes to zero as δ goes to zero. For $\delta > 0$, it follows from equation (5) that $\frac{x_{t+b}^0}{x_t^0} > \frac{x_{t+b}^c}{x_t^c}$, hence the sufficient condition holds. \square

Proposition 5. *If z^*/N is sufficiently small, $P_t^c(c) > P_t^c(c')$ for all $c' \in [c+1, T]$, for all $t \geq c'$.*

Proof. We will show by induction that $P_t^c(c) - P_t^c(c+b) \geq \sum_{\tau=0}^{b-1} [1 - \frac{P_{t-\tau-1}^c(c)}{p}] p x_{t-\tau}^c \frac{x_{t-\tau}^c}{L_{t-\tau}^c} > 0$, $b = 1, \dots, T - c$ and $t = c + b, \dots, T$. We first establish the base case

$$P_{c+b}^c(c) - P_{c+b}^c(c+b) \geq \sum_{\tau=0}^{b-1} [1 - \frac{P_{c+b-\tau-1}^c(c)}{p}] p x_{c+b-\tau}^c \frac{x_{c+b-\tau}^c}{L_{c+b-\tau}^c}.$$

From repeated applications of equation (7), we have

$$P_{c+b}^c(c) = P_c^c(c) + \sum_{\tau=0}^{b-1} [1 - \frac{P_{c+b-\tau-1}^c(c)}{p}] p x_{c+b-\tau}^c \frac{x_{c+b-\tau}^c}{L_{c+b-\tau}^c}$$

$$P_{c+b}^c(c+b) = p x_{c+b}^c \frac{x_{c+b}^c}{L_{c+b}^c}.$$

Hence, we must show $P_c^c(c) - p x_{c+b}^c \frac{x_{c+b}^c}{L_{c+b}^c} \geq 0$, or $p x_c^c \frac{x_c^c}{L_c^c} - p x_{c+b}^c \frac{x_{c+b}^c}{L_{c+b}^c} \geq 0$. Since $x_{c+b}^{c+b} = x_c^c$, this reduces to $\frac{x_c^c}{L_c^c} \geq \frac{x_{c+b}^c}{L_{c+b}^c}$ or $\frac{L_{c+b}^c}{L_c^c} \geq \frac{x_{c+b}^c}{x_c^c}$, which follows from Lemma 1.

Now consider the inductive step

$$P_t^c(c) - P_t^c(c+b) \geq \sum_{\tau=0}^{b-1} [1 - \frac{P_{t-\tau-1}^c(c)}{p}] p x_{t-\tau}^c \frac{x_{t-\tau}^c}{L_{t-\tau}^c} > 0.$$

Application of equation (7), and then of the inductive hypothesis, yields

$$\begin{aligned}
P_t^c(c) - P_t^c(c+b) &= P_{t-1}^c(c) - P_{t-1}^c(c+b) - \left[1 - \frac{P_{t-1}^c(c+b)}{p}\right] p x_t^c \frac{x_t^{c+b}}{L_t^c} + \left[1 - \frac{P_{t-1}^c(c)}{p}\right] p x_t^c \frac{x_t^c}{L_t^c} \\
&\geq \sum_{\tau=0}^{b-1} \left[1 - \frac{P_{t-\tau-2}^c(c)}{p}\right] p x_{t-\tau-1}^c \frac{x_{t-\tau-1}^c}{L_{t-\tau-1}^c} - \left[1 - \frac{P_{t-1}^c(c+b)}{p}\right] p x_t^c \frac{x_t^{c+b}}{L_t^c} + \left[1 - \frac{P_{t-1}^c(c)}{p}\right] p x_t^c \frac{x_t^c}{L_t^c} \\
&= \sum_{\tau=0}^{b-1} \left[1 - \frac{P_{t-\tau-1}^c(c)}{p}\right] p x_{t-\tau}^c \frac{x_{t-\tau}^c}{L_{t-\tau}^c} + \left[1 - \frac{P_{t-b-1}^c(c)}{p}\right] p x_{t-b}^c \frac{x_{t-b}^c}{L_{t-b}^c} - \left[1 - \frac{P_{t-1}^c(c+b)}{p}\right] p x_t^c \frac{x_t^{c+b}}{L_t^c}.
\end{aligned}$$

Therefore we must prove

$$\left[1 - \frac{P_{t-b-1}^c(c)}{p}\right] p x_{t-b}^c \frac{x_{t-b}^c}{L_{t-b}^c} - \left[1 - \frac{P_{t-1}^c(c+b)}{p}\right] p x_t^c \frac{x_t^{c+b}}{L_t^c} \geq 0.$$

Noting that $x_{t-b}^c = x_t^{c+b}$, this reduces to

$$\frac{\left[1 - \frac{P_{t-b-1}^c(c)}{p}\right] L_t^c}{\left[1 - \frac{P_{t-1}^c(c+b)}{p}\right] L_{t-b}^c} \geq \frac{x_t^c}{x_{t-b}^c}.$$

Letting z^*/N become arbitrarily small, we can make $\frac{1 - \frac{P_{t-b-1}^c(c)}{p}}{1 - \frac{P_{t-1}^c(c+b)}{p}}$ arbitrarily close to one. Since $\frac{L_t^c}{L_{t-b}^c} > \frac{x_t^c}{x_{t-b}^c}$ by Lemma 1, the result follows. \square

Proposition 6. *Consider a firm of age $T \geq 2$. If z^*/N is sufficiently small and $\delta[1 + \delta p/2(1-p)] < 1/2$, then $S_t^c(c')$ reaches its maximum over cohorts $c' \in [1, T]$ for cohort c for at least the two most recent cohorts, i.e., $c \in [T-1, T]$.*

Proof. For $c = T$, the proposition follows from Proposition 4 and equation (9). For $c = T-1$, the proposition follows from Proposition 4 and equation (9) for cohorts $c' \in [1, T-1]$. It remains to be shown that $S_T^{T-1}(T-1) > S_T^{T-1}(T)$. We have $\frac{S_T^{T-1}(T-1)}{S_T^{T-1}(T)} = \frac{P_T^{T-1}(T-1)}{P_T^{T-1}(T)}(1-\delta)$, so we need

$\frac{P_T^{T-1}(T-1)}{P_T^{T-1}(T)} > \frac{1}{1-\delta}$. Using equation (7), we have

$$\begin{aligned}
\frac{P_T^{T-1}(T-1)}{P_T^{T-1}(T)} &= \frac{P_{T-1}^{T-1}(T-1) + [1 - \frac{P_{T-1}^{T-1}(T-1)}{p}]px_T^{T-1}x_T^{T-1}/L_T^{T-1}}{P_T^{T-1}(T)} \\
&= \frac{px_{T-1}^{T-1}x_T^{T-1}/L_T^{T-1}}{px_T^{T-1}x_T^T/L_T^{T-1}} + \frac{[1 - \frac{P_{T-1}^{T-1}(T-1)}{p}]px_T^{T-1}x_T^{T-1}/L_T^{T-1}}{px_T^{T-1}x_T^T/L_T^{T-1}} \\
&= \frac{x_{T-1}^{T-1}L_T^{T-1}}{x_T^{T-1}L_T^{T-1}} + [1 - \frac{P_{T-1}^{T-1}(T-1)}{p}] \frac{x_T^{T-1}}{x_T^T} \\
&= \frac{x_{T-1}^{T-1}L_T^{T-1}}{x_T^{T-1}L_T^{T-1}} + [1 - \frac{P_{T-1}^{T-1}(T-1)}{p}][1 - (1-\delta)p],
\end{aligned}$$

where we have used equation (5). From Lemma 1, it follows that $\frac{P_T^{T-1}(T-1)}{P_T^{T-1}(T)} \geq 1 + 1 - (1-\delta)p$ for z^*/N sufficiently small. Algebra (available on request) then shows that the condition $2 - (1-\delta)p > \frac{1}{1-\delta}$ reduces to $\delta[1 + \frac{\delta p}{2(1-p)}] < \frac{1}{2}$. \square

Proposition 7. *If z^*/N is sufficiently small and $T - c$ is sufficiently large, $S_T^c(T) > S_T^c(c')$ for all cohorts $c' \in [1, c]$.*

Proof. Using equation (9), we can reduce the inequality for cohort c , $S_T^c(T) > S_T^c(c)$, to $P_T^c(T) > P_T^c(c)(1-\delta)^{T-c}$. The right-hand side of this inequality can be made arbitrarily small for $T - c$ sufficiently large because $P_T^c(c)$ is bounded from above by p . On the left-hand side of the inequality, we use equation (7) to obtain $P_T^c(T) = px_T^c \frac{x_T^T}{L_T^c} = px_T^c \frac{z^*}{L_T^c}$. x_T^c is bounded from below by \bar{x} . L_T^c is bounded from above by Nz^* , the maximal number of desired meetings with unknowns by all agents in the firm. Thus $P_T^c(T)$ is bounded from below by $p\bar{x} \frac{z^*}{Nz^*} = p\bar{x}/N$.

This establishes the proposition for cohort c . Intuitively, the proposition should extend to cohorts $c' \in [1, c-1]$: the number of agents remaining in each of these cohorts is smaller than for cohort c , hence the own-cohort share of ego's network should be even smaller than for cohort c . (Note that if we were to weaken Assumption 5 by decoupling firm hiring from layoffs, it would be possible for earlier cohorts to be larger than later cohorts so we could not extend Proposition 7 to all cohorts $c' \in [1, c-1]$.) We will show that $S_T^c(T) > S_T^c(c)$ implies $S_T^{c-1}(T) > S_T^{c-1}(c-1)$ for $T - c$ sufficiently large and z^*/N sufficiently small. The proof can then be repeated to show $S_T^{c-1}(T) > S_T^{c-1}(c-1)$ implies $S_T^{c-2}(T) > S_T^{c-2}(c-2)$, \dots , $S_T^2(T) > S_T^2(2)$ implies $S_T^1(T) > S_T^1(1)$.

From equation (9) we see that $S_T^c(T) > S_T^c(c)$ implies $P_T^c(T) > P_T^c(c)(1 - \delta)^{T-c}$. We will show that this implies $P_T^{c-1}(T) > P_T^{c-1}(c-1)(1 - \delta)^{T+1-c}$, from which $S_T^{c-1}(T) > S_T^{c-1}(c-1)$ follows from equation (9). Specifically, we will show that $P_T^{c-1}(T)$ can be made arbitrarily close to $P_T^c(T)$ and that $P_T^{c-1}(c-1) < P_T^c(c)$ or $P_T^{c-1}(c-1)$ can be made arbitrarily close to $P_T^c(c)$. Since $P_T^c(T)/P_T^c(c) > (1 - \delta)^{T-c}$, it follows that $P_T^{c-1}(T)/P_T^{c-1}(c-1) > (1 - \delta)(1 - \delta)^{T-c} = (1 - \delta)^{T+1-c}$.

From equation (7) we have $P_T^c(T) = px_T^c \frac{x_T^c}{L_T^c}$ and $P_T^{c-1}(T) = px_T^{c-1} \frac{x_T^c}{L_T^{c-1}}$, where

$$L_T^c = N(1 - \delta)^T \left[1 - \frac{P_{T-1}^c(0)}{p} \right] x_T^0 + \sum_{c'=1}^T \delta N(1 - \delta)^{T-c'} \left[1 - \frac{P_{T-1}^c(c')}{p} \right] x_T^{c'}$$

and

$$L_T^{c-1} = N(1 - \delta)^T \left[1 - \frac{P_{T-1}^{c-1}(0)}{p} \right] x_T^0 + \sum_{c'=1}^T \delta N(1 - \delta)^{T-c'} \left[1 - \frac{P_{T-1}^{c-1}(c')}{p} \right] x_T^{c'}$$

by equation (8). We then have $P_T^{c-1}(T)/P_T^c(T) = (x_T^{c-1}/x_T^c)(L_T^c/L_T^{c-1})$. From equation (5) we see that x_T^{c-1}/x_T^c becomes arbitrarily close to one as $T - c$ grows large. Now consider L_T^c/L_T^{c-1} . Note that N factors out of both the numerator and denominator of this ratio. Likewise, z^* factors out of every x , hence z^* factors out of both the numerator and denominator. Changes in N or z^* therefore affect L_T^c/L_T^{c-1} only through the terms containing P . From equation (7), we see that we can make any P arbitrarily small by shrinking z^*/N . It follows from our expressions for L_T^c and L_T^{c-1} that we can make the ratio L_T^c/L_T^{c-1} arbitrarily close to one by shrinking z^*/N .

Finally, consider the ratio $P_T^{c-1}(c-1)/P_T^c(c)$. From equation (7), we can write

$$\begin{aligned} P_T^{c-1}(c-1) &= px_{c-1}^{c-1} \frac{x_{c-1}^{c-1}}{L_{c-1}^{c-1}} + \left[1 - \frac{P_{c-1}^{c-1}(c-1)}{p} \right] px_c^{c-1} \frac{x_c^{c-1}}{L_c^{c-1}} + \left[1 - \frac{P_c^{c-1}(c-1)}{p} \right] px_{c+1}^{c-1} \frac{x_{c+1}^{c-1}}{L_{c+1}^{c-1}} + \dots \\ &+ \left[1 - \frac{P_{T-1}^{c-1}(c-1)}{p} \right] px_T^{c-1} \frac{x_T^{c-1}}{L_T^{c-1}} \end{aligned}$$

and

$$\begin{aligned} P_T^c(c) &= px_c^c \frac{x_c^c}{L_c^c} + \left[1 - \frac{P_c^c(c)}{p} \right] px_{c+1}^c \frac{x_{c+1}^c}{L_{c+1}^c} + \left[1 - \frac{P_{c+1}^c(c)}{p} \right] px_{c+2}^c \frac{x_{c+2}^c}{L_{c+2}^c} + \dots \\ &+ \left[1 - \frac{P_{T-1}^c(c)}{p} \right] px_T^c \frac{x_T^c}{L_T^c}. \end{aligned}$$

Note that each term in the expression for $P_T^c(c)$ corresponds to the term in the expression for $P_T^{c-1}(c-1)$ for the preceding period. Consider the corresponding terms $[1 - \frac{P_{c+b-2}^{c-1}(c-1)}{p}]px_{c+b-1}^{c-1} \frac{x_{c+b-1}^{c-1}}{L_{c+b-1}^{c-1}}$ and $[1 - \frac{P_{c+b-1}^c(c)}{p}]px_{c+b}^c \frac{x_{c+b}^c}{L_{c+b}^c}$, $b = 1, \dots, T - c$. Note that $x_{c+b-1}^{c-1} = x_{c+b}^c$. Also note that $(z^*)^2/z^*N = z^*/N$ factors out of every term, so that changes in N or z^* can only affect $P_T^{c-1}(c-1)/P_T^c(c)$ through $P_{c+b-2}^{c-1}(c-1)$ and $P_{c+b-1}^c(c)$. As above, we can make $P_{c+b-2}^{c-1}(c-1)$ and $P_{c+b-1}^c(c)$ arbitrarily small by shrinking z^*/N . The difference between the numerators of the corresponding terms can therefore be made arbitrarily small.

Now consider the difference between the denominators of the corresponding terms. From equation (8), we have

$$L_{c+b-1}^{c-1} = N(1-\delta)^{c+b-1} \left[1 - \frac{P_{c+b-2}^{c-1}(0)}{p} \right] x_{c+b-1}^0 + \sum_{c'=1}^{c+b-1} \delta N(1-\delta)^{c+b-1-c'} \left[1 - \frac{P_{c+b-2}^{c-1}(c')}{p} \right] x_{c+b-1}^{c'}$$

$$L_{c+b}^c = N(1-\delta)^{c+b} \left[1 - \frac{P_{c+b-1}^c(0)}{p} \right] x_{c+b}^0 + \sum_{c'=1}^{c+b} \delta N(1-\delta)^{c+b-c'} \left[1 - \frac{P_{c+b-1}^c(c')}{p} \right] x_{c+b}^{c'}$$

By shrinking z^*/N , L_{c+b-1}^{c-1} and L_{c+b}^c can be made arbitrarily close to \tilde{L}_{c+b-1}^{c-1} and \tilde{L}_{c+b}^c , respectively, which are given by

$$\tilde{L}_{c+b-1}^{c-1} = N(1-\delta)^{c+b-1} x_{c+b-1}^0 + \sum_{c'=1}^{c+b-1} \delta N(1-\delta)^{c+b-1-c'} x_{c+b-1}^{c'}$$

and

$$\tilde{L}_{c+b}^c = N(1-\delta)^{c+b} x_{c+b}^0 + \sum_{c'=1}^{c+b} \delta N(1-\delta)^{c+b-c'} x_{c+b}^{c'}$$

We have

$$\begin{aligned} \tilde{L}_{c+b-1}^{c-1} - \tilde{L}_{c+b}^c &= N(1-\delta)^{c+b-1} [x_{c+b-1}^0 - (1-\delta)x_{c+b}^0] - \delta N(1-\delta)^{c+b-1} x_{c+b}^1 \\ &= N(1-\delta)^{c+b-1} [x_{c+b-1}^0 - (1-\delta)x_{c+b}^0 - \delta x_{c+b}^1] > 0 \end{aligned}$$

since

$$\begin{aligned} x_{c+b-1}^0 - (1-\delta)x_{c+b}^0 - \delta x_{c+b}^1 &= (1-\delta)(x_{c+b-1}^0 - x_{c+b}^0) + \delta(x_{c+b-1}^0 - x_{c+b}^1) \\ &= (1-\delta)(x_{c+b-1}^0 - x_{c+b}^0) > 0. \end{aligned}$$

We have now shown that, for z^*/N sufficiently small,

$$\left[1 - \frac{P_{c+b-2}^{c-1}(c-1)}{p}\right] px_{c+b-1}^{c-1} \frac{x_{c+b-1}^{c-1}}{L_{c+b-1}^{c-1}} < \left[1 - \frac{P_{c+b-1}^c(c)}{p}\right] px_{c+b}^c \frac{x_{c+b}^c}{L_{c+b}^c}, \quad b = 1, \dots, T - c.$$

The same algebra that proves $\tilde{L}_{c+b-1}^{c-1} - \tilde{L}_{c+b}^c > 0$ also proves $L_{c-1}^{c-1} - L_c^c > 0$, hence $px_{c-1}^{c-1}x_{c-1}^{c-1}/L_{c-1}^{c-1} < px_c^c x_c^c / L_c^c$. Thus for z^*/N sufficiently small, all corresponding terms in the expressions for $P_T^{c-1}(c-1)$ and $P_T^c(c)$ are smaller for the former. However, the expression for $P_T^{c-1}(c-1)$ also contains the additional term $[1 - \frac{P_{T-1}^{c-1}(c-1)}{p}]px_T^{c-1}x_T^{c-1}/L_T^{c-1}$. Suppose that the presence of this term makes the ratio $P_T^{c-1}(c-1)/P_T^c(c)$ greater than one. In this case, we can make $P_T^{c-1}(c-1)/P_T^c(c)$ arbitrarily close to one by increasing $T - c$ and thereby increasing the number of terms in the expressions for $P_T^{c-1}(c-1)$ and $P_T^c(c)$. \square

C Incorporating Contacts and Referrals into the Model of Section 2⁴⁵

We introduce some new notation:

r_t = mass of ego's contacts hired by his employer at the beginning of period t

g_t = ego's stock of contacts at the beginning of period t .

We now have two state variables, n_t and g_t . The equations of motion are

$$n_t = (1 - \delta)n_{t-1} + r_t + px_t$$

$$g_t = (1 - \psi)g_{t-1} + \delta n_{t-1},$$

where ψ is the rate at which ego's contacts return to unknown match quality.

We must now distinguish between ego's value function when employed, $V(n_{t-1}, g_{t-1})$, and his

⁴⁵We thank, without implicating, Alexis Toda for help with this Appendix.

value function when unemployed, $V_u(g_t)$. We have

$$V(n_{t-1}, g_{t-1}) = \max_{x_t} \left\{ [(1-\delta)n_{t-1} + r_t] \frac{y_H}{2} + x_t \frac{py_H + (1-p)y_L}{2} - c[x_t + r_t + (1-\delta)n_{t-1}] \right. \\ \left. + \beta[(1-\delta)V(n_t, g_t) + \delta V_u(g_t)] \right\}. \quad (\text{C.1})$$

The first order condition yields

$$\frac{py_H + (1-p)y_L}{2} + \beta(1-\delta)p \frac{\partial V}{\partial n_t} = c'[x_t^* + r_t + (1-\delta)n_{t-1}]. \quad (\text{C.2})$$

We then have

$$\frac{\partial V}{\partial n_{t-1}} = (1-\delta) \frac{y_H}{2} - (1-\delta)c'[x_t^* + r_t + (1-\delta)n_{t-1}] + \beta(1-\delta) \left[(1-\delta) \frac{\partial V}{\partial n_t} + \delta \frac{\partial V}{\partial g_t} \right] + \beta\delta^2 V'_u,$$

where V'_u is the marginal value of adding contacts. We also have

$$\frac{\partial V}{\partial g_{t-1}} = \beta(1-\delta)(1-\psi) \frac{\partial V}{\partial g_t} + \beta\delta(1-\psi)V'_u. \quad (\text{C.3})$$

We assume that $V_u(g_t)$ is concave, so that $V''_u \leq 0$. In the case where V'_u is constant, we can find a closed-form solution for x_t as we did in Section 2. In the case where V'_u is decreasing, we cannot find a closed-form solution for x_t but can show that it declines monotonically with time.⁴⁶

It is straightforward to show that if V'_u is constant then an affine solution $V(n_{t-1}, g_{t-1}) = \frac{\partial V}{\partial n} n_{t-1} + \frac{\partial V}{\partial g} g_{t-1} + V(0, 0)$ exists where $\frac{\partial V}{\partial n}$ and $\frac{\partial V}{\partial g}$ are constants (hence $\frac{\partial V}{\partial n_t} = \frac{\partial V}{\partial n_{t-1}}$ and $\frac{\partial V}{\partial g_t} = \frac{\partial V}{\partial g_{t-1}}$). Moreover, since n_t and g_t are bounded this solution satisfies the transversality condition. We see that the equations above have a recursive structure. Given a constant V'_u , we can solve for a constant $\frac{\partial V}{\partial g}$. Given constants V'_u and $\frac{\partial V}{\partial g}$, we can solve for a constant $\frac{\partial V}{\partial n}$. We have

$$\frac{\partial V}{\partial n} = \frac{(1-\delta)(1-p) \frac{y_H - y_L}{2} + \beta\delta^2 V'_u \left[1 + \frac{\beta(1-\delta)(1-\psi)}{1-\beta(1-\delta)(1-\psi)} \right]}{1 - \beta(1-\delta)^2(1-p)}.$$

Except for the term including V'_u , the right-hand side of this expression is the same as the right-hand side of the expression for $V'(n)$ in Section 2. Substituting back into the first-order condition,

⁴⁶The marginal value of adding contacts will be influenced by the marginal impacts of contacts on the probability of finding a new job and on the value of a new job. Endogenizing this value in general equilibrium is left to future research.

we obtain the equivalent of equation (3):

$$\frac{py_H + (1-p)y_L}{2} + \beta(1-\delta)p \frac{(1-\delta)(1-p)\frac{y_H - y_L}{2} + \beta\delta^2 V'_u \left[1 + \frac{\beta(1-\delta)(1-\psi)}{1-\beta(1-\delta)(1-\psi)}\right]}{1 - \beta(1-\delta)^2(1-p)} = c'(x_t + (1-\delta)n_{t-1}) \equiv c'(z^*).$$

Since $V'_u > 0$, the solution for z^* increases relative to Section 2: the positive marginal value of adding contacts causes the constant number of matches ego seeks in every period to increase.

Using the fact that $z^* = x_t + r_t + (1-\delta)n_{t-1}$, we have $x_t = z^* - (1-\delta)n_{t-1} - r_t$ and $x_{t+1} - x_t = -(1-\delta)(n_t - n_{t-1}) - (r_{t+1} - r_t)$. It can be shown that $n_t - n_{t-1} = (1-\delta)^t(1-p)^t p z^* + (1-p)r_t$. Substituting, we have $x_{t+1} - x_t = -(1-\delta)^{t+1}(1-p)^t p z^* + [1 - (1-\delta)(1-p)]r_t - r_{t+1}$. Thus, if the firm's hiring of ego's contacts satisfies $[1 - (1-\delta)(1-p)]r_t - r_{t+1} < (1-\delta)^{t+1}(1-p)^t p z^*$, x_t decreases monotonically.

One way to put some structure on r_t is to note that while ego is employed at the firm his stock of contacts is decreasing at rate ψ . It is then reasonable to think that the firm's hiring of ego's contacts will decrease at the same rate: $r_{t+1} = r_t(1-\psi)$. In this case $\psi < (1-\delta)(1-p)$ is a sufficient condition for x_t to decrease monotonically.

We now consider the case where $V''_u < 0$. We immediately see that our solution above for $\partial V/\partial n$ can no longer hold since V'_u will decrease as g increases over time, causing $\partial V/\partial n$ to decrease rather than remain constant. Indeed, $\partial V/\partial n$ decreasing over time is precisely what we want to show, because we can see from the first order condition that this implies the desired number of matches z_t^* will decrease with time, which will reinforce the monotonic decrease of x_t .

Standard arguments show that V is concave, hence $\partial^2 V/\partial n^2 \leq 0$. What remains to be shown is $\partial^2 V/\partial n \partial g \leq 0$. Then, as n and g increase toward their steady state values, $\partial V/\partial n$ (weakly) decreases over time.

Since the presence of r_t has no bearing on whether V has the desired property, we drop it for simplicity. We further streamline notation by rewriting equation (C.1) as

$$V(n, g) = \max_x \{a(1-\delta)n + bx - c(x + (1-\delta)n) + \beta((1-\delta)V(n', g') + \delta V_u(g')\},$$

where $a = \frac{y_H}{2}$, $b = \frac{py_H + (1-p)y_L}{2}$, and

$$\begin{aligned} n' &= (1 - \delta)n + px, \\ g' &= (1 - \psi)g + \delta n. \end{aligned}$$

Letting TV be the right-hand side of this Bellman equation, the true value function satisfies $TV = V$, that is, it is a fixed point of T . It can be shown that n and g are bounded, so that $(n, g) \in [0, \bar{n}] \times [0, \bar{g}]$. Letting X be the space of all continuous functions on $[0, \bar{n}] \times [0, \bar{g}]$, it can be shown that $T : X \rightarrow X$ is a contraction. By the contraction mapping theorem, $TV = V$ has a unique solution, and $V = \lim_{m \rightarrow \infty} T^m V_0$, where V_0 is any function (typically chosen to be 0).

Now to show that $\partial^2 V / \partial n \partial g \leq 0$ for the true value function, it suffices to show that for any (concave) V , we have

$$\frac{\partial^2 V}{\partial n \partial g} \leq 0 \implies \frac{\partial^2 TV}{\partial n \partial g} \leq 0. \quad (\text{C.4})$$

To see this, suppose we show (C.4). Let $V_0 = 0$ and $V_m = TV_{m-1}$ for all $m \geq 1$. Since the zero function satisfies $\partial^2 V / \partial n \partial g \leq 0$, it follows from induction that $\partial^2 V_m / \partial n \partial g \leq 0$. Letting $m \rightarrow \infty$, since V_m converges to the true value function V , we obtain $\partial^2 V / \partial n \partial g \leq 0$.

Take any concave function V with $\partial^2 V / \partial n \partial g \leq 0$, and rewrite the first-order condition (C.2) as

$$0 = b - c'(x + (1 - \delta)n) + \beta(1 - \delta)p \frac{\partial V}{\partial n}(n', g'). \quad (\text{C.5})$$

Let $x^*(n, g)$ be the solution of this equation. We can also rewrite equation (C.3) as

$$\frac{\partial TV}{\partial g} = \beta(1 - \psi) \left((1 - \delta) \frac{\partial V}{\partial g}(n', g') + \delta V'_u(g') \right),$$

where $n' = px^*(n, g) + (1 - \delta)n$. Differentiating this expression with respect to n , we obtain

$$\frac{\partial^2 TV}{\partial n \partial g} = \beta(1 - \psi) \left((1 - \delta) \left(\frac{\partial^2 V}{\partial n \partial g}(n', g') \left(p \frac{\partial x^*}{\partial n} + 1 - \delta \right) + \delta \frac{\partial^2 V}{\partial g^2}(n', g') \right) + \delta^2 V''_u(g') \right).$$

For notational simplicity let $V_{11} \equiv \partial^2 V / \partial n^2$, $V_{12} \equiv \partial^2 V / \partial n \partial g$, and $V_{22} \equiv \partial^2 V / \partial g^2$. Then the above equation becomes

$$(TV)_{12} = \beta(1 - \psi) \left((1 - \delta) (V_{12} (p \partial x^* / \partial n + 1 - \delta) + \delta V_{22}) + \delta^2 V''_u(g') \right).$$

Since $\beta > 0$, $0 < \psi < 1$, $0 < \delta < 1$, and $V_u'' < 0$, to show $(TV)_{12} \leq 0$, it suffices to show

$$V_{12}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{22} \leq 0. \quad (\text{C.6})$$

We establish that (C.6) holds in four steps. First, we show that $\partial x^*/\partial n < 0$. Differentiating the first-order condition (C.5) with respect to n , we obtain

$$\begin{aligned} 0 &= -c''(\partial x^*/\partial n + 1 - \delta) + \beta(1 - \delta)p(V_{11}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{12}), \quad (\text{C.7}) \\ \text{or } (c'' - \beta(1 - \delta)p^2V_{11})\partial x^*/\partial n &= -(1 - \delta)(c'' - \beta(1 - \delta)pV_{11} - \beta\delta V_{12}). \end{aligned}$$

Since $c'' > 0$ (increasing marginal cost), $V_{11} < 0$ (concavity),⁴⁷ and $V_{12} \leq 0$ (assumption), it follows that $\partial x^*/\partial n < 0$.

Second, we show that $\partial x^*/\partial n + 1 - \delta < 0$. Since we already know $\partial x^*/\partial n < 0$, it follows from $0 < p < 1$ that

$$\partial x^*/\partial n + 1 - \delta < p\partial x^*/\partial n + 1 - \delta.$$

Using this inequality together with (C.7), since $V_{11} < 0$ we obtain

$$\begin{aligned} 0 &= -c''(\partial x^*/\partial n + 1 - \delta) + \beta(1 - \delta)p(V_{11}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{12}) \\ &< -c''(\partial x^*/\partial n + 1 - \delta) + \beta(1 - \delta)p(V_{11}(\partial x^*/\partial n + 1 - \delta) + \delta V_{12}), \text{ or} \\ (c'' - \beta(1 - \delta)pV_{11})(\partial x^*/\partial n + 1 - \delta) &< \beta(1 - \delta)p\delta V_{12}. \end{aligned}$$

Since $c'' > 0$, $V_{11} < 0$, and $V_{12} \leq 0$, we obtain $\partial x^*/\partial n + 1 - \delta < 0$.

Third, we show that $V_{11}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{12} < 0$. Since $c'' > 0$ and $\partial x^*/\partial n + 1 - \delta < 0$ by the previous step, it follows from (C.7) that

$$\beta(1 - \delta)p(V_{11}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{12}) = c''(\partial x^*/\partial n + 1 - \delta) < 0.$$

Therefore we obtain $V_{11}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{12} < 0$.

⁴⁷Suppose $V_{11} = 0$ (or $V_{22} = 0$). The determinant condition for concave functions $V_{11}V_{22} - V_{12}^2 \geq 0$ then implies $V_{12} = 0$, and (C.6) holds.

Finally, we rewrite the left-hand side of (C.6):

$$V_{12}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{22} = \frac{V_{12}}{V_{11}}(V_{11}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{12}) + \delta \frac{V_{11}V_{22} - V_{12}^2}{V_{11}}.$$

Since $V_{11} < 0$, $V_{12} \leq 0$ (assumption), and by the previous step $V_{11}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{12} < 0$, it follows that the first term is nonpositive. By the determinant condition for concave functions $V_{11}V_{22} - V_{12}^2 \geq 0$, hence the second term is also nonpositive. Therefore $V_{12}(p\partial x^*/\partial n + 1 - \delta) + \delta V_{22}$ is the sum of nonpositive terms, and (C.6) holds.

D Sample Selection

Table D.1: Selection Comparison

Dimension	Saygin et al WP	This paper
<u>Closure</u>		
	1980-2007 Austria	1998-1999 Brazil (Ceará, Acre, Santa Catarina, Mato Grosso do Sul, Espirito Santo)
Distinguish Exit by	worker-flow approach (Fink, Segalla, Weber, and Zulehner 2010)	last year observed in data
Period of Firm Exit	Quarter	Year
Min. Num. Employees in last per.	5	5
<u>Ego</u>		
	Blue or white collar workers, 20-55	Males, ≥ 20 hrs/week
	At closure in the period of firm exit	
	?	Not leaving for death or retirement
Tenure at closure	> 1 yr	> 3 months
Alters	-	> 1 employed alter
Re-employment censored at...	1 yr	2 yrs
Location of re-employment	same country as closure	same state as closure
<u>Alter</u>		
	?	Males, ≥ 20 hrs/week
Time Since last Co-worked	≤ 5 years	≤ 4 years
Overlap	> 30 days	> 3 months
If..	ego's hist. firm has ≤ 3000	ego's plant
Excluding..	egos from the same closure	alt.+egos from the closure firm
<u>Potential</u>		
Location of alters..	firm in closure qtr.	pl. in month ego leaves closure
Minimum tenure of alter	?	3 months

E Maps

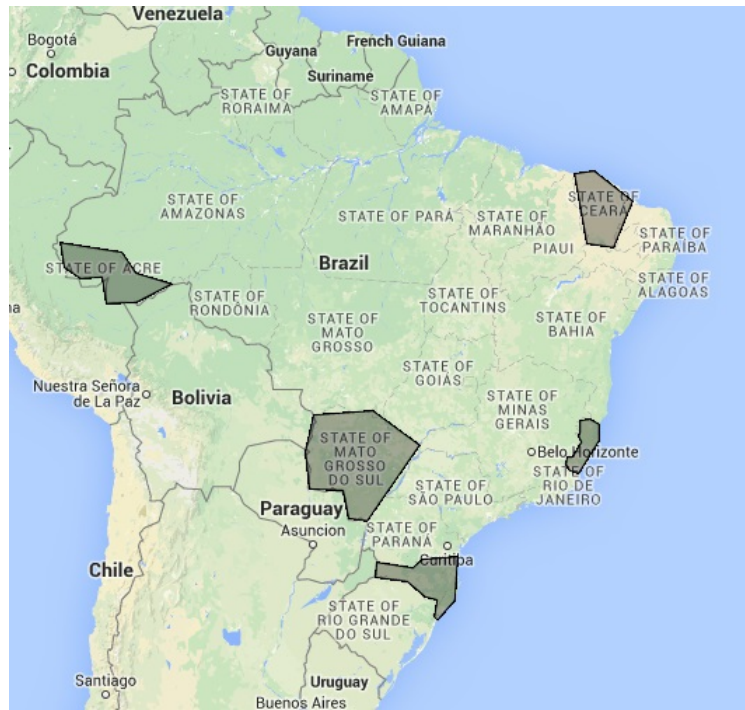


Figure E.1: Brazilian State Coverage - Source: Brazil. Map. Google Maps. 17 August 2014.

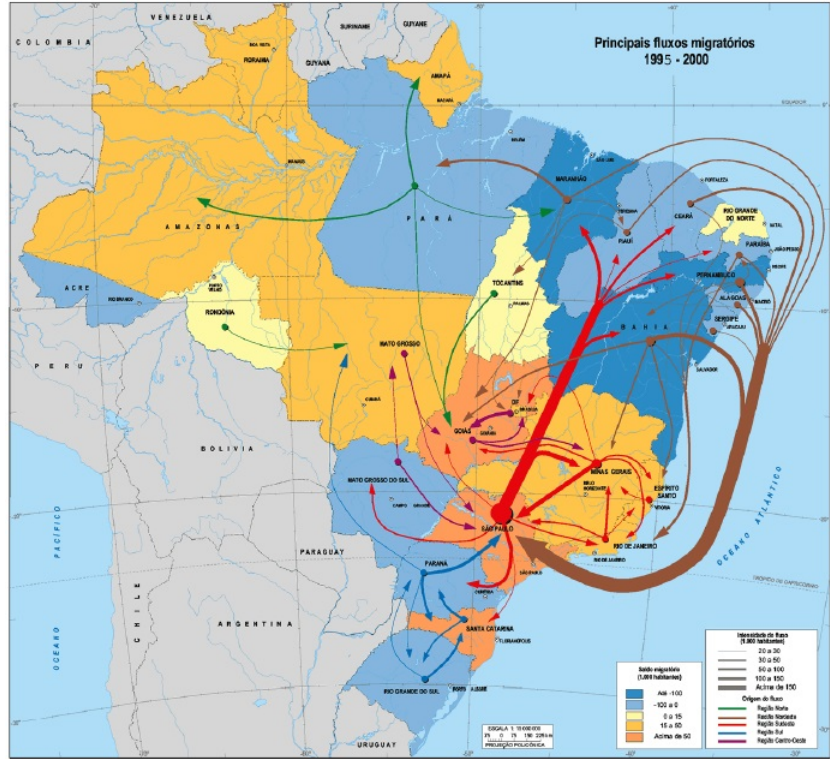


Figure E.2: Migration 2000 - Source: [IBGE 2000](#)